

Effect of Disease Transmission Coefficient on SEIRS Epidemic Model

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ABSTRACT

The paper extends our previous work on SEIR and investigates further the effect of the disease transmission coefficient on our model. The paper also studies the local and global stabilities of the disease free equilibrium using matrix and Lyapunov function methods when the basic reproduction number, R_0 . The results obtained are in good agreement with existing results in the literature. The proof of our theorem shows that when the endemic state is locally asymptotically stable.

Keywords: Reproductive Number, SEIRS, Saturated Incidence Rate.

Aims Research Journal Reference Format:

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1. INTRODUCTION

Mathematical modeling has remained an important and effective tool to represent and predict the spread of an epidemic disease in sample population. The technique has many significant advantages over the damages presented by infectious disease. It could be used as guide for making policy decision on how to act to limit the damage caused by an epidemic, or to prevent its future outbreak. More specifically, it could be used to answer question on health economic aspects of prevention, emergency planning and risk assessment and in evaluation of control programmers. More recently, mathematical modeling has been used to predict the spread of foot and mouth disease and to simulate the outbreak of severe acute respiratory ailment. These models can be separated into two distinct types; those that approximate the total population of the system as constraint, and those that allow a total population to vary, accounting for births and deaths due to natural causes.

A model is only ever as good as the assumptions made to build it. The basic premise of a model is that a small group of infected individuals is introduced to a wholly susceptible population. The assumption made about the transmission of infectious are crucial. With all of these models we assume that the population is homogeneously mixed, and that every pair of individuals has an equal probability of coming into constant with one another. Previous contributions such as [1,2] on mathematical modelling of biological problems have been found useful and resourceful. [3] investigated for the numerical solution effect of saturation term on the susceptible individual. [3, 4] discussed dynamical behavior of epidemiological models with nonlinear incidence rates. [6,7] studied the global dynamics of the SEIR models with a non-linear incidence rate and with a standard incidence, respectively. [8] considered SEIR models that incorporate density dependence in the death rate. [9] considered the global stability of the SEI and SEIR model with infectious force in latent and infected period with non permanent immunity. [10] studied the long time behavior of a non-autonomous SEIRS epidemic model. They obtained new sufficient conditions for the permanence (uniform persistence) and extinction of infectious population of the model. . In [11,12], recent works were also presented in the current trend of SEIRS epidemic model. Recently [3] studied numerical simulation on the effect of saturation terms on the susceptible individual in SEIRS Epidemic Model using the variational iteration method.

In this paper, we extend the work done by [3] to study the effect of disease transmission coefficient on the model. In addition to the above, theorems are formulated and proved in order to establish criteria for the local and global stabilities of the disease free equilibrium. Results are presented in the form of basic reproduction number.

2. MATHEMATICAL EQUATIONS

A population of size $N(t)$ is partitioned into subclasses of individuals who are susceptible, exposed (infected but not yet infectious) infectious and recovered with sizes denoted by $S(t)$, $E(t)$, $I(t)$ and $R(t)$ respectively. The sum $E(t) + I(t)$ is the total infected population. If it is assumed that all immigrant individuals are susceptible and vertical transmission can be assumed to acquire temporary immunity in which recovered individual goes back to the susceptible class again then following [3] whose SEIRS epidemic model is presented below;

$$\left. \begin{aligned} \frac{ds}{dt} &= \Lambda - \frac{\beta SI}{1 + m_1 s} - \mu s + \delta R \\ \frac{dE}{dt} &= \frac{\beta SI}{1 + m_1 s} - (\mu + \varepsilon) E \\ \frac{dI}{dt} &= \varepsilon E - (\mu + \gamma) I \\ \frac{dR}{dt} &= \gamma I - (\mu + \delta) R \end{aligned} \right] \quad (1)$$

The parameter $\Lambda(t) > 0$ is the birth rate, $\beta(t) > 0$ is the disease transmission coefficient, $\mu(t) > 0$ is the mortality/death rate, $\varepsilon(t) > 0$ is the rate of developing infectivity, $\gamma(t) > 0$ is the recovery rate, $\delta(t) > 0$ is the rate of losing immunity, with initial value

$$S(0) > 0, E(0) \geq 0, I(0) > 0, R(0) \geq 0$$

$$\text{At the equilibrium, } \frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \quad (2)$$

Equation (1) becomes

$$\left. \begin{aligned} \Lambda - \frac{\beta SI}{1 + m_1 s} - \mu S + \delta R &= 0 \\ \frac{\beta SI}{1 + m_1 s} - (\mu + \varepsilon) E &= 0 \\ \varepsilon E - (\mu + \gamma) I &= 0 \\ \gamma I - (\mu + \delta) R &= 0 \end{aligned} \right] \quad (3)$$

From equation (3); we obtained the disease free equilibrium;

$$(DFE) = (S^0, E^0, I^0, R^0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \quad (4)$$

For the endemic equilibrium $I \neq 0$ equation (3) implies that

$$R = \frac{(\mu + \varepsilon)(\mu + \gamma)(\mu + \delta)(\mu + \wedge m_1) - \wedge \varepsilon \beta (\mu + \delta) \gamma \varepsilon}{(\varepsilon \beta (\mu + \delta) - m_1 (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))(\gamma \varepsilon \delta - (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))} = R^{**}$$

$$I = \frac{(\mu + \delta)}{\gamma} \left(\frac{(\mu + \varepsilon)(\mu + \gamma)(\mu + \delta)(\mu + \wedge m_1) - \wedge \varepsilon \beta (\mu + \delta) \gamma \varepsilon}{(\varepsilon \beta (\mu + \delta) - m_1 (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))(\gamma \varepsilon \delta - (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))} \right) = I^{**}$$

$$E = \frac{(\mu + \gamma)(\mu + \delta)}{\varepsilon \gamma} \left(\frac{(\mu + \varepsilon)(\mu + \gamma)(\mu + \delta)(\mu + \wedge m_1) - \wedge \varepsilon \beta (\mu + \delta) \gamma \varepsilon}{(\varepsilon \beta (\mu + \delta) - m_1 (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))(\gamma \varepsilon \delta - (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta))} \right) = E^{**}$$

$$S = \frac{(\mu + \varepsilon)(\mu + \gamma)(\mu + \delta)}{\varepsilon \gamma \beta - m_1 (\mu + \varepsilon)(\mu + \gamma)(\mu + \delta)} = S^{**} \quad (5)$$

Therefore the endemic equilibrium points are $(S, E, I, R) = (S^{**}, E^{**}, I^{**}, R^{**})$

3. NEXT GENERATION MATRIX R_0

Let G be a next generation matrix. It comprises of two parts F and V^{-1} where

$$F = \left[\frac{\partial f_i(x_0)}{\partial x_j} \right] \quad (6)$$

$$V = \left[\frac{\partial V_i(x_0)}{\partial x_j} \right] \quad (7)$$

F_i is the new infections, while the V_i are transfers of infections from one compartment to another. X_0 is the disease free equilibrium state.

R_0 is the dominant Eigen value of the matrix.

$$\text{If } G = FV^{-1} \quad (8)$$

and

$$\left. \begin{aligned} \frac{dE}{dt} &= \frac{\beta SI}{1 + m_1 S} - (\mu + \varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu + \gamma)I \end{aligned} \right\} \quad (9)$$

Because we are interested in E and I compartment.

Suppose

$$|G - \lambda I| = 0 \tag{10}$$

Hence, the dominant eigenvalue is our R_0 therefore,

$$R_0 = \frac{\beta \wedge \varepsilon}{(\mu + m_1 \wedge)(\mu + \gamma)(\mu + \varepsilon)} \tag{11}$$

4. LOCAL STABILITY OF DISEASE FREE EQUILIBRIUM

The system of equation (1) was linearised by setting

$$S - S_1 = x, E = E, I = I, R = R \tag{12}$$

Therefore the resulting linearized equations are:

$$\left. \begin{aligned} \frac{dx}{dt} &= -\mu x + (\beta S_1^2 m_1 - \beta S_1)I + \delta R + \text{non-linear terms} \\ \frac{dE}{dt} &= -(\mu + \varepsilon)E + (\beta S_1 I - \beta S_1^2 m_1)I + \text{non-linear terms} \\ \frac{dI}{dt} &= \varepsilon E - (\mu + \gamma)I \\ \frac{dR}{dt} &= \lambda - (\mu + \delta)R \end{aligned} \right\} \tag{13}$$

Therefore the resulting characteristic equation is $|A - \lambda I| = 0$ i.e

$$\begin{vmatrix} -\mu - \lambda & 0 & \frac{\beta \wedge (\wedge m_1 - 1)}{\mu} & 0 \\ 0 & -(\mu + \varepsilon) - \lambda & -\frac{\beta \wedge (\wedge m_1 - 1)}{\mu} & 0 \\ 0 & \varepsilon & -(\mu + \gamma) - \lambda & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) - \lambda \end{vmatrix} = 0 \tag{14}$$

So that

$$(-\mu - \lambda)(-\mu + \delta) - \lambda \left[(-\mu + \varepsilon) - \lambda \right] (-\mu + \gamma) - \lambda + \frac{\varepsilon \beta \wedge (\wedge m_1 - 1)}{\mu} = 0 \tag{15}$$

Hence, $\lambda_1 = -\mu, \lambda_2 = -(\mu + \delta)$ (16)

and

$$\lambda^2 + (2\mu + \varepsilon + \gamma)\lambda + (\mu + \varepsilon)(\mu + \gamma) \left[1 - \frac{(\mu^2 - m_1^2 \wedge^2)}{\mu^2} R_0 \right] = 0$$
 (17)

Solving equation (17) gives

$$\left. \begin{aligned} \lambda_3 &= \frac{1}{\mu} \left(-\mu^2 - \frac{\mu\gamma}{2} - \frac{\varepsilon\mu}{2} + \frac{1}{2}\sqrt{D} \right) \\ \lambda_4 &= \frac{1}{\mu} \left(-\mu^2 - \frac{\mu\gamma}{2} - \frac{\varepsilon\mu}{2} - \frac{1}{2}\sqrt{D} \right) \end{aligned} \right\}$$
 (18)

where

$$D = \mu^2\gamma^2 - 2\varepsilon\gamma\mu^2 + \varepsilon^2\mu^2 + 4\varepsilon\gamma R_0\mu^2 - 4\varepsilon\gamma R_0\lambda^2 m_1^2 + 4\mu^4 R_0 + 4\mu^2 R_0\lambda^2 m_1^2 + 4\varepsilon\mu^2 R_0 + 4\mu^3\gamma R_0 - 4\mu\gamma R_0\lambda^2 m_1^2 - 4\varepsilon\mu R_0\lambda^2 m_1^2$$
 (19)

Theorem

Suppose $R_0 < \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} < 1$, then the disease free equilibrium is locally asymptotically stable, and if

$R_0 \geq \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} > 1$ then the disease free equilibrium is unstable

Proof:

Since $\mu > 0, \varepsilon > 0, \delta > 0, \gamma > 0, m > 0$ and if $R_0 < \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} < 1$

By Descartes rule of signs there is no sign change in equation (17). If $R_0 < \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} < 1$ which implies that

there are no positive roots in equation (17). Furthermore, if λ is replaced by $(-\lambda)$ in Equation (17) then the disease

free equilibrium point $(S_1, E_1, I_1, R_1) (\frac{\wedge}{\mu}, 0, 0, 0)$ is unstable provided $R_0 \geq \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} > 1$

5 . GLOBAL STABILITY OF THE DISEASE FREE EQUILIBRIUM

Consider the Lyapunov function defined thus

$$L = (\mu + \varepsilon)I + \varepsilon E \quad (\mu + \varepsilon)(\mu + \gamma) \tag{20}$$

$$L^1 = (\mu + \varepsilon)\varepsilon E - (\mu + \varepsilon)(\mu + \gamma)I + \frac{\varepsilon\beta SI}{1 + m_1 S} - (\mu + \varepsilon)\varepsilon E \tag{21}$$

At disease free equilibrium, $S = \frac{\Lambda}{\mu}$ and $I \geq 0$ then:

$$L^1 = (S_1, E_1, I_1, R_1) = \left(\frac{\Lambda}{\mu}, 0, 0, 0 \right) = \left(\frac{\varepsilon\beta \frac{\Lambda}{\mu}}{1 + m_1 \frac{\Lambda}{\mu}} - (\mu + \varepsilon)(\mu + \gamma) \right) I \tag{22}$$

Equation (22) simplified to;

$$L^1 = (\mu + \varepsilon)(\mu + \gamma)[R_0 - 1]I \tag{23}$$

Therefore if $R_0 \leq 1$

$$L^1 \leq 0 \tag{24}$$

Hence the disease free equilibrium is globally asymptotically stable

6. LOCAL STABILITY OF THE ENDEMIC EQUILIBRIUM

$$\text{Let } S - S^* = w, E - E^* = x, I - I^* = y, R - R^* = z \tag{25}$$

$$\frac{dS}{dt} = \frac{dw}{dt}, \frac{dE}{dt} = \frac{dx}{dt}, \frac{dI}{dt} = \frac{dy}{dt}, \frac{dR}{dt} = \frac{dz}{dt} \tag{26}$$

Hence

The linearised equations are as follows:

$$\left. \begin{aligned} \frac{dw}{dt} &= (-\beta I^* + 2\beta I^{*2} m_1 S^* - \mu)(w + (-\beta S^* + \beta S^* \gamma + \beta S^* m_1)\gamma + \delta z + \\ &\text{non linear terms} + \text{constant terms} \\ \frac{dx}{dt} &= (\beta I^* - 2\beta I^* m_1 S^*)w + (\beta S^* - \beta S^{*2} + \beta S^* m_1)\gamma - (\mu + \varepsilon)x + \\ &\text{non linear terms} + \text{constant terms} \\ \frac{dy}{dt} &= \varepsilon x - (\mu + \gamma)\gamma + \text{non linear terms} + \text{constant terms} \\ \frac{dz}{dt} &= \gamma y - (\mu + \delta)z + \text{non linear terms} + \text{constant terms} \end{aligned} \right\} \tag{27}$$

Equation (27) is equivalent to;

$$\begin{pmatrix} w^1 \\ x^1 \\ y^1 \\ z^1 \end{pmatrix} = \begin{pmatrix} (-\beta I^* + 2\beta I^* m_1 S^* - \mu) & 0 & (-\beta S^* + \beta S^* m_1) & \delta \\ (\beta I^* - 2\beta I^* m_1 S^*) & -(\mu + \varepsilon) & (\beta S^* - \beta S^* m_1) & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} + \text{non linear terms} \quad (28)$$

Let $f(\lambda) = \lambda^4 - A_3\lambda^3 - A_4\lambda^2 - A_5\lambda - A_6 = 0$, then (S^*, E^*, I^*, R) is locally asymptotically stable if $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$

The Jacobian matrix of above equation is

$$A^* = \begin{pmatrix} (-\beta I^* + 2\beta I^* m_1 S^* - \mu) & 0 & (-\beta S^* + \beta S^{*2} m_1) & \delta \\ (-\beta I^* - 2\beta I^* m_1 S^*) & -(\mu + \varepsilon) & \beta S^* - \beta S^* m_1 & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{pmatrix} \quad (29)$$

Therefore the characteristics equation is

$$|A^* - \lambda I| = 0 \quad (30)$$

$$\begin{vmatrix} (-\beta I^* + \beta I^* m_1 S^* - \mu) & 0 & (\beta S^* + \beta S^* m_1) & \delta \\ (\beta I^* + 2\beta I^* m_1 S^* - \mu) & -(\mu + \varepsilon) & (\beta S^* - \beta S^* m_1) & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{vmatrix} = 0 \quad (31)$$

$$\text{Evaluating (31) and let } A_0 = -\delta\varepsilon\gamma(\beta I^* + 2\beta I^* m_1 S^*), A_1 = (-\beta I^* + 2\beta I^* m_1 S^* - \mu), A_2 = \beta S^* - \beta S^* m_1 \quad (32)$$

$$A_0 - [(\mu + \delta) + \lambda] \left[-(\mu + \gamma) + \lambda \right] \left[-(\mu + \gamma) + \lambda \right] \left[(A_1 - \lambda) - \varepsilon(\beta S^* - 2\beta S^* m_1) \left[\frac{-A_0}{\varepsilon\gamma\delta} + (A_1 - \lambda) \right] \right] \\ A_0 - ((\mu + \delta) + \lambda) \left[-((\mu + \varepsilon) + \lambda) \left[-(\mu + \gamma)A_1 + (\mu + \gamma + A_1)\lambda - \lambda^2 \frac{A_0 A_2}{\varepsilon\gamma\delta} - A_2 A_1 + A_2 \lambda \right] \right] = 0 \quad (33)$$

Expanding, factorizing and re-arranging gives,

$$\lambda^4 - (\gamma + A_1 - \varepsilon + A_2 - (\mu + \delta))\lambda^4 - \left((\mu + \varepsilon)(\mu + \gamma + A_1) + A_2(\mu + \varepsilon) - (\mu + \gamma)A_1 \right) \lambda^2 \\ - \left(-\frac{A_0 A_2}{\varepsilon\gamma\delta} - A_2 A_1 + (\mu + \delta)(\gamma + A_1 - \varepsilon + A_2) \right) \lambda^2 \\ - (-(\mu + \varepsilon)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon\gamma\delta}(\mu + \varepsilon) - A_2 A_1(\mu + \varepsilon) + (\mu + \delta)(\mu + \varepsilon)(\mu + \gamma + A_1) + A_2(\mu + \delta)(\mu + \varepsilon) \\ - (\mu + \delta)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon\gamma\delta}(\mu + \delta) - (\mu + \delta)A_2 A_1) \\ \lambda - (-(\mu + \delta)(\mu + \varepsilon)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon\gamma\delta}(\mu + \varepsilon)(\mu + \delta) - A_2 A_1(\mu + \varepsilon)(\mu + \delta) + A_0) = 0 \quad (34)$$

Also by Descartes rule of signs, Let,

$$\begin{aligned}
 A_3 &= \gamma + A_1 - \varepsilon + A_2 - (\mu + \delta) < 0 \\
 A_4 &= \left((\mu + \varepsilon)(\mu + \gamma + A_1) + A_2(\mu + \varepsilon) - (\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon \gamma \delta} - A_2 A_1 + (\mu + \delta)(\gamma + A_1 - \varepsilon + A_2) \right) < 0 \\
 A_5 &= \left(-(\mu + \varepsilon)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon \gamma \delta} - (\mu + \varepsilon) - A_2 A_1 (\mu + \varepsilon)(\mu + \delta)(\mu + \varepsilon)(\mu + \gamma + A_1) + A_2)(\mu + \delta)(\mu + \varepsilon) \right. \\
 &\quad \left. - (\mu + \delta)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon \gamma \delta} - (\mu + \varepsilon) - (\mu + \delta)A_2 A_1 \right) < 0 \\
 A_6 &= \left(-(\mu + \delta)(\mu + \varepsilon)(\mu + \gamma)A_1 - \frac{A_0 A_2}{\varepsilon \gamma \delta} (\mu + \varepsilon)(\mu + \delta) - A_2 A_1 (\mu + \varepsilon)(\mu + \delta) + A_0 \right) < 0 \tag{35}
 \end{aligned}$$

Then equation (35) becomes

$$f(\lambda) = \lambda^4 - A_3 \lambda^3 - A_4 \lambda^2 - A_5 \lambda - A_6 = 0 \tag{36}$$

let $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$ in Equation (35) then $f(\lambda)$ have no change in sign meaning there are no positive roots of $f(\lambda)$.

Also if λ is replaced by $-\lambda$ in Equation (35), then

$$f(-\lambda) = \lambda^4 + A_3 \lambda^3 - A_4 \lambda^2 - A_5 \lambda - A_6 = 0 \tag{37}$$

So if $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$ in Equation (37) $f(-\lambda)$, have four sign changes which implies, that there are exactly four negative roots of $f(-\lambda)$. Since there is no positive roots for $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$ That is all eigenvalues are negatives, then the endemic or disease equilibrium is locally asymptotically stable if $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$

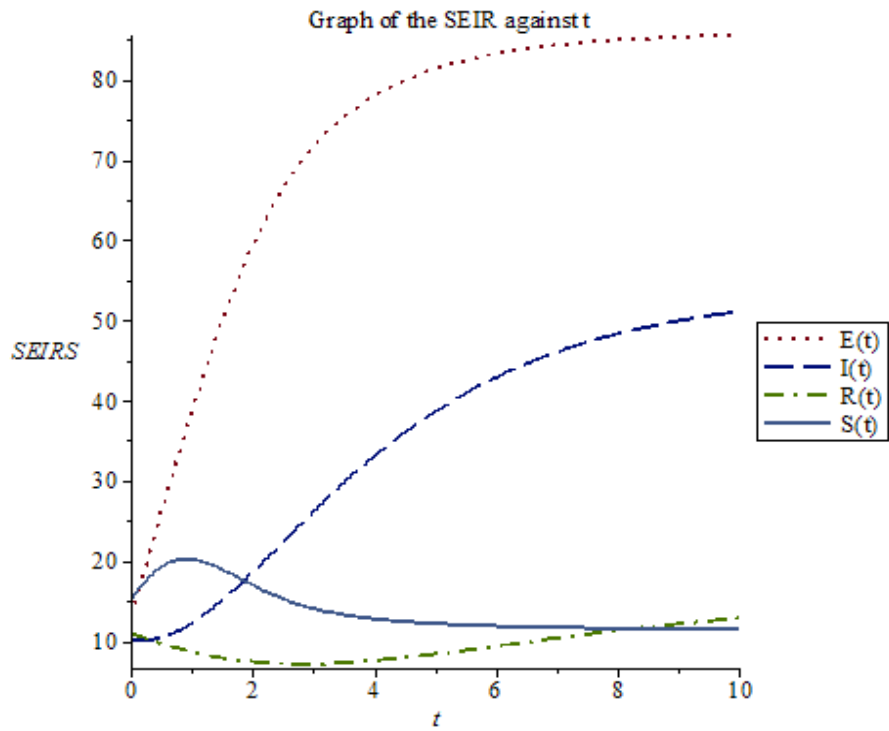


Fig. I: Graph of SEIRS against t when $\beta = 1$

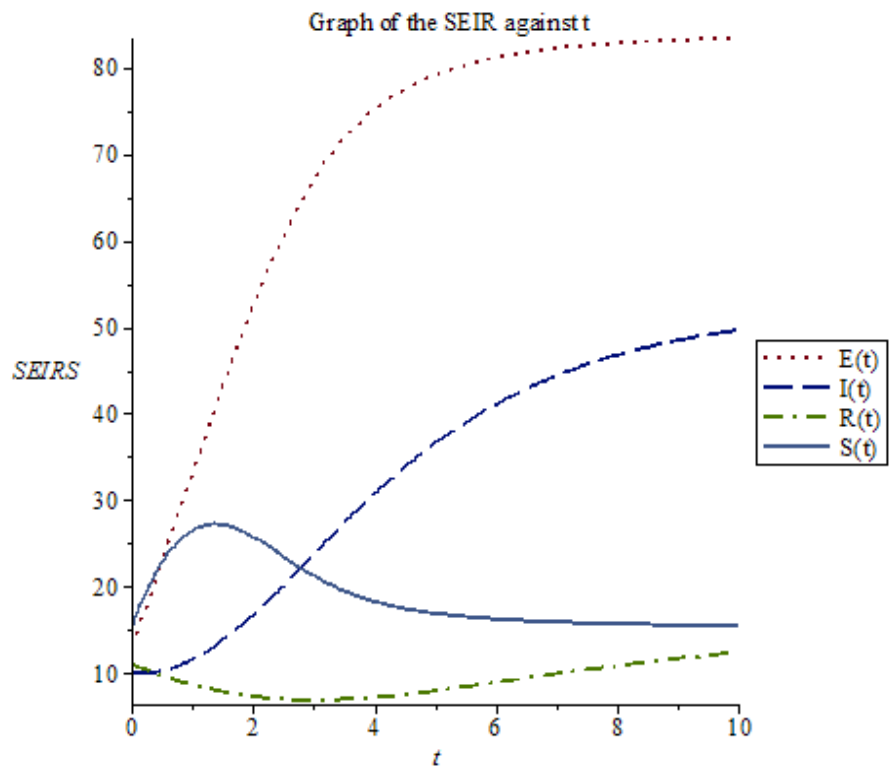


Fig. II: Graph of SEIRS against t when $\beta = 0.75$

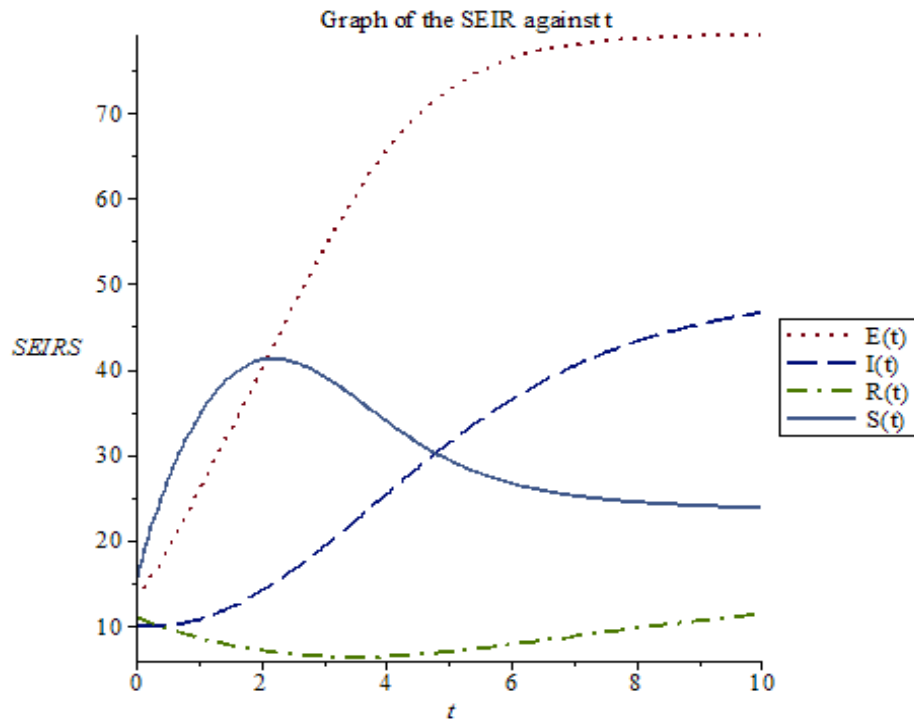


Fig. III: Graph of SEIRS against t when $\beta = 0.5$

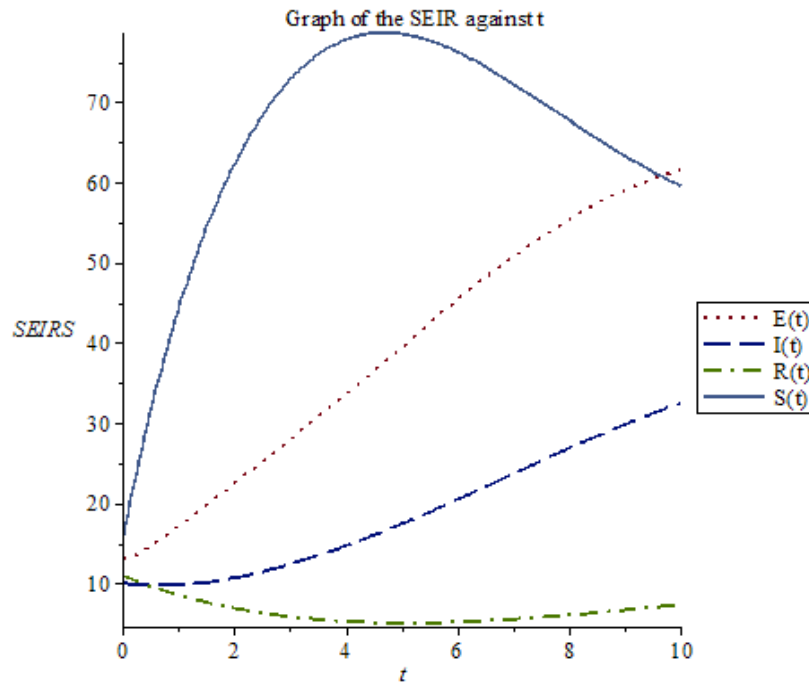


Fig. IV: Graph of SEIRS against t when $\beta = 0.25$

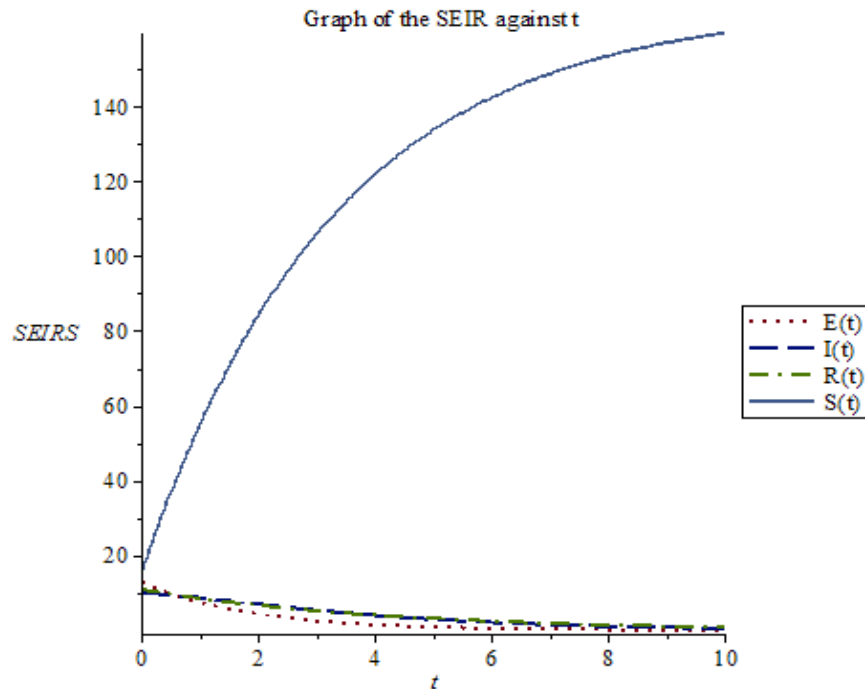


Fig. V: Graph of SEIRS against t when $\beta = 0.0005$

7. Discussion of Results and Conclusion

Fig I: shows the unstable nature of disease free equilibrium when $\beta = 1.0$. It is also observed that susceptible class and other classes decrease drastically while exposed compartment increases to a reasonable end. This shows the effect of β at $\beta = 1.0$. Fig II: shows the unstable nature of disease free equilibrium at $\beta = 0.75$. It is also observed that exposed compartment increases to a reasonable end while susceptible individuals begin to increase and little individual recovered. This also shows the effect of transmission coefficient β at $\beta = 0.75$ because susceptible class begins to increase but not as exposed individual. Fig III: shows the unstable nature of disease free equilibrium at $\beta = 0.50$ but susceptible individual increases more than when β was 0.75 but more individuals are still in the exposed compartment. This also shows the effect of transmission coefficient β at $\beta = 0.50$. Fig IV: shows a slight asymptotic stability nature of disease free equilibrium when $\beta = 0.25$ because susceptible individual increases to a reasonable end more than other compartment. This also show the effect of β at $\beta = 0.25$. Fig V: shows the stable nature of disease free equilibrium at $\beta = 0.0005$. Susceptible individuals increase more than other individuals at $\beta = 0.0005$. This shows the effect of transmission coefficient in disease eradication.

In conclusion, the simulation results show that disease transmission coefficient β plays appreciable role in the disease eradication. The lower the disease transmission coefficient, the better stability of disease free equilibrium and hence, the disease will be eradicated from the population. Therefore for better disease eradication in a population, transmission coefficients β should be so low and susceptible individuals should be given better orientations on how to reduce transmission coefficients for better eradication.

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