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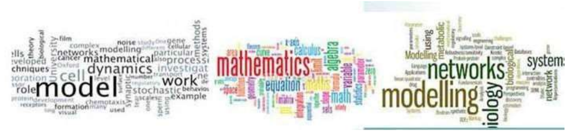

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First Order Differential Equations with Applications to Security Service Companies

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ABSTRACT

First order differential equations is of interest to Every sectors because of the possibility in it to investigate a wide variety of problems in the physical, biological and social sciences. First order differential equations are powerful tools in Sciences and engineering, such that if carefully and effectively applied to real life problems will yield good estimate results for prediction. The problem of a company engaging in security services is modeled into a first order differential equation and the resulting simplified mathematical equation is then solved. The company expected rate of decay (death, retirement etc.) is then substituted and estimated years of expansion is determined, this form the list of some tested years. It was discovered that the result obtained is a good estimation for prediction. This paper also gives room for any other rate of decay to be tested and expected number of years to be determined..

Keywords: First Order Differential Equations, Applications, Security Service Companies, Predictions

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1. INTRODUCTION

According to pack (1960), a first order differential equation is a relation between the dependent variable independent variable t and the derivative of the dependent variable with respect to the independent variable (i.e. $\frac{dx}{dt}$).

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Suppose we denote $\frac{dx}{dt}$ by $\dot{x}$, we assume that in some domain D, we may express $\dot{x}$ as a function of t and x, thus

$$\dot{x} = f(t, x) \qquad (1.1)$$

Where f is a given function of two variables. Any function $x = \phi(t)$. Which with its derivative y^1 identically satisfies equation (1.1) is called a solution, and our objective is to determine whether such functions exist and if so, how to find it.

Pack(1960), stated further that the simplest type of first order differential equation occurs when f depends only on t, in this case

$$\dot{x} = f(t) \qquad (1.2)$$

Therefore, we seek a function $x = \phi(t)$ whose derivative is the given function f. From the elementary calculus we know that ϕ is an anti-derivative of f and we write

$$x = \phi(t) = \int_{t_0}^{t} f(s)ds + c \qquad (1.3)$$

Where c is an arbitrary constant.

If we return to the solution of equation (1.1), Although there is no satisfactory general method of writing down the solutions of this equation, therefore, the solution depends on the class of the first order differential equation as a result, we will present few methods, each of which is applicable to aCertain class of equations of the form (1.1) .

1.1 Methods Of Solving First Order Differential Equation

1.1.1 Separation of Variables:

Diprime (1976) stressed that this method is one of the most commonly used method.

If $$\frac{dx}{dt} = f(t) \qquad (1.4)$$

Where $f(t)$ is a given function of t. We know from calculus that if

$$x(t) = \int_{a}^{t} f(t^1)d\dot{t} \qquad (1.5)$$

The solution can be immediately obtained in the form of

$$\int f(t)dt + \int g(x)dx = c$$

1.1.2 Equation Reducible to Separable Type

Burghes (1980) made it known that certain equations of the form

$$g(t,x)dx = f(t,x)dt$$

That are not separable can be made separable by a change of variable.

This can always be done, if the ratio of $\dfrac{f(t,x)}{g(t,x)}$ is a function of $\dfrac{x}{t}$.

Let $u = \dfrac{x}{t}$ and the function of $\dfrac{x}{t}$ be $h(u)$, so that

$$\frac{dx}{dt} = \frac{f(t,x)}{g(t,x)} = h(u)$$

Since f is function of t, so is u. it follows that

$x(t) = tu(t)$ and

$$\frac{dx}{dt} = u + t\frac{du}{dt}$$

Thus the differential equation can be written as

$$u + t\frac{du}{dt} = h(u) \text{ or } t\frac{du}{dt} = h(u) - u$$

Clearly, it is separable

$$\frac{du}{h(u) - u} = \frac{dt}{t}$$

So we can solve for $u(t)$ and the solution of the original differential equation is simply $x = tu(t)$.

1.1.3 Exact Differential Equations

Suppose we want to find a differential equation that represents the following family of curves

$$F(t,x) = c$$

First let us look at two nearby points (t,x) and $(t+\delta t, x+\delta x)$, both on a specific curve of the family. If the curve is characterized by $c=k$, then

$$F(t,x)=k,\ f(t+\delta t, x+\delta x)=k$$

Clearly, the difference between the two is equal to zero.i.e $f(t+\delta t, x+\delta x)-f(t,x)=0$

This difference can be written in the form of

$$f(t+\delta t, x+\delta x)-f(t,x)=f(t+\delta t,\ x+\delta x)-f(t,x+\delta x)+f(t,x+\delta x)-f(t,x) \qquad .(1.6)$$

With the understanding that δt and δx are approaching zero as a limit. We can equally use the definition of partial derivative

$$f(t+\delta t, x+\delta x)-f(t,x+\delta x)=\frac{\partial f}{\partial t}\delta t, \text{ and } f(t,x+\delta x)-f(t,x)=\frac{\partial f}{\partial x}\delta x$$

To write (1.6) as

$$\delta f=\frac{\partial f}{\partial t}\delta t+\frac{\partial f}{\partial x}\delta x \qquad or \qquad \partial f=\frac{\partial f}{\partial t}dt+\frac{\partial f}{\partial x}dx$$

This is known as total differential, since $\delta f=0$, so we have

$$\frac{\partial f}{\partial t}dt+\frac{\partial f}{\partial x}dx=0 \qquad (1.7)$$

This is the differential equation representing the family of curves $F(x,y)=c$, in other words the Solution of the differential equation in the form (1.7) is given by $F(x,y)=c$.

Now, let $\frac{\partial f}{\partial t}=f(t,x)$ and $\frac{\partial f}{\partial x}=g(t,x)$ (1.8)

Any equation of the form $f(t,x)dt+g(t,x)dx=0$ that satisfies
$\frac{\partial}{\partial x}f(t,x)=\frac{\partial}{\partial t}g(t,x)$ is known as an exact equation.

1.1.4 Integrating Factors

According to Balogun (2006), Factor, which when multiplied with a non-exact differential equation Makes exact, is known as an integrating factor.

For example, the equation
$xdt+(t^2x^3+t)dx=0$ is not exact, if , however , we multiply it by $(xy)^{-2}$, the

Resulting equation

$\frac{1}{t^2 x}dt + (x + \frac{1}{ty^2})dx = 0$ is exact, since $\frac{\partial}{\partial x}(\frac{1}{t^2 x}) = -\frac{1}{t^2 x^2}$ and

$\frac{\partial}{\partial t}(x + \frac{1}{ty^2}) = -\frac{1}{t^2 y^2}$

Hence by definition $(xy)^{-2}$ is an integrating factor.

Olagunju (2006): an integrating factor exists for every differential equation of the form

$f(t,x)dt + g(t,x)dx = 0$.

Unfortunately, no general rule is known to find it. For certain special type of differential equations, integrating factor can be found systematically.

We assume that $f(t,x)dt + g(t,x)dx = 0$ is not exact differential equation, we wish to find an integrating factor μ so that

$\mu f(t,x)dt + \mu g(t,x) = 0$ is exact. For this equation to be exact, it must satisfy the condition

$\frac{\partial}{\partial x}(\mu f) = \frac{\partial}{\partial t}(\mu g)$ which gives

$$\mu(\frac{\partial f}{\partial x} - \frac{\partial g}{\partial t}) = \frac{\partial \mu}{\partial t}g - \frac{\partial \mu}{\partial x}f \qquad (1.9)$$

Now we consider the following possibilities:

The integrating factor is a function of t only. In this case, (1.9) becomes

$\frac{1}{g}(\frac{\partial f}{\partial x} - \frac{\partial g}{\partial t}) = \frac{1}{\mu}\frac{\partial \mu}{\partial t}$.

If the left-hand of this equation is also a function of x only, then

$\frac{1}{g}(\frac{\partial f}{\partial x} - \frac{\partial g}{\partial t}) = G(t)$

Then clearly $\frac{d\mu}{\mu}G(t)dt$

Which gives

$In\mu = \int G(t)dt$ or $\mu = e^{\int G(t)dt}$

2. SECURITY SERVICE COMPANY

Application of first order differential equation to solution of practical problems depends largely on the ability to transform the problems into mathematical models. One can then treat these models for necessary predictions, using expected initial conditions together with other available information. One of these practical problems is the case of a company engaging in security services. The has been feeding some private and public companies with security services. Presently, the company has 250 security men in service. Surveys carried out indicate that the demand for their services will be doubled over certain number of years and the management of the company has decided to expand the strength of the security men year after year, spreading the employment of new security men out over each year.

The intention of the management is to have 10 new security men in the first year, 20 new security men in the second year, 30 security men in the third year, and so on until the tenth year, when the company anticipates having 100 new security men. The management is now trying to find a way of estimating when the target would be met. This planned expansion will exceed the required capacity of 500 security men at the end of the tenth year, but the management must allow for a drop in the services of security men through death rate, and retirement/ retrenchment rates.

It was found out that the company expects that this rate of decay in services "δ" will be $\frac{10}{250}$ =0.04 or 4% of the security men capacity per year. The parameter "δ" is very important, because a bad estimate could lead to serious under or over expansion. It therefore requires a model which will predict the current security service capacity y in terms of the time t (in years) and the decay factor "δ". The rate of plant expansion may then be formulated as $\frac{dy}{dt}$ or $\dot{y}$, which will reduce at a rate of δy. However, new security men are being added. The added capacity after t years will be.

$$10 + 20 + 30 + 40 + \ldots + 10t \quad where \quad t = 1 + 2 + 3 + \ldots + t$$

This can be written as

$$10\{1 + 2 + 3 + 4 + \ldots t\}$$

However, it is noted that this is an Arithmetic progression where the first term is 10, common difference is 10 and the number of terms is t.

Let the total number of security men for t years be S_t, then

$$S_t = \frac{t}{2}\{20 + (t-1)10\} = 5t^2 + 5t$$,Now, let $y = 5t^2 + 5t$

The rate at which capacity is added will be the derivative of this , i.e

$$\frac{dy}{dt} = 10t + 5 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .1$$

Since the rate of expansion is the rate of adding capacity less the rate of decay, we have

$$\frac{dy}{dt} = 10t + 5 - \delta y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .2$$

Or $\dot{y} + \delta y = 10t + 5$ 3

Since this is an initial value problem (i.e differential equation(s) with the initial condition(s) and some known information about the problems is known as initial value problems: Akinmuyise 2002.).

Therefore with initial condition $y_0 = 250,$ the homogeneous equation becomes

$\dot{y} + \delta y = 0$, has solution of the form $y_1 = Ae^{-\delta t}$, where A is an arbitrary constant.
This is known as complementary solution. We now need to obtain a particular solution of the nonhomogeneous problem.

We note that the R.H.S of (3) is a linear function of t ,we then try
$y = at + b \ as\ solution$, where a and b are constants. i.e.
if $y = at + b \ (is\ taken\ as\ solution)$. (3b)

Substituting this into equation (3), we have
$a + \delta(at + b) = 10t + 5$ . . . . .(4)

Opening brackets, collecting like terms and equating coefficients of powers of t on both sides gives
$a\delta = 10$ . . . . . .(i)
$a + b\delta = 5$ . . . . .(ii)

This enables us to solve for a and b in terms of i.e $a = \frac{10}{\delta}$

$$b = \frac{5\delta - 10}{\delta^2}$$

Leading to the particular integral

$$y_2 = \left(\frac{10}{\delta}\right)t + \frac{5\delta - 10}{\delta^2}$$ as in (3b above)

Therefore, the general solution is given by

$$x(t) = Ae^{-\delta t} + \left(\frac{10}{\delta}\right)t + \frac{(5\delta - 10)}{\delta^2} \quad . \quad . \quad . \quad . \quad .(5)$$

The arbitrary constant A is obtained by the introduction of the initial condition

$x_{(0)} = 250$ which gives

$$250 = A + \frac{5\delta - 10}{\delta^2} \qquad \Rightarrow A = 250 - (\frac{5\delta - 10}{\delta^2})$$

or

$$A = 250 + (\frac{10 - 5\delta}{\delta^2}) \qquad (6)$$

Since the solution depends on the given rate of decay, we can now introduce the subscript δ and substitute the value of A in (6) into (5). This becomes

$$x_\delta(t) = \left[250 + \frac{(10 - 5\delta}{\delta^2}\right]e^{-\delta t} + \left(\frac{10}{\delta}\right)t + \frac{5\delta - 10}{\delta^2} \text{.......................(7)}$$

We can now take the company's expected rate of decay $\delta = 0.04$, and substitute number of years (through testing), to estimate the number of years that the demand for their services will be doubled (i.e 500)

$$x_{0.04}(t) = 6375e^{-0.04t} + 250t - 6125 \ ,$$

Hence

$$x_{0.04}(5) = 6375e^{-0.04(5)} + 250(5) - 6125$$

$$= 5219.40855 + 1250 - 6125 = 344.41 \cong 344 \ \sec urity \ \ men$$

$$x_{0.04}(6) = 389.753 \approx 389$$

$$x_{0.04}(7) = 443 \ \sec urity\, men$$

$$x_{0.04}(8) = 504 \ \sec urity\, men$$

$$x_{0.04}(9) = 593 \ \sec urity\, men$$

$$x_{0.04}(10) = 648 \ \sec urity\, men$$

Thus, from the result above, the target will be met by the beginning of the 8th years, given that the rate of reduction (decay) is 0.04. That is to say that at the beginning of the 8th year we would have 504 security men which is very close to the target which is 500. Observation/challenges with this, the management discovered that the decision on the expansion is incorrect. What happens to the excess security men from the beginning of the 8th year through the tenth year?

Thus, the management is to be advised to reduce the rate of steady expansion per year, so as to meet his target at the tenth year when the number of security men on ground wiil be the same as those needed by his customers. This reduction rate in other to meet the target is the challenge ahead of the management. If the management finds it difficult then she could stop expansion after the 8th year and pray that the reduction factor δ tends to zero for the last two years, so that she has nearly 504 security men at the tenth year.

3. CONCLUSION

The importance of first order differential equations is clearly spelt out in this case study. In ordinary sense, the management's decision in expanding the security service steadily year by year, spreading the new security men out over each year as 10,20,30 . . . etc. up to the 10years and the demand projection survey that their demand will doubled over the next 10 years would have yielded incorrect result, if the weapon "first order differential equation" have not been employed.

Assuming the management's expected rate of decay is:

$$\delta = 0.03 \quad or \quad \delta = 0.05 \quad or \ \delta = 0.06 \quad etc$$

It is possible to find out the estimated number of years for the number of security men to be doubled or tripled

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