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Tools for Digital Control Design and Analysis

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ABSTRACT

Abstract
The design and analysis of digital control systems, a field known as digital control engineering, is a large and expansive area of study. Digital control engineering techniques have become a pervasive part of modern technical society. The power of microprocessors has continued to increase while the cost is reducing over the years. Many devices have embedded microprocessors in them and provide digital control designers great opportunities, yet there are some engineers who cannot discuss the tools used in the subject even at awareness level. This paper is therefore aimed at making it possible for engineers from various fields to be aware of many tools used in digital control engineering, by introducing the subject to non-digital-control engineering experts. The topics covered include tools used in classical and modern control systems and although the subject matter is large, the content of this paper has been restricted to the most essentials in the foundations of digital control engineering.

Keywords: Tools, Digital control design, analysis.

Aims Research Journal Reference Format:
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1. INTRODUCTION

This paper is required to serve one main purpose – it is intended to give an overview of the tools used to design and analyze modern digital control system so that none specialist can be conversant with it. It is clearly impossible to cover all aspects of digital systems theory and practice within the compass of an article but attempts will be made to cover all the key areas. The work is meant to provide a broad introduction to the techniques for the design of digital control systems for none specialist engineers, in their day to day work and those who would like to be informed as to how digital techniques may be put to use. Digital Control engineering went through a considerable period of transition in the 1970's and 1990's with the development of "modern" control theory. In essence the movement looked at control engineering from a somewhat different viewpoint to time of conventional frequency response/transfer function methods. The drive was for a better understanding of control systems and an improved quality of control. To achieve this much use was made of digital computers for the processing of data and this led to a move away from the Classical Laplace transform/calculus approach towards those techniques that could be readily programmed onto a computer.

The sheer unfamiliarity of the resulting mathematics (rather than its intrinsic difficulty) drove a wedge between the theoreticians and most practicing digital control engineers. This rift was rationalized by asserting that "modern control is impractical". Originally there may have been some truth in this, the early theories could not cope with the imperfections found in real plant, and the analog controllers were not suited to their implementation. However, the advent of inexpensive and virtually limitless computing power means that it is possible to implement even quite complex calculation cheaply and quickly. The techniques of "modern" control were put into practice. Much of the work presented herewith will, therefore, be concerned with the principle which underpin the theory and with the ways in which it may be put into practice by the agency of a digital computer. In addition there will be discussions in applying computers to real control system problems. Finally there will be number of examples showing a range of applications, from the simple to the complex.

Figure 1 provide "road map" of what every digital control engineer should follow in the design and analysis of digital control. It will be seen that there are three levels of understanding: introductory, theoretical and applications. These are applied to the four branches of the studies namely "modern control"

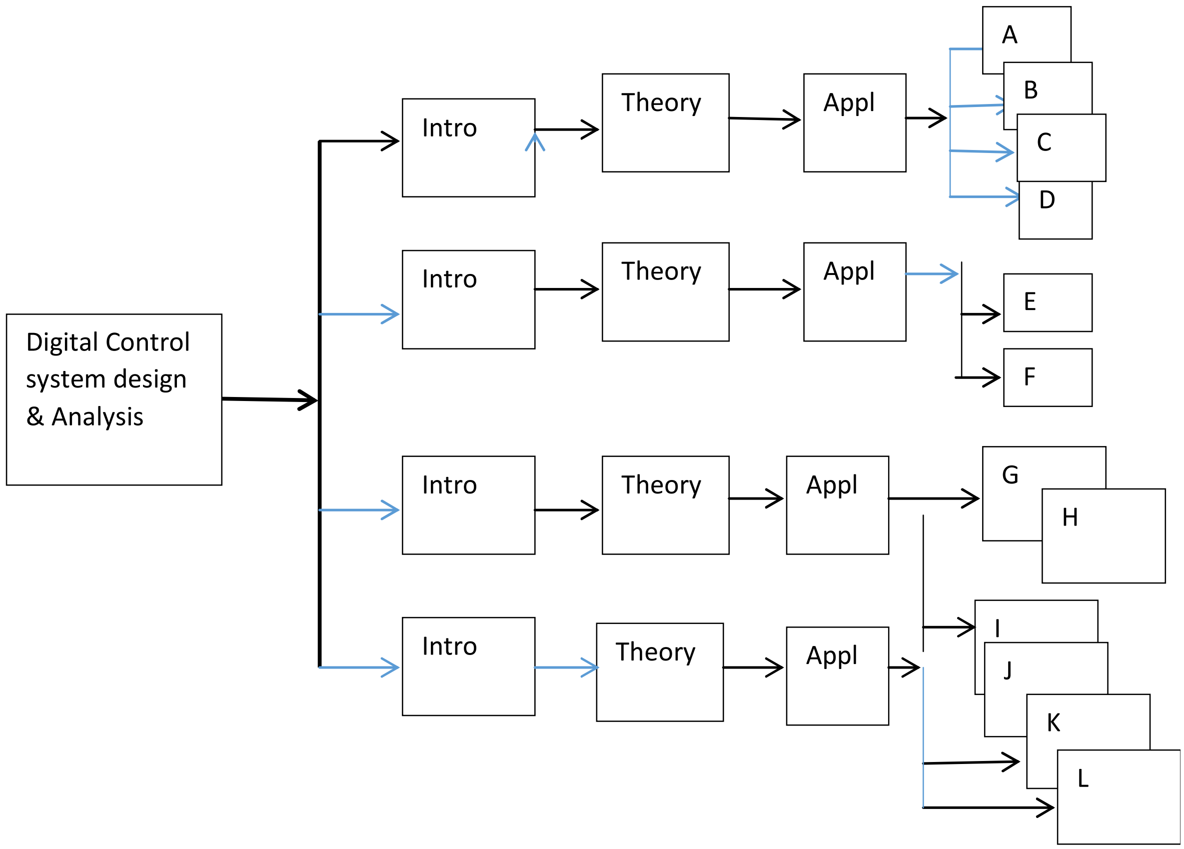

Fig. 1 showing various routes to the design and Analysis of Digital Control Systems

Comprising A – Identification, B – Optimal Control, C- Multivariable Control, and D – Self Tuning Control. The next branch is the classical control method where digital computers can offer performance improvements; E – conventional Control, and F – Time Delay Systems. The third branch leads to four industrial applications G- Robotics, H – Aerospace, I -Industrial and J- others (such as traction, Oil and Gas process). In addition, enabling technology such as K- Information Communication Technology (ICT), L – Software engineering are needed to implement the digital control systems.

In what follows some of the principal tools needed for design and analysis of digital control systems are identified and put in their context.

1.1 Simple Extensions o Continuous Control

It is intuitively satisfying to say that if a digital system has high sampling rate then it approximates to a continuous control system. Under these assumptions one may design controllers by relying on the vast store of classical control method. The justification for using digital control in these circumstances must be that the practical limitations of analog controllers (e.g. drift, inability to implement pure data, etc) are overcome, that the implementation is cheaper or that supervisory control is more easily implemented. However, the use of high sampling rates is wasteful of computer power and can lead to problems of arithmetic precision etc [33, 34]. One is therefore driven to find methods of design which take account of the sampling process and which may, therefore be used at low sampling rates;

2. MODELLING OF DISCRETE SYSTEMS

The first step in the design process is to develop a model of the plant and other continuous components. It is found adequate in most instances to characterize the digital input and digital output processes as switch closures of negligible duration occurring simultaneously; the so called synchronous sampling. As illustrated in figure 2, one may confine the analysis to the state of the variables at these sampling times, even though the continuous elements will have finite outputs between the samples.

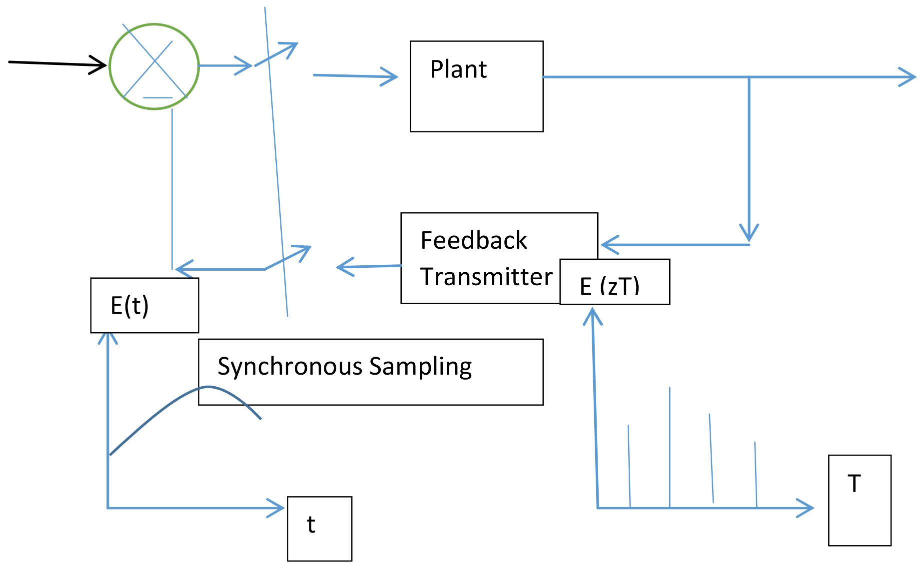

Fig. 2 Direct Conversion of Analog to Digital Control System.

2.1 Classical Control Approach [1-4, 6-7, 15-22]

When these assumptions are valid one may develop a transfer function calculus of discrete systems which parallels the classical Laplace approach. For a system of order n the output at the Kth sample time may be defined in terms of the input sequence (Uo, U1 ……) and the output sequence (Y0, Y1, …..)

$$Yk + A1\ Yk - 1 + A2\ Yk - 2 \ldots . AnYk - n = B0Uk + B1\ Uk - 1 + B2\ Uk - 2 \ldots Bn\ Uk - n \quad (1)$$

That is, a difference equation relating the current output to the n previous outputs, the current input and the n previous inputs.

Using the notation delay operator, this gives Z-1 = e -st

$$Y\ k + A1\ z - 1\ Y(z) \ldots . An\ z - nY(n) = B0\ U(z) + B1\ z - 1\ U(z) \ldots . Bnz - n\ U(z) \quad (2)$$

Or

$$\frac{Y(z)}{U(z)} = \frac{B0 + B1z - 1 \ldots \ldots Bnz - n}{1 + A1\ z - 1 \ldots \ldots \ldots Anz - n} \quad (3)$$

The system pulse transfer function. This transfer function is equally applicable to a continuous process defined by samples and to a purely discrete process such as computer program.

2.2 State Space Approach [3 – 7]

An alternative approach is to use the concept of "state". The system state vector being a sufficient definition of the plant at any given time.

The state space method has the following advantages:

i. It is a time domain approach
ii. It can be applied to linear / non-linear, Time variant / Time invariant systems
iii. It is easier to apply where the Laplace Transform cannot be applied
iv. The method is suitable for digital computer computation because it is time domain approach
v. Nth order differential equation (DE) can be simplified by "n" equations of first order, making the solution easier.

For example given a plant represented by

$$d^3y/dt^3 + 6d^2y/dt^2 + 11dy/dt + 6y = u \qquad (4)$$

Obtain the Laplace transform

$$s^3y(s) + 6s^2y(s) + 11sy(s) + 6y(s) = u(s) \qquad (5)$$

$$(s^3 + 6s^2 + 11s + 6)\, y(s)= u(s) \qquad (6)$$

Obtain inverse Laplace transform

$$Y^3(t) + 6Y^2(t) + 11Y^1(t) + 6Y(t) = u(t) \qquad (7)$$

Chose the state variables :

Let $Y(t) = x_1$ (8)
$Y^1(t) = x_2$ (9)
$Y^2(t) = x_3$ (10)
$Y^3(t) = 2u(t) - 6x_3 - 11x_2 - 6x_1$ (11)

Rewriting
$\dot{x}_1 = x_2$ (12)
$\dot{x}_2 = x_3$ (13)
$\dot{x}_3 = 2u(t) - 6x_3 - 11x_2 - 6x_1$ (14)

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u \qquad (15)$$

$$Y= [1 \;\; 0 \;\; 0]\, x (t) \qquad (16)$$

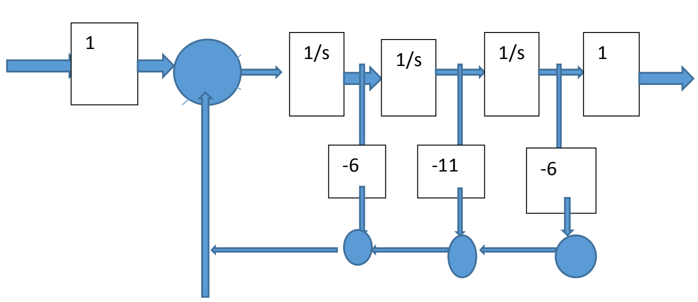

Figure 3: .Block Diagram representation of the state space equation

The modeling process requires the determination of A, B,C,D. this representation of a system easily encompassed m input, P output system of order n where
A = n x n (16)
B = n x m (16)
C = p x n (16)
D = p x m (16)

These matrices may be deduced directly from discrete identification procedures or from the continuous-time state model;
X = AX + BU (16)
Y = CX + DU (16)

Both design consists of devising a scheme whereby a system's input is manipulated so that an adequate speed of response and steady state accuracy is achieved. This also involves consideration of system stability, integrity and sensitivity.

2.3 Classical Method [33]
The classical transfer function approach to design involves the association of performance characteristics with particular system pole position (Or other equivalent measures). Feedback is then applied from measurements of the current and previous systems outputs YK, Yk-1 etc. to modify these pole position. Similarly the current and past values of system error EK, EK-1, etc may be combined in the forward path to obtain similar effects

2.4 State-Space Design Methods [33]
In the state formulation the eigenvalues of the A matrix assume the same significance as the roots of the characteristic polynomial. In classical design, the input forcing function it becomes possible to manipulate eigenvalues [3,6,7,14,28,30]

For example, obtain the eigenvalue and eigenvector of the matrix given below;

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad (16)$$

Recall
[sI – A] = 0 (16)

Where I is identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$SI = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} \quad (16)$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix} \quad (16)$$

Finding eigenvalue

$$S\begin{bmatrix} s & -1 \\ 11 & s+6 \end{bmatrix} + 1\begin{bmatrix} 0 & -1 \\ 6 & s+6 \end{bmatrix} + 0\begin{bmatrix} 0 & s \\ 6 & 11 \end{bmatrix} \quad (16)$$

$S^3 + 6s^2 + 11s + 6 = 0$ (16)
$A_1 = -1$ (16)
$A_2 = -2$ (16)
$A_3 = -3$ (16)

Eigenvector

[sI – A] [x]=0 (16)

At A_1 = (16)

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 6 & 11 & 5 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = 0 \qquad (16)$$

It is not always possible to vary particular eigenvalues via the given inputs and realization that this may occur leads to the concept of CONTOLLABILITY. Likewise the modes associated with a particular eigenvalue may not be visible through the available outputs; y, thus giving, rise to the concept of OBSERVABILITY. A mode must be both controllable and observable if feedback is to be effective in changing it.

For example, given a Multivariable Control System:

vi. Explain what it meant by system controllability and observability?

vii. Determine the controllability and observability of a system described by the state equation shown below and draw the block diagram that represent the system.

$$X(dot)(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \qquad (16)$$

$$Y(t) = [\,1 \;\; 0 \;\; +2\,]\, x(t)$$

Given the equation

$$X(dot)(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \qquad (16)$$

X1(dot) = -x1 + u (16)

X2(dot) = -2x2 + u (16)

X3(dot) = -x3 (16)

2.5 Block Diagram presentation

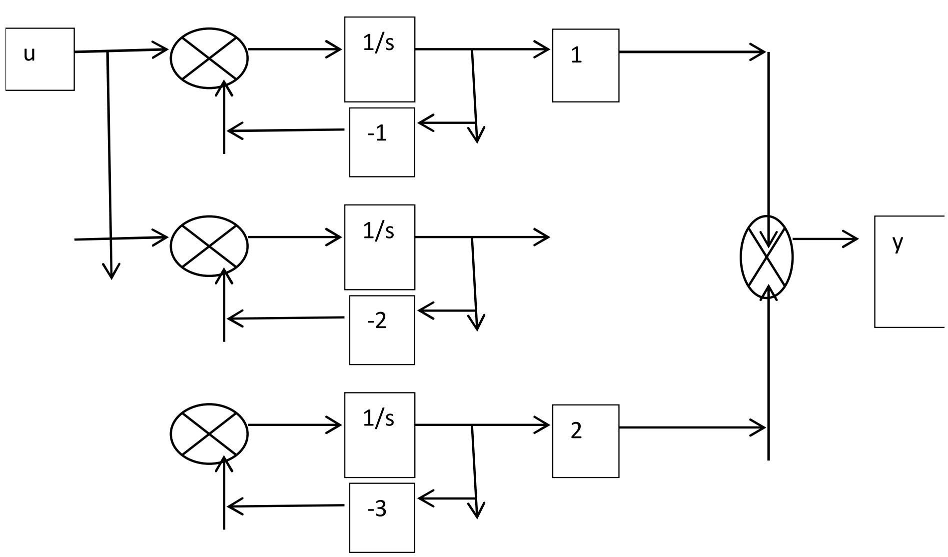

Figure 4: Block Diagram

2.6 Explanation of the block diagram

From the diagram, the first state is both controllable and observable, second state is controllable but NOT observable, the third state is uncontrollable and but observable. Since all the states are not observable and controllable, the entire system is uncontrollable and unobservable. While pole placement techniques may provide adequate designs, there is often economic pressure to produce a "best" or optimal design. The criteria applied includes energy usage, minimizing elapse time or some other combinnations.

2.7 Optimal Control [7,11-12, 19, 22, 27-29]

The method or tool used to achieve an optimal control scheme are Bellmane,"dynamic programming" or the "calculus of variations". In general, these approaches lead to open-loop solution. Sometimes it is possible to achieve a feedback formulation of the solution such solution ought to be more tolerant of plant variations than the open-loop solutions. However, they are bound to be sub-optimal in such circumstances. Although optimal controllers are mathematically deducible for continuous-time control schemes they cannot be implemented except via the agency of a digital computer. It is inducing cost of this computing power combined with increasing processed for efficiency which prompts the inclusion of optimal control in digital control.

2.8 Self-tuning and adaptive control

The techniques described so far all assume, implicitly, that the plant has been perfectly modeled. An imperfection in this area will result in non-ideal performance and the methods vary in their sensitivity to these inaccuracies. The smith predictor control scheme for instance can become totally unstable in the face of modeling errors. There exists some work by Owens et Al [38, 43, 46] which is attempting to deal with the general problem of design in the face of modeling errors. An alternative approach is to configure the control system so that it can adapt its control coefficients so as to achieve the closest possible match to some performance criterion. The problem here is to devise some method of choosing the coefficient adaption in a way which is effective and based on feasible plant measurements. Such controllers are termed adaptive and self-tuning [47,54]. This area of control engineering leans very heavily on the concepts of identification and optimal control couched in the notation of state space. Despite its apparent erudition and abstraction the technique of self-tuning control is becoming established industrially. Needless to say such control schemes require computer systems for their implementation though these need not always be very powerful.

3. CONCLUSION

This paper has set out the basic tools used in the design and analysis of digital control system without going into technical details and has explained the relevance of and relationship between the major topic areas. The work is meant to provide a broad introduction to the techniques for the design of digital control systems for none specialist engineers, in their day to day work and those who would like to be informed as to how digital techniques may be put to use. For research students and others in the field, large compilation of literature review have been included for further reading.

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