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# A Price-Based Grid Resources Pricing Approach for Non-Storable Real Assets

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## ABSTRACT

A grid-computing paradigm delivers the processing power of massively parallel computation to all subscribed users. Current trends, research, and developments in grid computing show that the available grid resources exist as non-storable compute commodities and are distributed geographically – the grid problem. To solve the grid problem, several initiatives have developed frameworks for grid economy and have proposed several algorithms towards an optimized resources scheduling in a grid environment. However, since the grid resources availability depends on the time of usage and are transient, such generic approaches lack the ability to capture the realistic valuation of the resources and fail to guarantee the certainty in their availability measured as Quality of Service (QoS). Uncertainties in grid resource availability do not guarantee a user expected QoS without over committing (e.g., storing the non-storable resources) resources to the users. To guarantee QoS (satisfy a users' computing needs), we propose a price-based and quality-aware model that captures the realistic value of the grid compute commodities. We use the financial option theory from a real option perspective to value grid resources by treating them as real assets. We discuss a set of results on pricing grid compute cycles. Our results are based on the compute cycle usage obtained from the WestGrid node at the University of Manitoba. We extend and generalize our study to any grid in general but with specific reference to the WestGrid.

**Keywords:** Financial Options, Price Stochasticity, Compute Cycles, Real Options, Quality of Service.

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## 1. INTRODUCTION

The grid infrastructure is the core for successful grid applications [1] which also provides support for other computing services. In analogy to physical infrastructures such as roads, power grids, telephone lines, and domestic utility systems, the grid infrastructure describes the pieces of distributed computer processes, information, computations, and technologies bundled together. These pieces of distributed components integrate to form a functional structure. Understanding the functionality of the grid infrastructure is relevant to developing any price-based grid application such as a pricing framework. Over the years, there has been a pervasive and growing need for computing infrastructure. This has led also, to a geometric growth rate in IT developments where major computer hardware and IT vendors such as Sun Microsystems [2], Hewlett-Packard (HP) [3], and IBM [4] had to move their initiatives towards developments in grid computing. Grid computing [5] aims at providing a higher resource availability. However, it faces a number of challenging issues as large as the number of applications across the industry and the academia that explores its ubiquitous benefits. Several of these challenges are grid resources pricing issues [6], security related issues [7], and infrastructure-based issues [5].

Specifically, valuing (pricing) resources potentials of a computational grid system is a challenging task when viewed as a generic pricing problem. In a grid system for example, the resources are distributed across wide geographical regions, owned by dissimilar organization with diverse access rights and usage polices, and they exist as compute cycles (one of the compute commodities) in a time-dimensional space; i.e., the availability of a grid compute commodity (*gcc*) (CPU cycles, memory, network bandwidths, throughput, computing power, disks, processor, and various measurements, instrumentation tools) at $t_0,\ t_1 \cdots,\ t_n$ may not necessarily be constant at all times. This is because the use of *gcc* includes many options for the customer (user of a grid resources) – the option to use *gcc* "now" if $t_n = 0$, the option to use *gcc* "later" if $t_n > 0 \,\middle|\, t_n \neq 1$ (this is also called the option to wait). Whenever $t_n = 1$, the flexibility option waits for a future known utilization date.

For example, consider a *gcc* component such as bandwidth. The flexibility opportunities could range from a decision to use all available bandwidth today at $t_n = 0$, or to use them tomorrow at any time $0 < t_n < 1$ or in the future at $t_n = 1$. To price the *gcc*, we may apply the standard Discounted Cash Flow (DCF) method (in the absence of flexibility opportunities) since the DCF and other traditional methods do not capture the realistic value of an investment [8]. The presence of these flexibilities grants the user an obligation-free *gcc* usage. Should the users' computing needs change in the future, the user may modify the requests. For example, in a grid computing environment, a user (holder of *gcc*) may speculate a fall in the level of available *gcc* and hence may place a request upfront. However, because the requested *gcc* are non-storable, the grid must orchestrate the requested gcc from available sources in order to meet the user's satisfaction. Therefore, to properly value these flexibilities, we treat the *gcc* as real assets and price them with our proposed real assest model. In the following section we provide a basic introduction on options.

### A. Financial Options Vs Real Options

A financial option is defined (see, for example [9]) as the right to buy or to sell an underlying asset that is granted in exchange for an agreed-upon sum. The right to buy or sell an option may expire if the right is not exercised on or before a specific period and the option buyer forfeits the premium paid at the beginning of the contract. The exercise price (strike price) specified in an option contract is the stated price at which the asset can be bought or sold at a future date. A call option grants the holder the right to purchase the underlying asset at the specified strike price. On the other hand, a put option grants the holder the right to sell the underlying asset at the specified strike price. An American option can be exercised at any time during the life of the option contract; a European option can only be exercised at expiry. Options are derivative securities because their value is a derived function from the price of some underlying asset upon which the option is written. They are also risky securities because the price of their underlying asset at any future time may not be predicted with certainty. This means the option holder has no assurance that the option will be in-the-money (i.e., yields a non-negative reward), before expiry.

As with all risky securities, an important question one might want to ask is, how such a security might be valued apriori. The value of an option may be determined using a number of variables that relates to the underlying assets. These include the current value of the underlying asset, volatility of the underlying asset, expected dividends on assets, and the strike price of the options. To recap, financial options represent the right to either buy or sell an asset. On the other hand, real options deal with the possibility of a choice from two decisions; the choice for or against an investment decision without necessarily binding oneself upfront. Table I compares the parametric characteristics of a financial options and real options. Real options have distinct characteristic behaviour when compared with financial options [8]. Due to flexibility opportunity in the asset prices which fit well with gcc-s, we treat the gccs as real asset and propose a model to price them. To our search for related work, this is the first study that employs a knowledge from real option theory to pricing the grid resources as assets.

### B. WestGrid Facilities Overview

Historically, WestGrid (Western Canada Research Grid) started in 2003 with the seven participating institutions; the Simon Fraser University, Banff Center, TRIUMF (Tri- University Meson Facility), the Universities of Alberta, British Columbia, Calgary, and Lethbridge. By April 2006, it expanded to include the Universities in Saskatchewan and Manitoba (the University of Manitoba’s Polaris shared memory system). The node at University of Manitoba will be expanded in the near future with a distributed shared memory multiprocessor facility. Augmenting that will be a Department of Computer Science’s own distributed memory facility. Currently, the WestGrid is Canada’s largest High-Performance Computing (HPC) consortium with 14 partner institutions in four provinces; British Columbia, Alberta, Saskatchewan, Manitoba, and the Banff Centre. The components of the WestGrid are distributed among the participating institutions. For example, the University of British Columbia houses a large cluster of commodity-based computers while TRIUMF has low-cost computers with a low-speed interconnect; for “coarse-grained” concurrency. The University of Calgary is responsible for the clump of small shared-memory machines (a tightly coupled cluster of multiprocessors; “medium-grained” concurrency). A large shared memory machine at the University of Alberta (high-speed processor interconnect; offering “fine-grained” concurrency). The Simon Fraser University management system and graphics. In addition, various collaborative visualization tools are located in all participating institutions.

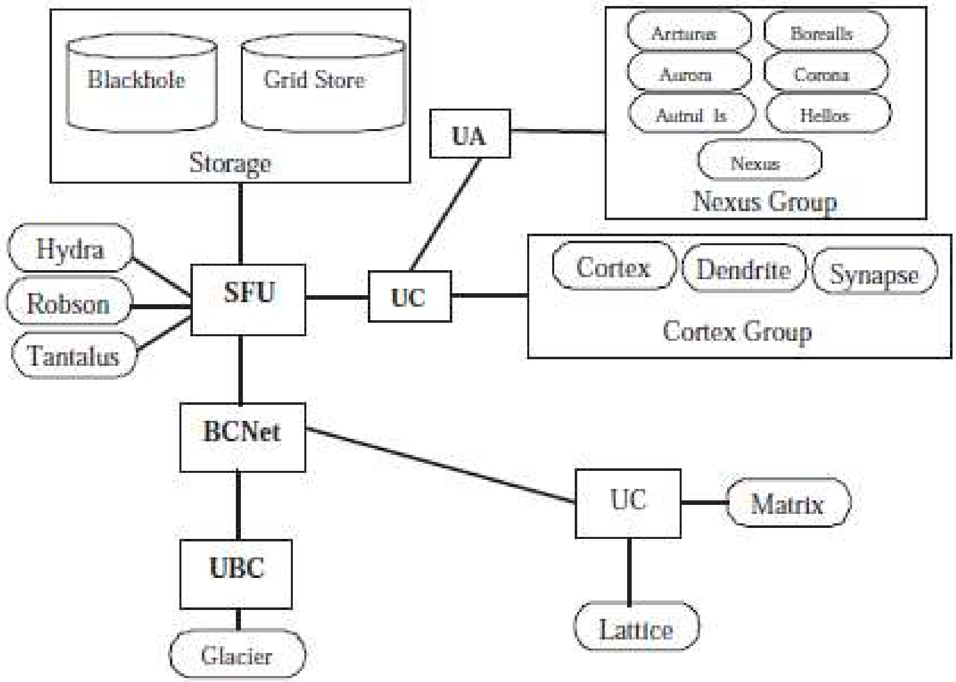


Figure 1. WestGrid Map [31].

Figure 1 shows an overview of the WestGrid infrastructures (map/facilities). Particularly, we study the state of the facilities available at the University of Manitoba HPC. The HPC facilities comprises of multiprocessor supercomputers or the clusters of multi-processor computer systems also called node. The nodes are linked together by high-speed interconnects. The HPC also require middleware tools, monitoring and management software. For these facilities, they require other data storage facilities. Given the enormous data requirements of modern scientific research, up to 0.5 terabyte of data or 1.5 petabyte of raw data per year may be required to carry out computation. Other components of the WestGrid include remote instrumentation and visualization tools. Remote instrumentation describes the science and art of gathering useful information about an object using analysis acquired by some other device not in contact with the object under investigation. Remote instrumentation application areas include atmospheric sciences, medical and biological sciences, and environmental sciences.

The rest of this paper is organized as follows. In Section II we review some related work. In Section III we introduce a conceptual approach to valuing grid compute commodities and pricing them as real options in a fuzzy logic framework and in a discrete time approach. A price-aware grid resources valuation model using real options is provided in Section IV. Section V reviews the WestGrid computing and infrastructural facilities. This work starts with WestGrid compute commodities in particular and scale the idea to the global grid in general. Section VI presents the results of our experiments and Section VII concludes the paper.

Table 1: Financial Options Vs. Real Options Parameters

| Parameter | Financial Asset | Real Asset |
|---|---|---|
| *P* | Stock price | Value of cash flow from the flexibility opportunities |
| X | Option strike price | Value of the option due to delay |
| r | Risk-free rate of interest | Compounded risk-free rate of interest |
| σ | Volatility | Volatility of stock's return |
| T | Time to expire (years) | Life of Option |

## 2. RELATED WORKS

Several works of literature in financial options ([xx]) shows diverse methodologies and approaches for pricing options. For example, closed-form equations such as the Black-Scholes [10] model, Monte Carlo path-dependent simulation methods (see for example [11] and [12]), lattices (binomial and trinomial (for example [13] and [14])), and other numerical techniques (for example, [15], [16], [17], [18], and [19]). These approaches focus specifically on financial options. In a computational grid system for example, resources must be priced as real assets but not as financial options because of the characteristics of gcc – their availability is transient. As a result, pricing them require the application of a scheme that is capable of estimating and valuing them as real assets using real options that captures users' flexibility opportunities.

A real options framework captures the set of assumptions, concepts, and methodologies for creating decision flexibility in a known future. Flexibilities in investment decisions are critical because not all of them have values in the future. Therefore, uncertainty abound in the decision to either exercise or not to exercise the options. This challenge in the real options concept has propelled several research efforts recently.

Current literature on real option approaches to valuing projects presents real options framework in eight categories [8]: option to defer, time-to-build option, option to alter, option to expand, option to abandon, option to switch, growth options, and multiple integrating options. There are also efforts reported towards improving the selection and decision methods used in the prediction of the capital that an investment may consume. Carlsson and Full´er in [21] apply a hybrid approach to valuing real options. Their method incorporates real option and fuzzy logic and some aspects of probability to account for the uncertainty involved in the valuation of future cash flow estimates. The results of the research efforts given in [20] and [21] have no formal reference to the QoS that characterize a decision system. Carlsson and Full´er [21] apply fuzzy methods to measure the level of decision uncertainties, however, there is a lack of indication on how accurate the decisions could be.

## 3. GRID COMPUTE COMMODITIES AS REAL OPTIONS IN A FUZZY FRAMEWORK

### A. A Fuzzy Logic Framework

We express the value of the gcc flexibility opportunities as previously given in [32] (and applied in [22] and [33]) as:

$$gcc: t_n = t_{ut} \qquad (1)$$

where gcc is called grid computing commodity, $t_n$ denotes the time-dimensional space and given as $0 \leq t_n \leq 1$ and $t_n$ describes the corresponding utilization time. If $t_n = 0$, gcc usage is “now” or “today”, if $t_n = 1$, gcc has a usage flexibility opportunity for “the future” where future is not to exceed 6 months (say). Users often request and utilize gcc at extremely high computing power but only for a short time for $t_{ut} = t_n \approx 0$. Therefore, disbursing the gcc on-demand and satisfying users’ Quality of Service (QoS), requires that the distributed resources be over-committed or under-committed for $t_n = 1$ or $t_n = 0$) respectively in order to satisfy the conditions specified in the Service Level Agreements (SLAs) document. Such extreme conditions (for example, holding gcc over a long time) requires some cost in the form of storage. Therefore, we express utilization time tn as a membership function of a fuzzy set T. A fuzzy set is defined (see for example [23]) as:

$$T = \big(t, \mu(t)\big) \,\big|\, \mathrm{t} \in \mathrm{T}, \mu_T(t) \in = [0,1]. \qquad (2)$$

Thus, given that T is a fuzzy set in a time domain (the time-dimensional space), then $\mu_T(t_n)$ is called the membership function of the fuzzy set T which specifies the degree of membership (between 0 and 1) to which $t_n$belongs to the fuzzy set T. We express the triangular fuzzy membership function as follows:

$$\mu_T(t_n) = \begin{cases} 1 & \text{for } x = b \\ \dfrac{x-a}{b-a} & \text{for } a \leq x \leq b \\ \dfrac{c-x}{c-b} & \text{for } b \leq x \leq c \\ 0 \text{ otherwise i.e., if } x \notin [a,b] \end{cases} \qquad (3)$$

where [a, c] is called the universe of discourse or the entire life of the option. Therefore, for every gcc at utilization time $t_n$, availability of the gcc expressed as membership function is the value compared to stated QoS conditions given in the SLA document. Figure 2 shows the triangular fuzzy membership function for the grid resources utilization corresponding to Equation (3). An SLA document [24] describes the agreed upon services provided by an application system to ensure that it is reliable, secure, and available to meet the needs of the business it supports.

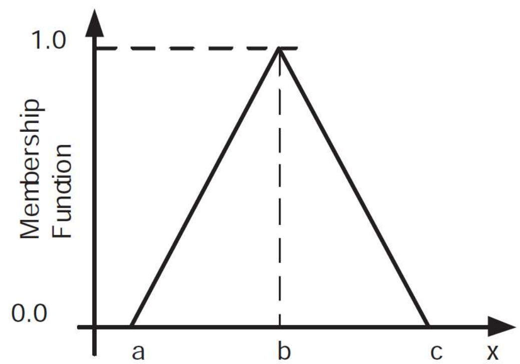


Figure 2. Triangular Fuzzy Membership Function for gcc Utilization Time.

The SLA document consists of the technicalities and requirements specific to service provisioning e.g., the expected processor cycles, QoS parameters, some legal and financial/business aspects such as violation penalties, and utilization charges for resources use. The implication of a service constraint that guarantees QoS and meets the specified SLA conditions within a set of intermittently available gcc is a system that compromises the basic underlying design objective of the grid as a commercial computing service resource [25]. Therefore, when Equation (1) is re-written, it becomes:

$$gcc: t_{ut} = t_n \big|_{QoS \approx SL} \qquad (4)$$

To satisfy QoS-SLA requirements, we apply a real options pricing scheme, which differs from a generic market-based resource sharing where all jobs are expected to receive some resource [26] based on the offered price or the application of demand and supply to set prices. In this paper, our work focuses on the valuation of a computational grid system resources – "grid compute commodities" gcc, with specific emphasis on the provision of a satisfaction guarantee in terms of the QoS requirements for grid resource users and resource owners through a regulated Service Level Agreements (SLAs)-based resource pricing infrastructure using a real option approach. The main contribution of this paper is the design and development of real option-based model for pricing grid resources. This contribution is expected to facilitate achieving some general grid system objectives such as:

i. To keep the grid "busy" for optimal gain (profitability wise); this ensures that no wait states or idle compute commodities exist in the grid.
ii. To provide an assessment for the evaluation the cost of use of grid resources and applications in order to justify its design and estimate the future benefits.
iii. To provide a generic plan for managing the resources/infrastructure of a grid that are essential for meeting the peak demands of grid resources utilization.
iv. To facilitate continuous research activities through timely upgrades on the grid infrastructure.
v. To improve resource utilization and justify IT investments in WestGrid.
vi. To provide a satisfaction guarantee in terms of the QoS requirements for grid resource users and resource owners through a regulated Service Level Agreements (SLAs)-based resource pricing infrastructure.

We base our evaluation results on the actual data available on the Polaris system at the University of Manitoba HPC [27] facilities and compare using the results of our simulation. We capture the possible price changes in the gcc with a discrete time approach and use it for pricing the real option that comprises of the gcc-s. Several schemes exist to price financial options; (i) application of the Black-Scholes model [10] which requires that the solution of the partial differential equation of the option price be satisfied, (ii) application of a discrete time and state binomial model of the underlying asset price which requires the application of the discounted expectations [13], [28]. In our simulation, we use the trinomial model [9], [29] to solve the real option pricing problem manifested as partial differential equation. This is a discrete time approach to calculate the discounted expectations in a trinomial-tree structure.

B. Real Option Pricing in a Discrete Time Approach

Consider a trinomial model of asset price in a small-time interval $\delta t$, we set the asset price changes by $\delta x$. Suppose this change remain the same or changes by $\delta x$, with probabilities of an up movement $p_u$, probability of steady move (without a change) $p_m$, and probability of a downward movement $p_d$. Figure 2 shows a one-step trinomial lattice expressed in terms of $\delta x$ and $\delta t$. The drift (due to known factors) and stochastic volatility ($\sigma$, due to unknown factors) parameters of the asset price can be captured in the simplified discrete process using $\delta x, p_u, p_m$, and $p_d$.

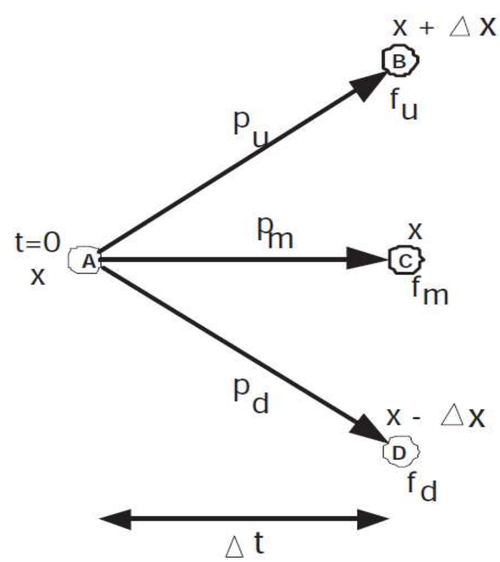


Figure 2. One-Step Trinomial Lattice.

The space step can be computed (with a choice) using $\sigma x = \sigma\sqrt{3\sigma t}$. A relationship between the parameters of the continuous time process and trinomial process (a discretization of the Geometric Brownian Motion (GBM)) is obtained by equating the mean and variance over the time interval $\delta t$ and imposing the unitary sum of probabilities. That is,

$$E[\delta x] = p_u(\delta x) + p_m(0) + p_d(-\delta x) \ = \ v\delta t \qquad (5)$$

From Equation (5),

$$E[\delta x^2] = p_u(\delta x^2) + p_m(0) + p_d(\delta x^2) \ = \ \sigma^2\delta t \ + v^2\delta t^2 \qquad (6)$$

where the unitary sum of probabilities can be presented as:

$$p_u + \ p_m + p_d = 1 \qquad (7)$$

Solving Equations (5), (6), and (7) yields the transitional probabilities;

$$p_u = 0.5\left(\frac{\sigma^2\delta t + v^2\delta t^2}{\delta x^2} + \frac{v\delta t}{\delta x}\right) \qquad (8)$$

$$p_m = 1 - \frac{\sigma^2\delta t + v^2\delta t^2}{\delta x^2} \qquad (9)$$

$$p_u = 0.5\left(\frac{\sigma^2\delta t + v^2\delta t^2}{\delta x^2} - \frac{v\delta t}{\delta x}\right) \qquad (10)$$

Suppose we consider the grid as a multi-resource system with grid compute commodities $gcc_i = \{gcc_1, gcc_2, \cdots, gcc_n\}$ where n is a finite number (the number of available grid commodities). These gcc's fits into the trinomial lattice if we let a real option to depend on some other variables such as $\mu_{gcc}$ and $\sigma_{gcc}$, the expected growth rate and volatility respectively, then we can let:

$$\frac{dgcc_i}{gcc_i} = \mu_{gcc}dt + \sigma_{gcc}dz \qquad (11)$$

Therefore, for any number of derivatives of gcc such as $(gcc_1, gcc_2, \cdots, gcc_n)$ with prices $p = (p_1, p_2, \cdots, p_n)$ respectively, we have

$$d\ lnS = \frac{dp_i}{p_i} = \mu_i dt + \sigma_i dz \qquad (12)$$

where S is a general variable for gcc pricing and the variables $gcc = \{$the set of $gcc_i\}$. That is, if there exist some gcc-s, then we have:

$$d\ lnS = [gcc(t) - a\ ln\ S]dt + [\text{stochastic term}] \qquad (13)$$

where the stochastic term (due to volatility) is given as $\sigma dz$ and a is called the mean reverting factor which dampens the fluctuating characteristics price-based models. For non-storable commodities such as electricity, and in particular, grid compute commodities the prices tend to drift about the mean. The application of the drifting parameter to the pricing model regulates excessive price drifts. The strength of the mean reverting process is determined by the value of a (high for a > 0). For a multi-asset problem, we have:

$$d\ lnS = [gcc_i(t) - a\ ln\ S_i]dt + \sigma_i dz_i\ \Big|_{i=1,2,\cdots,\ n} \qquad (14)$$

The value of gcc(t) is determined at time t when the expected value of S is equal to the future price i.e.,

$\mathrm{F(t)} = \widehat{\mathrm{E}}\,[S(\mathrm{t})]$. For example, suppose a computation requires more 'memory' (one of the gcc-s) in six months, the user may decide to pay some amount, $p upfront to hold a position for the expected increase. Illustrating as a 3-step trinomial process for a spot price of $P per byte per second and the projected month future prices of $$p_1$, $$p_2$, and $$p_3$ for 2, 4, and 6 months in the future respectively, there is a high rate of uncertainty of the actual amount of memory that will be available at the future date. The initial node corresponds to a "memory" price of $p per byte per cycle. Suppose $M_{i.j}$ represents the option values at i for i = 0, 1, ..., n-1 level and j node for j = 1, 2, 3 (for a trinomial lattice only); i.e., $M_{1,1}$ represents the option value at level 1 and at $p_u$. Therefore, the displacement for the node is $M_{i,j} + \alpha_j$. If there are displacements $\alpha p_u, \alpha p_m,$ and $\alpha p_d$ for $p_u, p_m,$ and $p_d$ respectively, we have the expected future price for memory given as:

$$E[S(t)] = p_u e^{M_{i,j}} + p_m e^{M_{i,n}} + p_d e^{M_{i,n}} \qquad (15)$$

Please note that, the I/O component of the application that is using the gcc dictates the memory usage, where "memory" is one of the many commodities in a gcc. This analysis can be extended and repeated to other commodities with appropriate functional changes.

## 4. A REAL ASSET PRICE-AWARE VALUATION

Figure 4 shows an abstract representation of a price-based grid infrastructure as presented in [12]. The infrastructure comprises of a four-level price-based infrastructure model. Level-0 contains the pools of available grid compute commodities *gcc*. A typical grid infrastructure consists of the following: (1) Resource modelling: The grid infrastructure provides a description of the available resources, application capabilities, and defines inter-component relationships between the various clusters that comprise the grid. The grid resources modelling approach facilitates resource discovery, provisioning, and quality of service management. (2) Monitoring and Notification: At any time during a grid computation, the infrastructure ensures that it provides updates regarding the state of use of resources.

These include notifications for changes in projected utilization levels and application notification regarding services changes. The monitoring capability also helps to maintain resource discovery and maintain QoS needed to support accounting and billing functions on resources pricing. Following notification and resources monitoring, resources may be re-deployed to ensure resource availability using some form of reservation. (3) Accounting and auditing: This aspect of the grid provides a log for the usage of shared resources. This transforms resources usage into cost for charging resource use by applications and users.

In the presented infrastructure, services are price-based and they range from buying grid compute commodities, such as bandwidth, processor cycles or memory to retailing computes time. The infrastructure is a four-level system. The functions executed at Level-3 are user-application based. At this level, several authenticated users log in to the grid to assess compute commodities. The objective of a logged-on user is to gain access to the computing commodities as soon and quickly as possible for a small price while occupying the highest level of QoS as defined in the SLA. To achieve this objective, the Grid Resources Broker (GRB) maps physical resources onto virtual resources (the *gcc*) while guaranteeing a service agreement between the QoS and SLA. Level-2 of the RoM provides functionality for selling (pricing) the *gcc* – accounting and charging for the resources usage by user is actually measured against the SLA for payments. The Globus toolkit and Nimrod-G provides the necessary Grid Security Infrastructure (GSI), that does the job submission and makes data repository among the various leveled-modules. A high-level search scheme to find resources in the grid environment is enabled by the Meta-computing Discovery Service (MDS). The actual grid compute commodities to be priced occupies the Level-0 of the infrastructure.

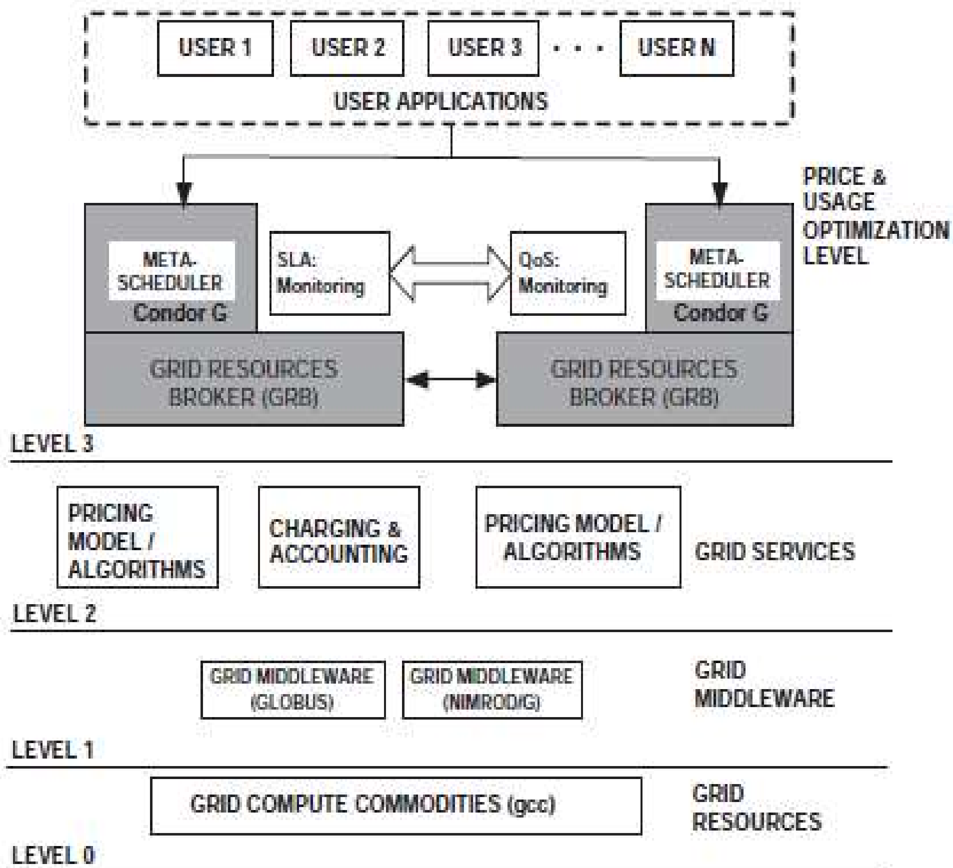


Figure 4. Grid Resources Pricing Infrastructure [22].

A. Grid Resources Utilization for WestGrid

This section focuses on the Polaris resources usage at the University of Manitoba's node of WestGrid in Canada. The available gcc-s in the Polaris include a 48 GB of memory, 24 UltraSparc-III processors (CPUs) with a bandwidth of 1050MHz, 1 Terabyte of high-speed disk storage. We base our assessment on the monthly grid commodities and the daily compute commodities demand/usage.

B. Price Variant Factor

Tables II and III capture usage patterns of the ACN Polaris comprehensively for the purpose of reference. (They are included here for the ease of review process without having to go to the ACN web site [27]). The monthly CPU utilization data was gathered from the day-to-day summary for one month. The Academic Computing and Networking (ACN) [27] gathers the monthly CPU utilization data from day-to-day and summarizes as monthly CPU utilization for January through December. The total CPU cycles ($gcc_{total}$) describes the monthly available and the parallel cycles ($gcc_{used}$) describes the effective monthly usage. For optimum gain and justification for grid infrastructures, the under-utilized ($gcc_{total} - gcc_{used}$) for the month should be as close to 0 as possible.

Therefore, the best optimum use of *gcc* occurred during the months of June, July, September, April, and March where the value of $gcc_{total} - gcc_{used}$ is 0 or CPU utilization is at least 97.72% in all cases. Using these data, grid "operators" attempt to normalize (i.e., keep the grid busy always) *gcc* by ensuring *gcc* availability for all peak period demands. Similarly, the ACN [27] shows the actual data of the monthly *gcc* demand and actual usage. The summary for January to December indicates the actual *gcc* demand (in %) where the CPU demand describes the request made by users and CPU usage (%) displays the actual value of *gcc* released to the user. The percentage CPU utilization indicates that services that meet actual QoS requirements (i.e., satisfy 100%) for acceptable QoS as observed in June, July, and August shows that the grid satisfies "just" 25% of *gcc* requests and a gross under-usage in January, May, November, February, October, and December.

Table 2: Monthly CPU Utilization – Polaris

| Months | Parallel CPU Cycles | Total CPU Cycles | Underutilized CPU Cycles | CPU % Utilization |
|---|---|---|---|---|
| Jan | 859 | 1095 | 236 | 78.45 |
| Feb | 655 | 728 | 73 | 89.97 |
| Mar | 770 | 788 | 18 | 97.72 |
| Apr | 549 | 550 | 1 | 99.82 |
| May | 801 | 909 | 108 | 88.12 |
| Jun | 755 | 755 | 0 | 100.00 |
| Jul | 950 | 950 | 0 | 100.00 |
| Aug | 381 | 437 | 56 | 87.19 |
| Sep | 601 | 601 | 0 | 100.00 |
| Oct | 613 | 710 | 97 | 86.33 |
| Nov | 278 | 467 | 189 | 59.53 |
| Dec | 744 | 855 | 111 | 87.02 |

Table 3: Daily CPU Demand Vs. Actual Usage. Source [27]

| Months | CPU Demand (%) | CPU Usage (%) |
|---|---|---|
| Jan | 135 | 75 |
| Feb | 105 | 95 |
| Mar | 125 | 80 |
| Apr | 140 | 80 |
| May | 165 | 95 |
| Jun | 175 | 100 |
| Jul | 200 | 100 |
| Aug | 125 | 100 |
| Sep | 75 | 75 |
| Oct | 65 | 65 |
| Nov | 50 | 50 |
| Dec | 75 | 75 |

The general purpose of our model (described in Section III and IV) is to keep the grid busy (without idle compute cycles). To achieve this objective, we define and apply a price variant factor (*PV F*). The *PVF* (*pf*) is a multiplier and a real number such as $0 \leq pf \leq 1$. Its value depends on some trends of technological developments which may become uncertain. Hence, we capture uncertainties in *pf* using fuzzy logic and treat *pf* as a fuzzy number. A Fuzzy Number [30] $A$, is a fuzzy set of the real line with a normal, convex and continuous membership function of bounded support.

For example, the grid resources may become under-utilized if users find better and faster ways (improved algorithms and technology) to solve their computing problems. Therefore, to maximize the grid resources utilization with more computing facilities and same technology, the value of *pf* is set to 0.1 and with new technology, the *pf* = 1.0. Our model therefore, adjusts the profitability in the use of grid resources by $(pf)^{-1}$ (for the grid operator) while providing quality of service set at SLA.

## 5. RESULTS AND DISCUSSIONS

The WestGrid node at University of Manitoba provides data for the gcc-s in the form of usage pattern. We simulated our model for pricing this commodity only in the study. However, we can use our model for pricing other commodities in the grid as well to quote prices for them. Figure 5 shows the *gcc* under-utilization obtained using set the dataset in [27]. The high level of under-utilization observed in Figure 5 is not due to high cost of *gcc*. However, a high under-utilization (idle resources or wait states) results from the grids inability to satisfy on-going requests for *gcc*. As a result, users are not willing to pay upfront to acquire *gcc*.

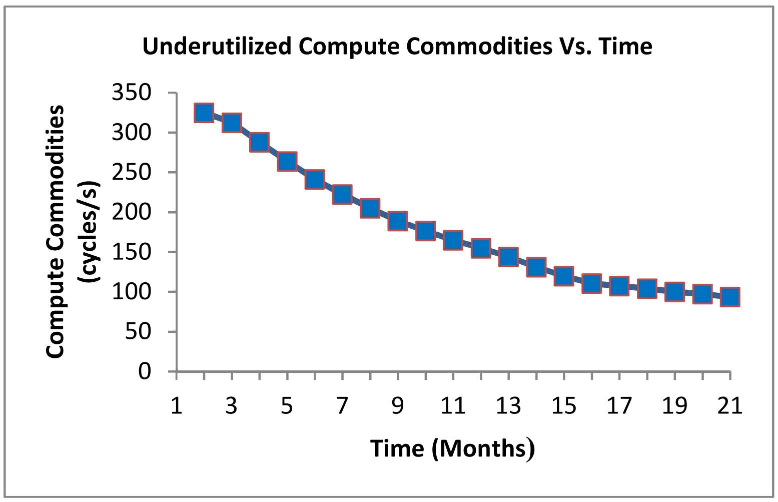


Figure 5. GCC Underutilization and Time (months).

Figure 6 shows the *gcc* under-utilization obtained using set "utilized compute commodities" from [27]. Figure 7 shows the *gcc* effective utilization obtained using set utilization rate of CPU cycles "CPU Percentage Utilization" from [27]. The figure shows an increasing rate of *gcc* usage. From the utilization trends in WestGrid, the generic grid problem – satisfying diverse and multiple requests and guaranteeing resources availability using non-storable reservoir of compute commodities persists. In our experiments, we simulated the grid compute COMMODITIES (*gcc*) and monitor users' request for utilization. We obtain option values (prices) and study the variation in a space of 6 months to determine the effects of time of exercise on option value. Time of exercise here means the time at which the grid compute commodities are going to be utilized, up to six months in the future.

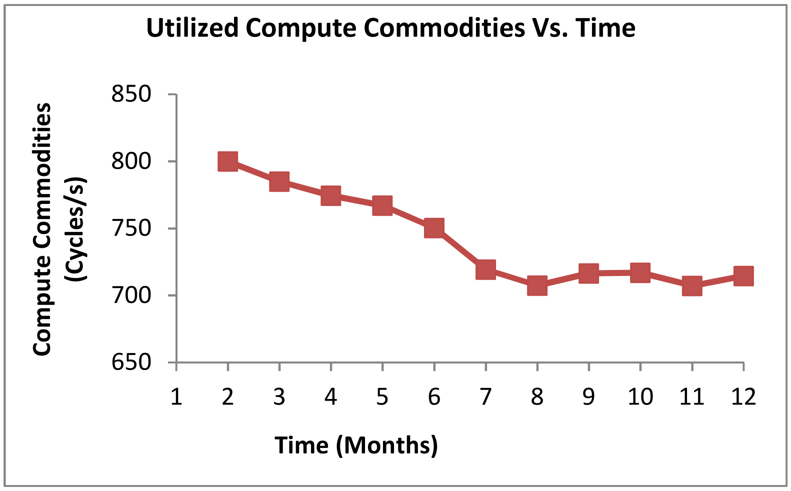


Figure 6. Price-Aware Grid Infrastructure

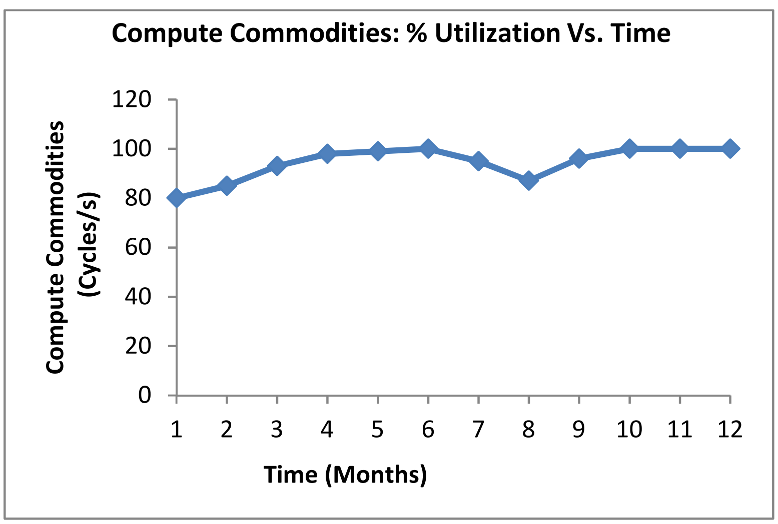


Figure 7. Effective gcc Usage.

Figure 8 shows the effects of time of exercise on *gcc*. We study some selected *cc* such as CPU cycles, memory, bandwidth, throughput, disk capacity, and processors depicted as $cc_1$, $cc_2$, $cc_3$, $cc_4$, $cc_5$, and $cc_6$ respectively. The effects of applying $PV F = (pf)^{-1}$ shows that at any given time, the grid satisfies the users' computing needs by granting computing requests at lower prices during off-peak demands for *cc* so that users take advantage of the low prices and use more of the *cc*. The resource prices are increased during peak demand (to make more profits) period to provide all the available resources at its full without compromising on the QoS. Similar results are observed for the other infrastructures in the WestGrid.

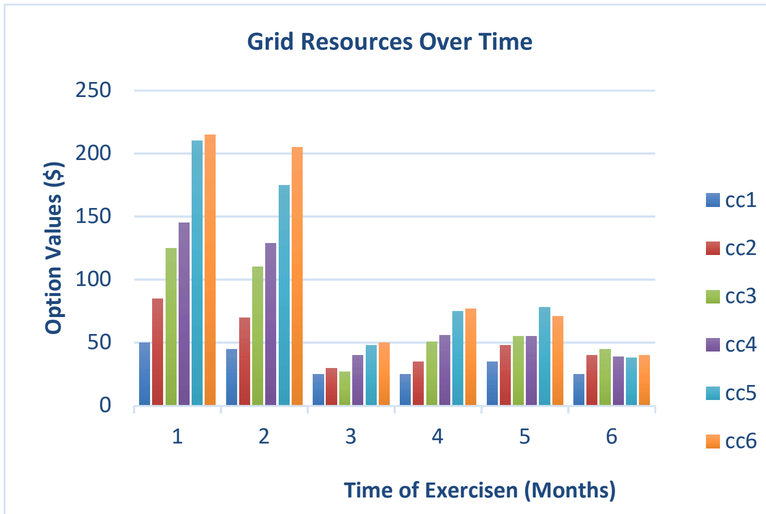


Figure 8. Effects of Time of Exercise on Grid Compute Cycles.

## 6. CONCLUSIONS

From the utilization trends in WestGrid, the generic grid problem – satisfying diverse and multiple requests and guaranteeing resources (non-storable reservoir of compute commodities) availability - persists. Our model provides a price-utilization control system that ensures optimal resources utilization while meeting the users' satisfaction guarantee under increased profitability. We proved this with one real grid node. Therefore, this paper has both academic and industrial value/contributions. Grid economy, as a new emerging research area, presents several issues to be considered by a service provider as well as a user of grid computing commodities – provision of virtual grid resources that are in much demand and this meets expected level of user QoS. For the industry, our model and results would provide a mechanism for assessing the profitability of the grid and demonstrates the need to manage the grid infrastructure in order to meet the peak load demand for grid resources. We also show that over-committing grid resources does not necessarily mean satisfying user QoS, but a remodelling of existing bargain systems through the application of a price variant factor.

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