



Mathematical Modeling of Oil Shale Pyrolysis

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ABSTRACT

A mathematical model of the oil shale Pyrolysis process is described by a nonlinear partial differential equation. The paper presents an analytical solution to describe this phenomenon. The nonlinear coupled governing equations are decoupled using parameter expanding method and solved analytically. The results obtained revealed that the heat transfer coefficient have significant effect on temperature and concentration of fuel.

Keywords: Temperature, Non-dimensionalization, Gas, Oil, Simulation, Oil shale, Pyrolysis, Fuel and Heat capacity

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1. INTRODUCTION

Oil shale is commonly defined as a fine-grained sedimentary rock containing matter that yields substantial amounts of oil and combustible gas upon destructive distillation. Most of the organic matter is insoluble in ordinary organic solvents therefore it must be decomposed by heating to release such minerals. Oil shale is distributed throughout the world, although the richest and most abundant identified deposits are contained in the Green River Formation Utah and Colorado. Anderson (1985). Oil shale can be exploited to extract oil and gas by retorting power generation by direct combustion. Shale oil, a synthetic crude oil which was produced by low temperature retorting of oil shale may be used as fuel oil directly, also may be transformed as the gasoline, the diesel oil by further treatment. Coal gas and semi-coke is the by-product of oil shale production after low-temperature retorting which can be uses as power plant fuel; the oil shale ash may be used as the as building materials and soil fertilization matter. Gouquan (2006).

Oil shale pyrolysis was developed in France, where in 1832, a method for producing lighting oil was realised. However, the plants were later closed because of the rapid development of the crude oil industry. Data from 1860 indicate that oil shale from the Volga basin in Russia was industrially mined and used as fuel. During the 19th century oil shale thermal processing factories also operated in Australia, the United State, Brazil, Germany, and Scotland. During the 20th century oil shale processing factories were built in several countries, including China and Israel. However, later most of them were closed. In Estonia and Germany oil shale has been used also in cement production both as fuel and as constituent of the clinkers. Today, considerable quantities of oil shale are mined in Estonia, Russia, China, Brazil, Australia, and Germany. Estonia's oil shale industry is currently the most developed in the wor4ld. Bauert (1994).



Oil shale can be used for several purposes; to obtain heat by direct combustion (for example, in the generation of electricity; to produce shale oil (SO); and as a source of other valuable chemicals. Underlying most definitions of oil shale is its potential for the economic recovery of energy, including shale oil and combustion gas, as well as a number of by-products. A deposit of oil shale having economic potential is generally one that is at or near enough to the surface to be developed by open-pit or conventional underground mining or by in-situ methods. Retorting of oil shale can be performed in different environment. Pyrolysis is a process of heating oil shale in an inert environment. Primary products of pyrolysis are liquid, gas and coke. The extent of decomposition (yield) and the quality of pyrolysis products depend on the composition of the source material, the temperature-time history, pressure, residence time and presence of other reactants such as water, methane etc. Because of the chemical composition of the oil produced, moderate to significant upgrading (nitrogen removal and or hydrogen addition) may be required to convert the oil into a refinery feedstock. Nuttal(1983).

Oil shale is currently an under-utilized energy resource. One reason is that petroleum crude oil and other energy sources are cheaper than shale oil. Other reason includes high investments, addition costs of mining and unfavourable environment problems. However, with the development of global economic growth accompanied by increasing costs of petroleum and coal, the utilization of oil shale will become increasingly important. Heating the oil shale to pyrolysis temperature and completing the endothermic kerogen decomposition reactions required a source of energy. Some technologies burn other fossil fuels such as natural gas, oil or coal to generate this heat and experimental methods has used electricity, radio waves, microwaves, or reactive fluid for this purpose. The combustion process used in the retort is of the type known as "forward" that is, the high temperature (combustion) reaction is propagated through the retort in the same direction as the injected oxidant gas stream. The combustion zone is a relatively narrow region of the retort. Residual fuel deposited ahead of the combustion zone is all consumed in this region (provided that the combustion temperature is sufficiently high). The combustion zone advances seeking new fuel. Heating by conduction and convection occurs ahead of the combustion zone. This heating causes pyrolysis of the oil shale to occur. The carbon residue formed during that pyrolysis is thought to be the principal source of fuel for the combustion process. The oil and gas product formed ahead of the combustion zone are driven forward and recovered. The carbon residue formed remains in place and undergo combustion.

Cooling, also by conduction and convection, occurs behind the combustion zone, Thomas (1963). The other basic type of combustion process is known as "reverse combustion". In such a process, the combustion zone is propagated in a direction opposite to the flow of oxidant gas. All oxygen is consumed in the combustion zone, the combustion zone move toward the oxygen supply. Combustion thus takes place with an excess of fuel. A thorough discussion of the reverse combustion process. Torrente (2001)

1.1. Aim and Objective of the Study

The aim of this work is to mathematically solve the Oil Shale Pyrolysis model analytically using parameter Expanding Method.

The objectives are;

- i. To formulate mathematical method for oil shale Pyrolysis
- ii. To solve the equation analytically using parameter Expanding Method
- iii. To provide the graphical summaries of the system response.



1.2. Definition of terms

- ❖ **Temperature:** a measure of hotness or coldness of an object or substance. It is measured in any of the various temperature scales as Celsius, Fahrenheit, and Kelvin.
- ❖ **Non-dimensionalization:** this is the partial or full removal of units from an equation involving physical quantities by a suitable substitution of variable.
- ❖ **Kerogen:** the portion of the organic material in oil shale that is insoluble in ordinary solvents for petroleum and that, upon the application of heat, may yield some or all of the following product: pyrolytic, bitumen, gas, oil, and a carbon residue.
- ❖ **Natural bitumen:** the soluble organic material naturally present in oil shale.
- ❖ **Pyrolytic bitumen:** the soluble organic material that forms during heating of an oil shale.
- ❖ **Gas:** the non-condensed vapours that form from the organic matter during heating of an oil shale.
- ❖ **Oil:** the hydrocarbon and other organic compounds that form as vapour during heating of an oil shale and that subsequently condense.
- ❖ **Carbon residue:** the insoluble portion of kerogen remaining in the spent shale after destructive distillation; this residue is similar to fixed carbon in coal.
- ❖ **Semi-coke:** is a solid waste material left after oil shale retorting in oil shale chemical.
- ❖ **Simulation:** is the imitation of the operation of a real-world process or relationship in a mathematical form in which equations are used to simulate the behaviour of the system or process under study.
- ❖ **Oil shale:** refer to any sedimentary rock that contains solid bituminous materials (called kerogen) that are released as petroleum-like liquid when the rock is heated in the chemical process of pyrolysis.
- ❖ **Pyrolysis:** is a thermo chemical decomposition of organic material at elevated temperature in the absence of oxygen. It involves the simultaneous change of chemical composition and physical phase, and is irreversible.
- ❖ **Fuel:** is any material that can be made to react with other substance so that it releases chemical or nuclear energy as heat or to be used for work.
- ❖ **Heat capacity:** is the amount of heat required to change its temperature by one degree.

2. REVIEW OF RELATED LITERATURE

Abu-Khamsin et al. (1987) observed that pyrolysis of crude oil in porous media went through three overlapping stages: distillation, visbreaking and coking. Jabber and Probert (2000) studied oil shale samples by non-isothermal thermogravimetry. The controlling parameters studied were the final temperature and the influence of particle size as well as the heating rate employed during the process of thermal degradation of oil shale sample. An integral method was used in the analysis of weight loss data to determine the pyrolysis and gasification kinetics. The activation energy decreased slightly as the shale particle size was reduced. Many oil shale deposits contain small, but ubiquitous, amounts of sulphides including pyrite and marcasite, indicating that the sediments probably accumulated in dysoxic to anoxic waters that prevented the destruction of the organic matter by burrowing organisms and oxidation. Dyni (1989).

Adegbesan (1987) provided low temperature oxidation reaction models that would be suitable for the use in numerical simulators of in-situ combustion for bitumen and heavy oil reservoirs. Two main types of reaction models were proposed; a non-steady state kinetic model representing the overall rate of oxygen consumption, and four non-steady state multi-response kinetic models representing oxidation reactions of the liquid-phase components. Thakur and Nuttal (1987) studied the pyrolysis kinetics of thermal decomposition of Moroccan oil shale by isothermal and non-isothermal thermogravimetry.



The combine use of these measurements has shown that thermal decomposition of oil shale involves two consecutive reactions with bitumen as an intermediate. Both reactions followed the first-order kinetics Dabbous (2012). In order to be able to apply the kinetics model to oil shale retorting problems, one must know something about how the temperature distribution in the retort behave as a function of time and position. An investigation of this problem has been reported by Fausert (1974). Davies (1990). Who tracked an in-situ combustion front using thin flame technique. Schechter and Marchesin (2003). Who construct a two-phase model for oxidation, involving air or oxygen and oil that include heat loss to the rock formation Souza et al.(2006) who studied the Riemann problem with forward combustion due to injection of air into a porous medium containing solid fuel.

Da Mota et al.(2002) used this approach to study combustion front in a two-phase (oil and oxygen) model. Combustion front were identified with travelling waves connecting an unburned state ahead of the front to a burned state behind it. Element analysis and pyrolysis kinetics of oil shales from all over the world were summarized by Nuttal et al (1983). Who observed that there were considerable variations in the kinetic parameter of the different shales. Burnham (2010). Note that these differences are primarily due to the use of poor kinetic analysis methods. The kinetics analysis round table convened to study the kinetic of reactions involving complex solid materials concluded that it was inappropriate to use a single heating rate and a prescribed kinetic model to derive kinetic parameters. The basic flaw in methods which followed this procedure was they resulted in activation energies that were heating rate dependent. By using a variety of computational methods, the panel observed that isoconversion and multi-heating methods were particularly useful in describing kinetics of complex material reactions.

Hubbard and Robinson (1950) interpreted their data by assuming that the total amount of kerogen that decomposed was equal to the total amount of gas, oil, and bitumen formed. Allred (1966) reinterpreted some of Hubbard and Robinson's data by assuming that the decomposition of kerogen and the appearance of carbon residue follow the same functional relation. Burnham and Braun (2005) reviewed various kinetic analysis approaches for obtaining kinetic parameters for reactions involving complex materials. They argue for the use of well-chosen models that are able to fit the data and extrapolate beyond the time-temperature range of the data. For complex materials such as kerogen, generalized distributed reactivity models were found to be suitable. Most of these studies recommended the use of distributed reactivity or similar methods, where the reaction rate is inherently independent of heating rates.

2.1 Methods of Solutions

Here we present the description of the method used in this paper work

2.2 Parameter Expanding Method

Parameter-Expanding method is a powerful tool to solve strongly nonlinear equation.

Recently, considerable attention has been directed towards analytical solution for nonlinear equations without small parameters. There are many analytical methods and numerical schemes to handle non-linear problems, such as variational iteration method He (2006), Swelian and Khader (2007), Swelian et al. (2008), homotopy perturbation method He (2006) Turgut and Yildirim (2007), the expansion function method He and Wu (2006), standard Lindstedt-Poincare He.(2002) and parameter-expansion method Darvishi et al. (2008), Xu (2007). Swelian and Khader (2009). The parameter-expansion method (PEM) which proposed by He is proved to be a very effective and convenient way for handling the non-linear problem.



He.(2001) developed a techniques where a bookkeeping parameter is introduced to the original differential equation. Recently parameter-expanding methods Xu (2007); Mohyud and Noor (2009). Including bookkeeping parameter method He (2001) and modified Lindstedt-Poincare methods Ozis and Yildirm (2007), Ramos (2007) have been caught much attention. The parameter expansion can also be applied to homotopy perturbation method Ozis and Akci (2011).however, there is a class of nonlinear oscillators represent a new class of nonlinear oscillating system which are called non-smooth oscillators play an important role in non-linear dynamics. Conservative non-smooth oscillators such as the ones considered here are governed by which the restoring force is inversely proportional to the dependent variable or restoring force are in form of rational function of dependent variable and they don't include any small parameter. Many new methods, such as variation method by He (2005) and Liu (2005), variation iterations method by J.H. He, X.H.

Abbasbandy (2007). Employed the He's homotopy perturbian technique to solve functional integral equations and the results obtained by Lagrange interpolation formula and homotopy perturbation method were compared. Biazer (2008) reported solution of Helmholtz equation by homotopy perturbation method Faeli et al (2008) investigated the solution of fourth order nonlinear parabolic equation using homotopy perturbation method. Also Ghotbi et al.(2008) obtained the solution of burgers and coupled burgers equation by homotopy perturbation method.

2.3 Oil Shale Extraction Methods

Extraction oil from oil shale required conversion of the solid hydrocarbon in the rock to liquid form, so that they can pumped or processed. This is done by heating the rock to a high temperature, and separating and collecting of the resultant liquid. This heating process is called retorting

2.4 Surface Processing

Surface processing also commonly referred to as surface retorting has traditionally been the more common of the two processes. The process basically entails three step:

- i. Mining of the oil shale: mining of the oil shale can be done using traditional mining methods, either by open pit mining or underground mining (sometimes called room-and-pillar method)
- ii. Thermal processing or retorting above ground
- iii. Processing of the shale oil to obtain a refinery feedstock and value added byproducts, disposal of the spent shale

Disadvantage of Surface processing

- i. Both mining methods have a large land impact and consume large amount of water (as the process require water for operation and also requires pumping out groundwater to prevent flooding of the mine)
- ii. Surface (or open-pit) mining involves a considerable land impact while Room-and-pillar mining methods are considered inefficient: approximately one-third of the resources are left behind in pillars and/or unmined areas. In fact, the thicker the resources the less efficient the mining process becomes.
- iii. Disposal of the waste shale is a major problem for some processes, requiring large quantities of water

2.5 In-Situ Retorting

In-situ is the technology for processing oil shale underground. This process obviates the problem of mining, handling, and disposing of large quantities of material, which occur for above ground retorting. In-situ retorting also offers the potential of recovering deeply deposited oil shale. In-situ process, the oil shale is slowly heated underground and the resultant liquid and gas are being extracted directly from the reservoir, in a manner not different from pumping crude



oil. Utilizing slow heating methods and lower heating requirements than those used in surface retorting holds another key advantage: the shale oil produced will be of a superior quality that produced in above-ground retort facilities, materially reducing the upgrade requirements before delivery to refineries can take place. In-situ technology has been successfully demonstrated on a small scale. Various technologies differ by the method used to introduce heat underground, but follow the same basic principle.

3. MATERIAL AND METHODS

3.1 Mathematical Formulation

The kinetic parameters of evolution during oil shale pyrolysis were obtained by the method of expansion techniques. In the case of a simple first order reaction such as the rate of product evolution is

$$\frac{dc}{dT} = \left[\frac{Ac_0}{H} \right] \exp \left[\frac{-E}{RT} - \frac{A}{H} \int_0^T \exp \left[\frac{-E}{RT} \right] dT \right] \quad (3.1)$$

Where T is the temperature, H is the heating rate, E and A represent the activation energy and Arrhenius pre-exponential factor respectively and R is the gas constant.

Equation (3.1) is typically simplified using the approximation:

$$\int_0^T \exp \left[\frac{-E}{RT} \right] dT = \left[\frac{RT^2}{E} \right] \exp \left[\frac{-E}{RT} \right] \quad (3.2)$$

Substituting equation (3.2) into (3.1) we have

$$\frac{dc}{dT} = \left[\frac{Ac_0}{H} \right] \exp \left[\frac{-E}{RT} - \left[\frac{ART^2}{HE} \right] \exp \left[\frac{-E}{RT} \right] \right] \quad (3.3)$$

The initial condition is formulated as

$$c(T_0) = c_0 \quad (3.4)$$

Where;

- A Arrhenius pre-exponential factor
- E Activation energy
- H Heating rate
- R Gas constant
- T Temperature [°C]



3.2. Non-Dimensionilization

Here, we non-dimensionalize equation (3.1)-(3.4) using the following dimensionless variables

$$\phi = \frac{c}{c_0}, \theta = \frac{E}{RT_0^2}(T - T_0) \quad (3.5)$$

Then;

$$T = \frac{R}{E}T_0^2\theta + T_0, \partial T = \epsilon T_0\partial\theta \quad (3.6)$$

$$c = \phi c_0, \partial c = c_0\partial\phi \quad (3.7)$$

Now

$$\begin{aligned} T &= ET_0\theta + T_0 \\ e^{\frac{-E}{RT}} &= e^{\frac{-E}{RT_0}} \cdot e^{\frac{-E}{RT}} \cdot e^{\frac{-E}{RT_0}} \\ e^{\frac{-E}{RT}} &= e^{\frac{-E}{RT_0}} \cdot e^{\frac{E}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)} \\ e^{\frac{-E}{RT}} &= e^{\frac{-E}{RT_0}} \cdot e^{\frac{E}{R}\left(\frac{T-T_0}{T_0T}\right)} \\ e^{\frac{-E}{RT}} &= e^{\frac{-E}{RT_0}} \cdot e^{\frac{E}{R}\left(\frac{ET_0\theta}{T_0(1+E\theta)}\right)} \\ e^{\frac{-E}{RT}} &= e^{\frac{-E}{RT_0}} \cdot e^{\frac{\theta}{1+E\theta}} \end{aligned} \quad (3.8)$$



Also

$$\begin{aligned}
 -\frac{E}{RT} &= -\frac{E}{RT} + \frac{E}{RT} - \frac{E}{RT} \\
 -\frac{E}{RT} &= -\frac{E}{RT} + \frac{E}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \\
 -\frac{E}{RT} &= -\frac{E}{RT} + \frac{E}{R} \left(\frac{T - T_0}{TT_0} \right) \\
 -\frac{E}{RT} &= -\frac{E}{RT} + \frac{E}{RT_0} \left(\frac{ET_0\theta}{ET_0\theta + T_0} \right) \\
 -\frac{E}{RT} &= -\frac{E}{RT} + \epsilon \left(\frac{\theta}{1 + \epsilon\theta} \right) \\
 -\frac{E}{RT} &= -\frac{1}{\epsilon} + \epsilon \left(\frac{\theta}{1 + \epsilon\theta} \right)
 \end{aligned}
 \tag{3.9}$$

Now

$$\begin{aligned}
 \frac{ART^2}{HE} &= \frac{AR}{HE} (\epsilon T_0\theta + T)^2 \\
 \frac{ART^2}{HE} &= \frac{AR}{HE} ((\epsilon T_0)^2 \theta^2 + 2\epsilon T_0^2 \theta + T_0^2) \\
 \frac{ART^2}{HE} &= \frac{ART_0^2}{HE} (\epsilon^2 \theta^2 + 2\epsilon\theta + 1) \\
 \frac{ART^2}{HE} &= \frac{ART_0^2}{HE} (1 + 2\epsilon\theta + \epsilon^2 \theta^2)
 \end{aligned}
 \tag{3.10}$$



Substituting equation (3.6)-(3.10) into (3.3), we have

$$\frac{c_0}{\epsilon T_0} \frac{\partial \phi}{\partial \theta} = \left[\frac{Ac_0}{H} \right] \exp \left(-\frac{1}{\epsilon} + \frac{\theta}{1+\epsilon\theta} - \beta(1+2\epsilon\theta+\epsilon^2\theta^2) e^{\frac{\theta}{1+\epsilon\theta}} \right)$$

ie

$$\frac{\partial \phi}{\partial \theta} = \left[\frac{A\epsilon T_0}{H} \right] \exp \left(-\frac{1}{\epsilon} + \frac{\theta}{1+\epsilon\theta} - \beta(1+2\epsilon\theta+\epsilon^2\theta^2) e^{\frac{\theta}{1+\epsilon\theta}} \right)$$

ie

$$\frac{\partial \phi}{\partial \theta} = \alpha \exp \left(-\frac{1}{\epsilon} + \frac{\theta}{1+\epsilon\theta} - \beta(1+2\epsilon\theta+\epsilon^2\theta^2) e^{\frac{\theta}{1+\epsilon\theta}} \right) \tag{3.11}$$

Where

$$\alpha = \frac{A\epsilon T_0}{H}, \beta = \frac{ART_0^2}{HE} e^{-\frac{E}{RT_0}} \tag{3.12}$$

Therefore, the dimensionalized equation and boundary conditions are

$$\left. \begin{aligned} \frac{\partial \phi}{\partial \theta} &= \alpha \exp \left(-\frac{1}{\epsilon} + \frac{\theta}{1+\epsilon\theta} - \beta(1+2\epsilon\theta+\epsilon^2\theta^2) e^{\frac{\theta}{1+\epsilon\theta}} \right) \\ c(T_0) &= c_0 \\ \theta &= \frac{E}{RT_0^2} (T_0 - T) = 0 \\ c(T_0) &= c_0, \phi(0) = c_0 \\ \text{ie} \\ \phi(0) &= 1 \end{aligned} \right\} \tag{3.13}$$



3.3. Expansion Method

Expanding the right hand side of equation (3.13) by taking

$$e^{1+\epsilon\theta} \approx 1 + (e-2)\theta \quad (3.14)$$

We have

$$\frac{\partial\phi}{\partial\theta} = \alpha \exp\left(-\frac{1}{\epsilon} + \frac{\theta}{1+\epsilon\theta} - \beta(1+2\epsilon\theta+\epsilon^2\theta^2)(1+(e-2)\theta)\right) \quad (3.15)$$

i.e

$$\frac{\partial\phi}{\partial\theta} = \alpha \exp\left(-\frac{1}{\epsilon} + \theta(1-\epsilon\theta) - \beta(1+(e-2)\theta+2\epsilon\theta+2\epsilon\theta(e-2)\theta+\epsilon^2\theta^2+(e-2)\theta\epsilon^2\theta^2)\right) \quad (3.16)$$

$$\frac{\partial\phi}{\partial\theta} = \alpha \exp\left(-\frac{1}{\epsilon} + \theta - \epsilon\theta^2 - \beta - \beta(e-2+2\epsilon)\theta + (2\epsilon(e-2)+\epsilon^2)\theta^2 + \epsilon^2(e-2)\theta^3\right) \quad (3.17)$$

Integrating both sides we have

$$\phi(\theta) = \int \alpha \exp\left(-\frac{1}{\epsilon} + \theta - \epsilon\theta^2 - \beta - \beta(e-2+2\epsilon)\theta + (2\epsilon(e-2)+\epsilon^2)\theta^2 + \epsilon^2(e-2)\theta^3\right) d\theta + C_1 \quad (3.18)$$

Neglecting θ^3 , we have

$$\phi(\theta) = \int \alpha e^{(-A+B\theta+C\theta^2)} d\theta + C_1 \quad (3.19)$$

Where

$$\begin{aligned} A &= \beta + \frac{1}{\epsilon} \\ B &= (1 - \beta(e-2) + 2\epsilon) \\ C &= (\epsilon^2 + 2\epsilon(e-2) - \epsilon) \end{aligned} \quad (3.20)$$

$$\phi(\theta) = -\frac{1}{2} \frac{\alpha \sqrt{\pi} e^{-A - \frac{1}{4} \frac{B^2}{C}} \operatorname{erf}\left(-\sqrt{-C}\theta + \frac{1}{2} \frac{B}{\sqrt{-C}}\right)}{\sqrt{-C}} + C_1 \quad (3.21)$$



At

$$\phi(0) = 1$$

$$C_1 = 1 + \frac{1}{2} \frac{\alpha \sqrt{\pi} e^{-A - \frac{1}{4} \frac{B^2}{C}} \operatorname{erf}\left(-\sqrt{-C}\theta + \frac{1}{2} \frac{B}{\sqrt{-C}}\right)}{\sqrt{-C}} \quad (3.22)$$

Therefore the general solution to equation (3.13) is

$$\phi(\theta) = -\frac{1}{2} \frac{\alpha \sqrt{\pi} e^{-A - \frac{1}{4} \frac{B^2}{C}} \operatorname{erf}\left(-\sqrt{-C}\theta + \frac{1}{2} \frac{B}{\sqrt{-C}}\right)}{\sqrt{-C}} + 1 + \frac{1}{2} \frac{\alpha \sqrt{\pi} e^{-A - \frac{1}{4} \frac{B^2}{C}} \operatorname{erf}\left(-\sqrt{-C}\theta + \frac{1}{2} \frac{B}{\sqrt{-C}}\right)}{\sqrt{-C}} \quad (3.23)$$

4. RESULT AND DISCUSSION

4.1. Analysis of Results

The system of partial differential equations describing the modelling of Oil Shale Pyrolysis which are solved analytically using expansion method technique. The analytical solution of equation (3.13) are computed for the values of $\beta = 0.1$, $\epsilon = 0.4$, $\alpha = 0.1$ and $\theta = 0..1$

The numerical result for the model of Oil Shale Pyrolysis using computer algebraic symbolic package MAPLE 16 are displayed in figure 4.1—4.3

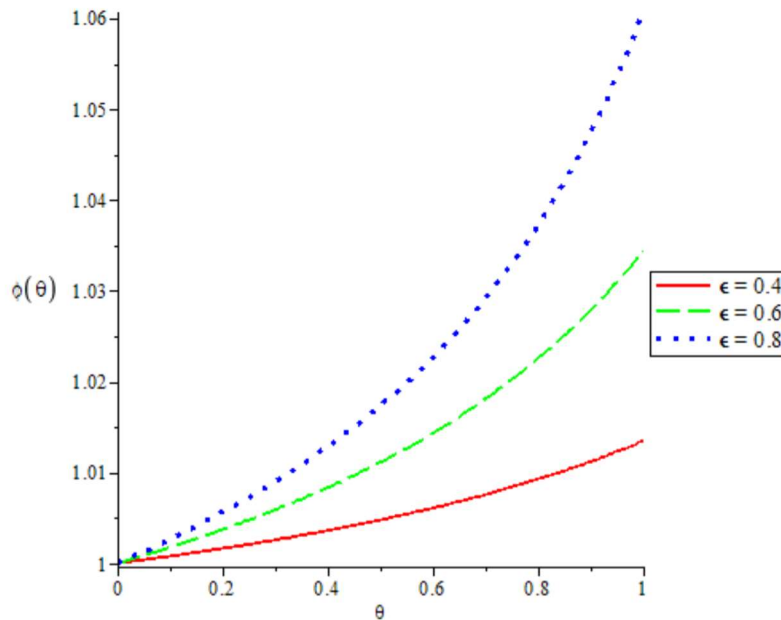


Figure 4.1: variation of the concentration of oil shale pyrolysis $\phi(\theta)$ with dimensional activation energy ϵ



Figure 4.1 shows the effect of dimensional activation energy ϵ on the concentration of oil shale pyrolysis. It is observed that the concentration of oil shale pyrolysis $\varphi(\theta)$ increases with temperature θ and increases with dimensional activation energy ϵ .

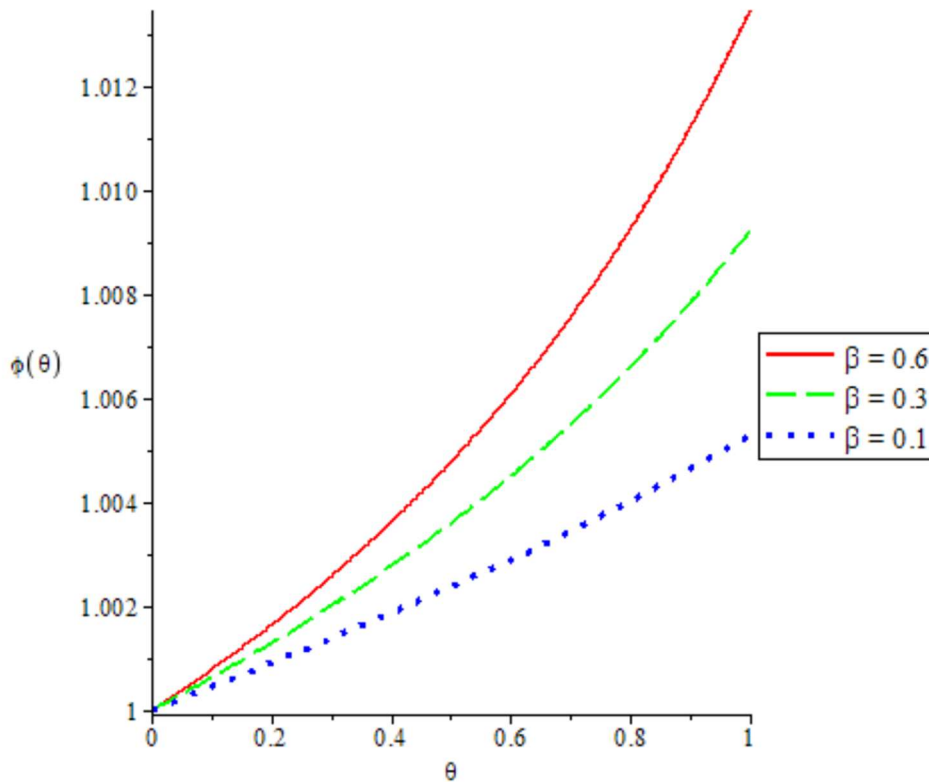


Figure 4.2: variation of the concentration oil shale pyrolysis $\varphi(\theta)$ with heating rate parameter β

Figure 4.2 shows the effect of heating rate parameter β on the concentration of oil shale pyrolysis. It is observed that the concentration of oil shale pyrolysis $\varphi(\theta)$ increases with temperature θ and increases with heating rate parameter β .

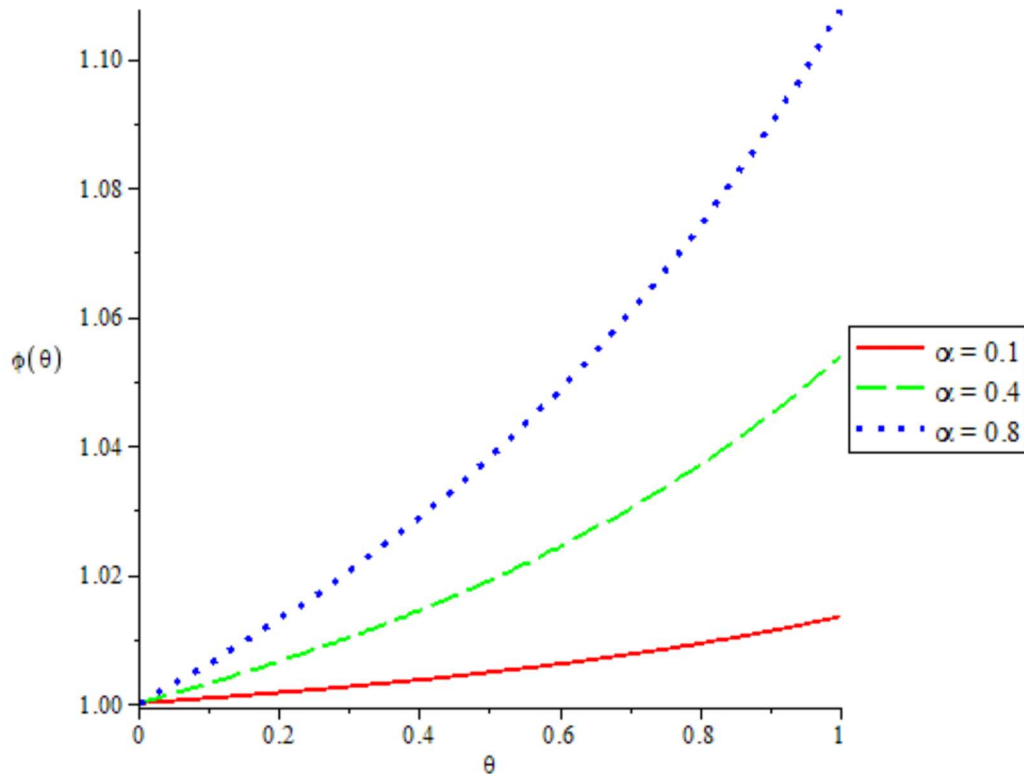


Figure 4.3: variation of the concentration oil shale pyrolysis $\phi(\theta)$ with Arrhenius pre-exponential factor α

Figure 4.2 shows the effect of Arrhenius pre-exponential factor α on the concentration of oil shale pyrolysis. It is observed that the concentration of oil shale pyrolysis $\phi(\theta)$ increases with temperature θ and increases with Arrhenius pre-exponential factor α .

5. CONCLUSION AND RECOMMENDATIONS

5.1. Conclusion

In this study we have formulated and solved analytically a mathematical model of oil shale pyrolysis. In particular, we solved the resulting equations using the expansion method techniques. Finally, we have proved the graphical summaries of the system response.

5.2. Recommendation

This work is open to researchers to study the effect of other organic solvent on the yield of oil produced, and also applying of other techniques to extract the oil from the shale (e.g steam).



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