

## A Financial Option Based Price and Risk Management Model for Pricing Electrical Energy in Nigeria

Allenotor, D. & Ojugo, A.A.

Department of Mathematics/Computer Science  
Federal University of Petroleum Resources  
Effurun, Delta State, Nigeria

[allenotor.david@fupre.edu.ng](mailto:allenotor.david@fupre.edu.ng) and [ojugo.arnold@fupre.edu.ng](mailto:ojugo.arnold@fupre.edu.ng)

### ABSTRACT

Abstract—An option based financial risk assessment is a requirement for users as well as managers of deregulated electricity markets. This is as a result of the extensive price and volume risks associated with the deregulated energy industry. Additionally, the presence of high volatility at times of peak demand and supply shortage increases the complexity of pricing wholesale electricity markets which are partly dependent on characteristics such as generation, distribution, heterogeneous demand, weather patterns, and more importantly, pricing. In Nigeria for example, the electricity market is volatile especially during peak demand with heavy supply shortages. This condition puts many users into several hours/days without electricity. The energy sector (electricity) was privatized in an attempt to solve this problem, but it did not capture flexibility in pricing of electricity. Rather, tariffs and static payment schemes were re-calibrated. One of the problems associated with static pricing of electricity is that the prices charged are not commensurate to the Quality of Service (QoS) derived by the consumers. To mitigate the risk in pricing electricity, we design Transmission Rights (TRs) for electricity in Nigeria and model it as a financial option pricing problem. We then use a nature inspired meta-heuristic algorithm, Ant Colony Optimization (ACO) to compute option prices and determine future TRs payouts.

**Keywords** – Financial option, Electrical Energy, Ant Colony Optimization, Transmission Rights, Option Value.

### Aims Research Journal Reference Format:

Allenotor, D. & Ojugo, A.A. (2017): A Financial Option Based Price and Risk Management Model for Pricing Electrical Energy in Nigeria. *Advances in Multidisciplinary & Scientific Research Journal*. Vol. 3. No.2, Pp 79-90

### 1. INTRODUCTION

Two key issues characterize the Nigerian power sector; (a) power outages leading to a poor Quality of Service (QoS) and (b) the lack of ability of the energy operators to bill electricity consumers' charges that are commensurate to the power they use. These challenges have compelled the power sector to take immediate reformation action. As a result, the Electric Power Sector Reform Act of 2005 unbundled the national power utility company into a series of 18 successor companies: six generation companies, 12 distribution companies covering all 36 Nigerian states, and a national power transmission company. In September 2013, the Federal Government of Nigeria initiated a privatization process for the energy industry. The result of the privatization process caused the Power Holding Company of Nigeria (PHCN) to cease to exist and originated the independent Nigerian Electricity Regulatory Commission (NERC). The NERC regulatory agency, as provided in the Electric Power Sector Reform Act of 2005 is tasked with monitoring and regulating the Nigerian electricity industry; with authority to issue licenses to market participants, and to ensure compliance with market rules and operating guidelines. This operating guideline in nature also documents the Service Level Agreements (SLAs) for the electricity markets.

These include tariffs, charges, and market rules. Additional statutory functions of the commission as contained in Section 32 (d) of the Electric Power Sector Reform (EPSR) Act, 2005 is to ensure a state of equilibrium for the provider and the consumer. That is, the prices charged by licensees should be fair to customers and sufficient enough to allow the licensees to finance their activities and to allow for reasonable earnings for efficient operation. To meet this regulatory mandate, the commission established a methodology for regulating electricity prices called the Multi-Year Tariff Order (MYTO).

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<sup>1</sup>We acknowledge gratefully the Tertiary Education Trust Fund (TetFund) for the full funding of this research under the Institution Based Research (IBR) research efforts.

The MYTO system provides a 15-year tariff pathway for the electricity industry with minor reviews each year [18]. Such reviews with stakeholder focus on changes in a limited number of parameters (such as inflation and gas prices). While developing the energy infrastructures, the government statement on privatization did not include flexible pricing model. They only indirectly indicated the cost of electricity by way of tariffs.

In the real world, trading in wholesale deregulated electricity market involves risk and financial risk management and it is at high priority for market participants. Many other economies (such as the US, Canada, and United Kingdom) that record successful energy markets modeled their energy markets using Financial Transmission Right (FTR). FTRs are financial instruments that entitle the holder to rebates of congestion charges paid by the Firm Transmission Service Customers. Market participants acquire FTRs in the form of financial options or obligations. They do not represent a right for physical delivery of power. The holder of the FTR is not required to deliver energy in order to receive a congestion credit. If a constraint exists on the transmission system in the day-ahead market, the holders of FTRs receive a credit based on the FTR Mega Watt (MW) reservation and the congestion price difference between point of delivery and point of receipt.

This credit is paid to the holder regardless of who delivered energy or the amount delivered across the path designated in the FTR. The consequences of the complexity associated with the wholesales in electricity market is that spot prices often present high volatility during peak activities of demand and supply (electricity markets are characterized generation, demand and supply, policy, weather patterns, and usage behavior). This is one of the challenges that face the Nigerian energy market.

### 1.1 Non-Storability Property of Electricity

Electricity, by its nature is difficult to store. However, consumers expect electricity to be available on demand. As a consequence, unlike the other energy and non-energy commodities, it is not possible, under normal operating conditions, to keep electricity in stock, ration it or have consumers queue for it. Continuously varying demand and supply further exacerbates the situation. In more technical terms, the transmission system operators are the results of the physical requirement for a controlling agency to coordinate the power generator facilities to meet the expected demand in the marketplace. In case there is a mismatch between supply and demand, the generators will speed up or slow down. In spite of advanced technologies available on the production side, still, the laws of physics rule the market.

From the foregoing, electricity market is characterized by similar attributes of stocks, shares, and bonds. Hence, we apply FTRs which provide a financial hedge for markets participants in a deregulated electricity market to price electrical energy consumed. FTRs are financial instruments that entitle the holder to a stream of revenues based on the hourly congestion price difference across a transmission path. FTR pricing is challenging because traditional methods such as market forces of demand and supply does not capture the realistic prices of electrical energy. The novelty of this research is two-fold; (a) to model FTR as a financial option pricing problem to price electrical energy for the Nigerian economy and (b) to reach equilibrium between the electricity provider and the consumers for profitability in terms of revenue to the electricity provider and profitability to the consumers in terms of Quality of Service (QoS) in the Service Level Agreement (SLA) document.

The remaining parts of this research is organized in the following sections. In Section 2, we explore the related research efforts of methodology of pricing electricity markets and establish why pricing FTRs in Nigeria is an important problem. Section 3 provides the relationship between financial option pricing and electricity markets. In Section 4, we propose the FTRs model as a special case of the Nigerian economy. We also do a mapping of the FTRs to financial option pricing algorithm. Section 5 describes our implementation and in Section 6, we present and discuss our results.

## 2. RELATED WORK

A large corpus of related work has applied financial option to price financial derivatives. One of the most prominent is the popular Black-Scholes Model (BSM) [2]. The BSM is applied where we experience continuous price jumps (i.e., where the price volatility of the price is high). The BSM requires solving a partial differential equation before the price movements can be computed. The pricing issues of the options on commodity forwards were first addressed in [3]. The stochastic behavior of the commodity prices was examined by means of a mean-reverting model in [4], however, the model excluded jumps which is an important factor to pricing. Lucia and Schwartz in [5] further extends this model to account for a deterministic seasonality while Clewlow and Strickland in [6] incorporated both mean-reversion and jumps without a closed-form solution for the forward. A model that captures the most important characteristics of electricity spot prices at the same time was presented in [7]. The binomial lattice has also been used to approximate discrete jump in price. One of the areas where the binomial has been used is in the deregulation of electricity market in order to design an efficient distributed resource given uncertainty in availability.

Allenator and Thulasiram in [8] considered the risk factors associated with pricing grid resources. They focused on obtaining an equilibrium between the provider of the grid resources and the user of the grid resources. In a similar manner, Wang et al. [17] proposed to apply a combination of spot instances as well as option instances to mitigate price risks. Wang et al. in [17] characterized the cost of European options. European options cannot be exercised any time before the expiration date even when the option value at the prevailing time is optimal or profitable. However, the American option that we propose to apply in this research, gives the user a handle of opportunity to exercise the option at any time before the expiration date.

### 2.1 Current Cost of Electricity in Nigeria

The current cost of electricity for customers in Nigeria is computed on static basis using four parameters: (i) The location of the customer – there are 11 electricity Distribution Companies (DisCos) operating in Nigeria, (ii) The tariff class of the customer – every electricity customer must belong to a specific tariff class. There are 5 major tariff classes each of which is divided into sub-classes based on a range of energy consumption. Table 1 shows the 5 major tariff classes. (iii) Tariff rate – this depends on the customer’s tariff class. Each tariff class has a unique tariff rate applicable to that class, and (iv) The quantity of energy consumed in KWh.

**Table 1: Tariff Classes**

Tariff Code	Class	Description
A	Residential	A customer who uses his premise exclusively as a residence - house, flat, or multi-storied house.
B	Commercial	A customer who uses his premise for any purpose other than exclusively as a residence or as a factory for manufacturing goods.
C	Industrial	A customer who uses his premises for manufacturing goods including welding and ironmongery.
D	Special	Customers such as agriculture and agro-allied industries, water boards, religious houses, government and teaching hospitals, government research institutes and educational establishments.
E	Street Lights	Street Lights.

In Nigeria, the electricity market is volatile especially during peak demand with heavy supply shortages. In the first instance therefore, supply does not equal demand (demand crushes above supply). This condition puts many communities and cities into several hours/days without electricity. When the government policy of privatization of the electrical sector was declared, it did not include pricing model. Instead, it calibrated tariffs and static payment schemes. However, the electricity markets around the world and in Nigeria in particular is characterized by volatility. The tariffing system is depended on the forces of policies as documented by the NERC regulatory agency in the Electric Power Sector Reform Act of 2005 [16]. The NERC is only tasked with monitoring and regulating the Nigerian electricity industry; with authority to issue licenses to market participants, and to ensure compliance with market rules and operating guidelines. It does not have a model to price the energy levels comparative to what is obtainable in the US, Canada, and UK. Therefore, the problem with the existing charging scheme is that the prices charged are not commensurate to the QoS derived by the consumers. In most cases, the prevailing condition us used to determine the value of energy demanded and hence, charged in what is popularly called estimated billing.

### 3. FINANCIAL OPTION PRICING

In financial option pricing, the prevailing condition does not determine price. This means that while a contract to buy and sell electricity is entered with an expiring date, the holder of the option cannot change the price (in contrast to the estimated billing system). This enable the holder of a put option to actually use the contract price to make business decision for the future without fear that it might rise. Existing pricing policies cannot determine the volatility of the energy market. The general idea or concept of financial option has been around for long time. In early writings, owners of olive presses sold rights to use olive presses during harvest months. Selling these rights provided necessary income during winter months. On other hand, buyer of the rights ensured time to use olive presses during busy months [15]. For decades, financial options have been used by traders and were meant for people with specialized needs and information. A financial option is a contract in which the buyer (holder of an option) has the right but no obligation to buy (with call option) or sell (with put option) an underlying asset (for example, a stock) at a predetermined price (strike price) on or before a specified date (expiration date). The seller (known as writer) has the obligation to honor the terms specified in the financial option contract. The holder pays a premium to the writer ([1], [13], and [14]). There are two types of financial options (call and put) and financial options can be exercised in various styles that are broadly grouped into two categories, vanilla options and exotic options:

A European option can be exercised only at the expiration date whereas an American option may be exercised on any date before the expiration date. Since the American options provides added flexibility, their premium is generally higher than European options. These two generally comprise vanilla options due to their simpler nature. Exotic options are complex. These include Asian (based on some average asset price), barrier (looking for first stopping time), Bermudan (having few exercise points during the contract period), Russian (expiration time itself is floating) and many other styles. These and other comprise exotic options. Most of the strategies for increasing assets involve investing in financial markets, which allows an investor to invest in equities such as company stocks.

### 4. BINOMIAL LATTICE OPTION PRICING MODEL

One of the early models for financial option pricing is the Nobel prize winning Black-Scholes-Merton (BSM) model [9] [10]. Following a continuous-time approach, they developed a model to alleviate risk involved in financial investments and rendered it as a stochastic Partial Differential Equation (PDE) (Equation 1).

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \quad (1)$$

where  $V$  is the price of the option as a function of asset price  $S$  and time  $t$ ,  $r$  is the risk-free interest rate, and  $\sigma$  is the volatility of the asset. This model makes some assumptions as follows:

- i. Financial options are European and can only be exercised at expiration.
- ii. No dividends are paid out during the life of the contract.
- iii. Efficient markets (market movements cannot be predicted).
- iv. No commissions are charged.
- v. Risk-free interest rate and volatility of the underlying asset are known and constant.
- vi. Follows a lognormal distribution (returns on the underlying asset are normally distributed).

This model provided closed-form solution for simplified option for solving PDE based on the Cox, Ross and Rubenstein (CRR) [10]. The CRR binomial model is a simple and intuitive numerical technique designed to price financial options. The binomial option pricing model has proved over time to be the most flexible, and popular approach to price financial options. If constructed assuming the same initial conditions, binomial model agrees asymptotically with the Black-Scholes model. Moreover, binomial model can be used for pricing American style options to determine early exercise possibilities.

An American option can be exercised anytime during contract period, compared to European options which can only be exercised at expiration time. The standard binomial option pricing model assumes that the binomial tree recombines with constant volatility, constant risk-free interest and constant payout return. However, using the CRR model these conditions could be relaxed unlike in the BSM Model. As a result of these flexibilities, we considered the CRR model to map TRs to a financial option pricing.

The CRR model uses a binomial tree structure to price financial options. The time between valuation (current) date and expiration date is divided into a certain number of (time) steps. Each node in the tree represents a possible price of the stock (underlying asset) at a particular time. In binomial method, knowing the asset price is the basis for computing the financial option value. After building the tree with asset price distribution, it starts from the leaf nodes and works backwards towards the root node, which represents the valuation date. The financial option price at the valuation date is calculated by computing pay-off at all the intermediate nodes between expiration date and the valuation date.

We compute the financial option using the following parameters: asset price  $S$ , strike price  $K$ , time to maturity  $T$ , interest rate  $r$ , number of steps  $N$ , interval time between two steps  $\Delta t$ , volatility  $\sigma$ , probability  $p$ , upward movement  $u$ , and the downward movement:  $d$ . Using Equation (2) and Equation (3) which comes from the expectation that over a small period of time the binomial model should behave in the same way as an asset in a risk neutral world, we build the binomial lattice.

$$pu + (1 - p)d = e^{r\Delta t} \quad (2)$$

Equation 2 makes sure that over the small period of time ( $\Delta t$ ) the expected return of the binomial model matches the expected return in a risk-neutral world.

Equation 3 ensures that over the small period of time ( $\Delta t$ ) the expected variance of the binomial model matches the expected variance of a risk-neutral world.

$$pu^2 + (1 - p)d^2 - (e^{r\Delta t})^2 = \sigma^2 \quad (3)$$

The CRR model [10] is the discretized version of Black-Scholes-Merton (BSM) model [2]. We start with the basic CRR [10] model and use it to analyze price fluctuation. The time period  $T$  is divided into  $N$  smaller intervals:  $t_1, t_2, \dots, t_N$ . Suppose  $S_n$  is the price at the  $n$ -th step, then the CRR model defines the price at the next step  $S_{n+1}$  such that:

$$S_{n+1} = \begin{cases} uS_n & \text{with a probability } p \\ dS_n & \text{with a probability } q = 1 - p \end{cases} \quad (4)$$

where  $u (> 1)$  and  $0 < d < 1$  are the two price movement factors. we compute the option price by building a discrete time and state binomial model of the asset price and then apply the discounted expectations. Figure 1 shows the one-step binomial.  $S_0$  is the current asset price (today at the node A). In a given short duration,  $\Delta t$ , the asset price  $S_0$  can either go up with a probability of  $pu$  to a new level  $uS_0$  (of node B) or go down with a probability  $pd$  to a new level  $dS_0$  (of node C). When the price moves up to  $uS_0$ , the pay-off for a call option at node B is  $f_u$  ( $\max(0, uS_0 - K)$ , where  $K$  is the strike price of the option) for a call option and when the price moves down to  $dS_0$ , the pay-off at node C  $f_d$  ( $\max(0, dS_0 - K)$ , where  $K$  is the strike price of the option). The BSM model sets up a riskless portfolio that consists of  $\Delta$ , the number of underlying assets, and an option such that the portfolio value will be the same regardless of whether the asset price goes up or down over the period  $\delta t$ . That is, the option value at the end of the contract term  $T$  ( $= \delta t$  in one-step binomial tree) for the up movement is  $S_0 u \delta - f_u$  and for the downward movement, its given as:

$$S_0 d \delta - f_d.$$

That is:

$$S_0 u \delta - f_u = S_0 d \delta - f_d \quad (5)$$

$$\Delta = \frac{f_u - f_d}{S_0(u - d)} \quad (6)$$

where  $\delta$  is the ratio of the change in option price to the change in asset price. The variables  $u$  and  $d$  are the factors by which the asset price moves up or down and are expressed as  $u = e^{\sigma \delta t}$  and  $d = e^{-\sigma \delta t}$  [10]. The current value of the portfolio for this future option price can be given by the expression,  $(S_0 \Delta - f_u) e^{r \delta t}$  where  $r$  is the risk-free interest rate. Given that the cost of setting up the portfolio is  $(S_0 \delta - f)$ , the riskless portfolio is:

$$S_0 \Delta - f = (S_0 \Delta - f_u) e^{-r \Delta t} \quad (7)$$

and substituting in  $\Delta$  in  $u = e^{\sigma \Delta t}$  into

$$S_0 \Delta - f = (S_0 \Delta - f_u) e^{-r \Delta t}, \text{ we have:}$$

$$f = e^{-r \Delta t} (p f_u + (1 - p) f_d) \quad (8)$$

where  $p = \frac{e^{r \Delta t} - d}{(u - d)}$  is called the probability of an up movement in the asset price and  $p f_u + (1 - p) f_d$  is the weighted sum of the future pay-off and  $f$  is the current value of the option. In other words, the value of an option is the discounted value of the weighted sum of the future pay-off. Figure 2 shows a two-step binomial tree and the pay-off and option values are given by:

$$f_u = e^{-r \Delta t} (p f_{uu} + (1 - p) f_{ud}) \quad (9)$$

$$f_d = e^{-r \Delta t} (p f_{ud} + (1 - p) f_{dd}) \quad (10)$$

$$f = e^{-2r \Delta t} (p^2 f_{uu} + 2p(1 - p) f_{ud}) + (1 - p)^2 f_{dd} \quad (11)$$

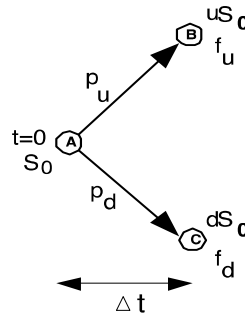


Figure 1: One –Step Binomial.

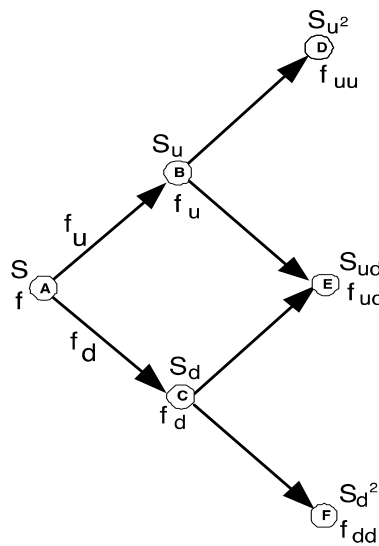


Figure 2: Two-Step Binomial Tree.



It is easy to observe that the down movement is assumed to be inversely proportional to the up movement, to generate a recombining tree. Financial option value at the expiration date: The computation of financial option value starts at the leaf nodes (maturity date) of the binomial tree. The value of the financial option at the leaf nodes for a call option is computed by  $\max [(S - K); 0]$  and for a put option it is  $\max [(K - S); 0]$ . Computation of the financial option at intermediate nodes –the value of the financial option is computed at the leaf nodes by working backward (Figure 1) towards the root node (valuation date) gives the value of the financial option. The financial option value at a node is computed using the financial option values of the two children nodes (Sud and fud) weighted by respective probabilities in Equation (9), Equation (10), and Equation (11). Figure 2 shows the extended binomial lattice for two-stop tree.

For the European option, the value at each node is simply the financial option value computed using Equation (9) and Equation (10). The value at the root node is the value of the financial option on the valuation day, computed using Equation (11). For the American option, the value at each node is  $\max [\text{option value based on discounting}; \text{local pay-off}]$  where local pay-off is  $\max (S-K,0)$  for a call option or  $\max (K-S,0)$  for a put option. This step allows to identify the best possible time to exercise the American option.

#### 4.1 Pricing Transmission Rights as an Option Pricing Problem

An option is a contract in which the buyer (holder of an option) has the right but no obligation to buy (with call option) or sell (with put option) an underlying asset (for example, a stock) at a predetermined price (strike price) on or before a specified date (expiration date). The seller (also known as writer) has the obligation to honor the holder's right specified in the option contract. The holder pays a premium to the writer (see for example [1]). We formulate TR as a financial option pricing problem in the solution/problem domain is described by nodes that capture price at various times. Each node represents a time instance with same electricity price at source and at sink for a TR. Every node (except at expiration time) is connected to a set of possible next instances of times (nodes) which may occur based on electricity price forecast. In this way, a TR can be represented from start time to expiration time using nodes and their relationship to next time interval with two electricity prices. In ideal world with no transmission constraints these two prices will be same all the time during start and expiration. However, congestion on transmission line is a reality which can be caused by many reasons. Normally congestion occurs because of de-rating of transmission line, during abnormal load or generation times etc. In our study, we observed that when congestion occurs, it spans across many hours even days rather than a short event.

To model congestion therefore, a random node is selected in solution space. This particular node and its neighbors are marked as congested time instances. Marking nodes as congested nodes means that electricity price at source is lower compared to electricity price at sink. Electricity price at source is changed for congested nodes to reflect congestion. This process is repeated until a desired level of congestion is achieved based on historical analysis of wholesale electricity market data. This special condition may be visualized as a financial Bermudan<sup>1</sup> option with electricity price at source as strike price. However, unlike Bermudan option, exercise times are not previously known in case of a TR. There will be multiple instances between start time and expiration time where this financial Bermudan option will be exercised. Algorithm need to be designed to search this problem space to find payouts to determine price of a TR.

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<sup>1</sup> Bermudan option is a type of exotic option which can be exercised at predetermined times.

## 5. FEASIBILITY STUDY AND IMPLEMENTATION

The data used for input to our model was collected on site. We obtained electricity billing and usage in the Abuja zone (Abuja Electricity Distribution Plc (AEDC)) and the Benin zone (Benin Electricity Distribution Plc (BEDC)). The Abuja zone capture billing for the Nassarawa, Niger, Kogi states and FCT while for the Benin zone, we captured usage and billing for Edo, Delta, Ondo, and Ekiti states. Table 2 shows the Benin Distribution Company (Benin DisCo) for class of tariff for up to 2024 and Table 3 shows the Abuja zone DisCo class of tariff for up to 2024.

**Table 2: Benin DisCo Tariffs – ₦/kWh**

Class	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
R1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
R2S	14.82	24.08	31.27	31.26	30.98	30.88	27.29	24.49	24.34	24.14
R2T	14.82	24.45	34.40	34.40	34.08	33.97	30.02	26.94	26.78	26.56
R3	26.52	38.56	40.46	40.46	40.08	39.96	35.32	31.69	31.50	31.24
R4	26.52	38.56	40.46	40.46	40.08	39.96	35.32	31.69	31.50	31.24
C1S	20.72	33.87	34.90	34.90	34.58	34.47	30.46	27.34	27.17	26.95
C1T	20.72	34.19	36.27	36.27	35.94	35.83	31.66	28.41	28.24	28.01
C2	24.65	40.67	38.11	38.11	37.76	37.64	33.27	29.85	29.67	29.43
C3	24.65	40.67	38.11	38.11	37.76	37.64	33.27	29.85	29.67	29.43
D1S	19.89	32.51	35.62	35.62	35.29	35.18	31.09	27.90	27.73	27.50
D1T	19.89	32.82	37.94	37.94	37.59	37.47	33.12	29.72	29.54	29.30
D2	25.84	38.56	39.29	39.29	38.92	38.80	34.29	30.77	30.59	30.34
D3	25.84	38.56	37.71	39.29	38.92	38.80	34.29	30.77	30.59	30.34
A1S	19.04	31.12	33.00	33.00	32.70	32.59	28.81	25.85	25.69	25.48
A1T	19.04	31.42	33.97	33.97	33.66	33.55	29.66	26.61	26.45	26.24
A2	19.04	31.42	35.27	35.27	34.95	34.84	30.79	27.63	27.46	27.24
A3	19.04	31.42	35.27	35.27	34.95	34.84	30.79	27.63	27.46	27.24
L1	19.62	32.07	36.26	36.26	35.93	35.81	31.65	28.40	28.23	28.00

**Table 3: Abuja DisCo Tariffs – ₦/kWh**

Class	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
R1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
R2	14.70	24.30	24.30	24.30	20.40	19.69	19.74	19.51	19.40	19.25
R3	32.25	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
R4	32.25	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
C1	23.61	36.65	37.39	36.25	30.78	29.7	29.78	29.43	29.26	29.03
C2	29.98	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
C3	29.98	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
D1	24.19	35.35	36.07	34.96	29.68	28.64	28.72	28.38	28.22	28.00
D2	31.43	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
D3	31.43	46.23	47.09	45.72	38.82	37.46	37.56	37.12	36.91	36.62
A1	23.16	35.02	35.74	34.63	29.40	28.38	28.45	28.12	27.96	27.74
A2	23.16	35.02	35.74	34.63	29.40	28.38	28.45	28.12	27.96	27.74
A3	23.16	35.02	35.74	34.63	29.40	28.38	28.45	28.12	27.96	27.74
S1	19.11	26.84	27.14	26.54	22.53	21.75	21.8	21.55	21.43	21.26

We considered pricing FTRs as an offline problem. Our approach is a variant from online approaches where all inputs are not known and algorithm makes certain decisions which may not be optimal. After mapping the TR pricing problem to a financial option pricing problem, we explored a nature-inspired algorithm, Ant Colony Optimization (ACO) to price TRs. The ACO strategy is based on the foraging behavior of ants. Many research efforts have indicated interests to determine the pattern ants in nature used to find shortest paths between their nest and food sources.



Research efforts have shown that ants use indirect communication to collaborate with other ants and this indirect communication involves modifying their environment, known as stigmergic<sup>2</sup> communication. In stigmergic communication, ants move to food source from their nest, depositing on the path (ground) a chemical substance excreted from their body known as pheromone. Other ants can sense and perceive these pheromones and tend to follow the path with highest pheromone concentration. Using this communication, ants are able to forage and collect their food in an efficient way. ACO involves a number of artificial ants (that behave like real ants) to build solution to an optimization problem. These ants exchange information about their own solution to other ants using a communication scheme similar to the one used by real ants [12].

We parallelized the ACO algorithm for TR pricing using all features of TR pricing problem (congestion, multiple strike prices and multiple exercise times). Parallel implementation and experiments were carried out on a cluster available on the West Grid Consortium [11] using OpenMP. OpenMP based parallel jobs on West Grid are mostly computed on Breezy and Hungabee computing facilities Breezy is an Appro AMD cluster with quad-socket, 6-core AMD Istanbul processors (24 cores @ 2.4 GHz) per node. This blade runs a 256 GB memory per compute node. Hungabee comprises an SGI UV100 login node and an SGI UV1000 computational node. The UV1000 is a large cache-coherent non-uniform memory access (ccNUMA) shared memory multiprocessor machine with 2048 cores (Intel Xeon E7 CPU family) and 16TB of memory.

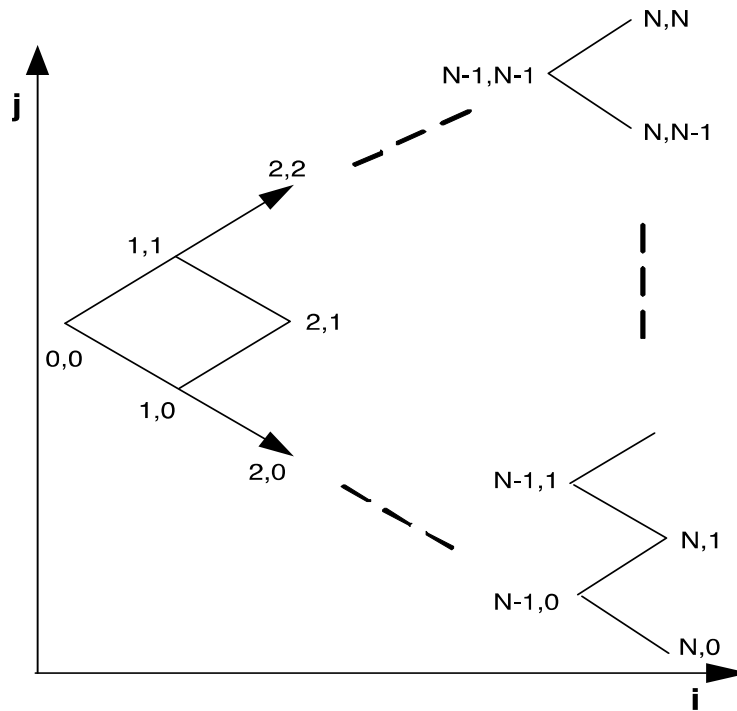


Figure 3: Solution space for implementation.

We followed the scheduling policy for shared memory nodes available on WestGrid. Each node in solution space contains electricity price and congested price. Nodes are connected to each other (edges) so ants can move from start time to expiration time. These edges store pheromone values, which ants can update while traversing these edges during execution of proposed algorithm. The proposed framework and algorithm will work with any type of data structure. For experiments, an array is used to implement complete binary tree. Figure 3 the implementation solution space shows. In this figure, we depict a complete binary tree is a set of nodes where every node other than leaf nodes has two children. For each node  $i$ ,  $2 * i + 1$  is left child,  $2 * i + 2$  is right child and  $(i * 1) / 2$  is parent.

<sup>2</sup> Stigmergy – a consensus social network mechanism of indirect coordination, through the environment, between agents or actions. The principle is that the trace left in the environment by an action stimulates the performance of a next action, by the same or a different agent.

## 6. RESULTS AND DISCUSSIONS

As an initial step, we carried out a feasibility study as a proof of concept. We carefully selected the TR constraints. These include congestion and single strike price. We ran a parallel code for the pricing algorithm to ascertain the level of speed up. In Table 4 and Table 5, our findings show an increase in the execution time as the number of time steps were increased.

**Table 4: Time Steps vs. Execution Time**

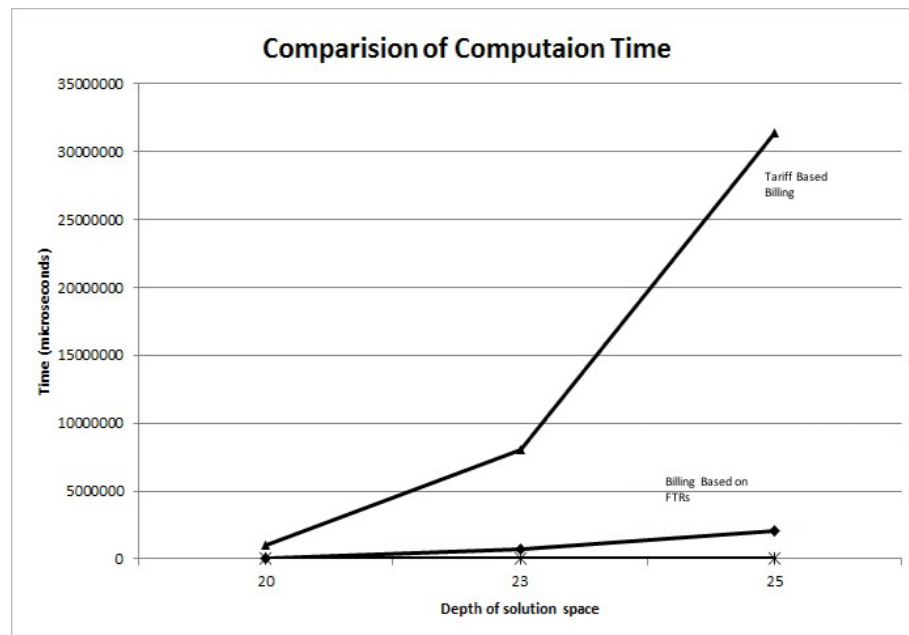
Time Steps	1024	2048	4096	8192
Execution Time (ms)	12.63	33.21	95.30	198.90

**Table 5: Volatility vs. Execution Time**

Volatility	40%	60%	80%	90%
Execution Time (ms)	45.35	45.30	43.75	44.32

Varying the volatility in steps of 40%, 60%, etc. helped in the simulation of the various types of scenarios such as peak hours (when volatility is high) or off peak hours (when volatility is lower). Table 5 shows that varying the volatility does not necessarily change the execution time. This is because the number of steps remains same within the solution space. We used a shared memory system to implement the algorithm since all processors share same use of data and communication can be as fast as time taken to access memory location. Although the issue of scalability of shared memory systems becomes an issue as we increase the number of processors.

Still on the issue of shared memory system; is cache coherence. This was not a major issue of concern for our experiment since each FTR (an ant) are required to have their own copy of memory we assume that there was no communication interference during the iteration. We were able to avoid communication error by allowing communication to only take place at the completion of each iteration to compare results from other ants in order to compute the best path for the iteration. Figure 4 shows the comparison of the simulated tariff billing and the computed option value based on TRs. It shows that it takes more computational steps to converge. Meaning that the solution space required for the tariff-based billing is higher than when we apply TRs.



**Figure 4: Comparison of Computation Time.**

Financial risk management is high priority for participants in wholesale deregulated electricity markets due to substantial price and volume risks. As a result of the high complexity of a wholesale electricity market, prices can exhibit high volatility at times of peak demand and supply shortages. Electricity markets are dependent on physical characteristics such as generation, distribution, demand, and weather patterns. In this research, we looked at the prices of electricity in Nigeria. We observed that the current model of charging electricity which is based on static process through tariffs and location does not capture the users' QoS and hence poor availability.

We model the electricity prices using TRs which provides a financial hedge in a de-regulated electricity market against price risk. TRs are financial instruments that entitle the holder to a stream of revenues based on congested price differences across a transmission path. Through this research, we also identified the various similarities between the TRs and financial options (derivatives) in financial market and few differences as well. We started by formulating TRs as a financial option pricing problem. We then applied a nature-inspired meta-heuristic algorithm, Ant Colony Optimization (ACO).

## **7. ACKNOWLEDGEMENT**

We acknowledge gratefully the Tertiary Education Trust Fund (TetFund) for the full funding of this research under the Institution Based Research (IBR) research efforts. We also appreciate our Vice Chancellor Prof. Akii O. A. Ibadode who has made research a way of life in Federal University of Petroleum Resources, Effurun. Thank you for the vision. We thank the Dean, College of Science, Prof. I. E. Agbozu and FUPRE TetFund Desk officer, Dr. Ogagaruwe for all your supports. We acknowledge the efforts and cooperation of the staff of the Benin Electricity Distribution Plc (BEDC) and the Abuja Electricity Distribution for providing us with many of useful data/information for this research.

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