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# Some Remarks on the flow of a Newtonian Fluid with Memory in the Presence of Suction/ Blowing

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**ABSTRACT**

The paper examines a Newtonian fluid flow in the presence of suction/blowing parameter. Of particular interest are the criteria for a similarity solution of the problem. The paper provides an analytical solution to the problem in the absence of suction/blowing. It also discusses the existence of unique solution of the problem in the presence of suction/blowing parameters. The paper provides numerical results which show the effects of memory, suction / blowing parameter on the flow.

**Keywords-** *Newtonian medium; Raleigh-Stokes; Suction; Blowing;*

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## 1. INTRODUCTION

The research into Newtonian and non-Newtonian fluids flows have attracted many fluid dynamists in recent times because of inevitable demands in areas of engineering and other relevant industries.

Olajuwon & Oyelakin (2012) investigated convection heat and mass transfer in power law fluid flow with non relaxation time past a vertical porous plate in the presence of chemical reaction, thermo and thermal diffusion. The authors provided the numerical solutions to the resulting equations. Their results presented the velocity, temperature and concentration profiles for pseudo plastic fluids and for different values of the parameters governing the equations. Zierep et al (2007) studied a Raleigh stoke problem for non Newtonian medium with memory. They discussed the analytical solution by Fourier method of the governing equation for a second grade fluid. They found an interesting memory behavior for the Raleigh-stokes problem. Ayeni et al, (2007) revisited a Raleigh-stokes problem for non-Newtonian fluid with memory. The revisited literature showed that there was a jump. They transformed by similarity technique, the partial differential equation governing the flow into ordinary differential equation. The resulting equation was solved analytically by asymptotic technique. The result showed that jumps at the boundary in the referenced paper are due to numerical technique.

In this paper, a Newtonian fluid flow in the presence of suction/blowing parameter is investigated. Of particular interest are the criteria for a similarity solution of the problems. We provide an analytical solution to the problem in the absence of suction/blowing and also provide numerical results that show the behavior of suction / blowing parameter on the flow.

## 2. MATHEMATICAL EQUATIONS

### *A. Governing Equation*

Following [1] and [3], the velocity distribution $u = u(y,t)$ for a Newtonian fluid satisfies the differential equations

$$\frac{\partial V}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial t \partial y^2} \qquad (2)$$

Together with the initial and boundary conditions

$$u(y,0) = 0, \quad u(0,t) = U, \qquad u(\infty,t) = o \qquad (3)$$

***B. Similarity Transformation [4]***

We seek a similarity variable of the form $u(y,t) = f(\eta)$ *such that* $\eta = \dfrac{y}{2\sqrt{\nu t}}$. (4)

The equation (1) gives the velocity $V = g(t)$. along the y-direction which is equal to the suction velocity.

In this paper the velocity is written as

$$g(t) = \beta\sqrt{\frac{\nu}{t}}. \tag{5}$$

**Remark:** For similarity, $\alpha = \alpha_0 t$ .

The equation (2) together with the boundary conditions in (3) gives

$$f''' + \frac{2}{\eta}\left(1 - \frac{\nu}{\alpha_o}\right)f'' + \frac{4\nu}{\alpha_o}\left(\frac{\beta}{\eta} - 1\right)f' = 0 \tag{6}$$

$$f(0) = U, \; f(\infty) = 0, \; f'(0) = 0 \tag{7}$$

A. ***Case 1:*** $\beta \neq 0$ ***(Suction / blowing model [5] )***

Existence of Unique Solution:

Following [6], [7] and [8], we establish the criteria for the existence of unique solution.

Theorem 1: *For* $0 < \alpha_o \le N$, $0 < y_1 \le M$, $\nu, \alpha_o, M, N, \sigma, U, > 0$, *problem (6) which satisfies conditions (7) and for which* $f''(0)$ *is fixed, has a unique solution.*

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \eta \\ f \\ f' \\ f'' \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ y_4 \\ -\left[\frac{2}{y_1}\left(1 - \frac{\nu}{\alpha_o}\right)y_4 + \frac{4\nu}{\alpha_o}\left(\frac{\beta}{y_1} - 1\right)y_3\right] \end{pmatrix} = \begin{pmatrix} \psi_1(y_1, y_2, y_3, y_4) \\ \psi_2(y_1, y_2, y_3, y_4) \\ \psi_3(y_1, y_2, y_3, y_4) \\ \psi_4(y_1, y_2, y_3, y_4) \end{pmatrix} \tag{9}$$

Satisfying the initial conditions

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \end{pmatrix} = \begin{pmatrix} 0 \\ U \\ 0 \\ \sigma \end{pmatrix} \tag{10}$$

<u>Remark:</u> $\sigma$ is guessed such that the boundary condition $y_2(M) = 0$.

Theorem 2:

*For* $0 < \alpha_o \le N$, $0 < y_1 \le M$, $0 \le y_2 \le N^*$, $0 \le y_3 \le M^*$, $-\sigma \le y_4 \le \sigma^*$, $M, N, M^*, N^* > 0$, the functions $\psi(i = 1,2,3,4)$ are Lipschitz continuous.

Proof:

$$\left|\frac{\partial \psi_1}{\partial y_1}\right| = 0, \left|\frac{\partial \psi_1}{\partial y_2}\right| = 0, \left|\frac{\partial \psi_1}{\partial y_3}\right| = 0, \left|\frac{\partial \psi_1}{\partial y_4}\right| = 0, \left|\frac{\partial \psi_2}{\partial y_1}\right| = 0, \left|\frac{\partial \psi_2}{\partial y_2}\right| = 0, \left|\frac{\partial \psi_2}{\partial y_3}\right| = 1, \left|\frac{\partial \psi_2}{\partial y_4}\right| = 0,$$

$$\left|\frac{\partial \psi_3}{\partial y_1}\right| = 0, \left|\frac{\partial \psi_3}{\partial y_2}\right| = 0, \left|\frac{\partial \psi_3}{\partial y_3}\right| = 0, \left|\frac{\partial \psi_3}{\partial y_4}\right| = 1,$$

$$\left|\frac{\partial \psi_4}{\partial y_1}\right| \le \left|\frac{2}{y_1^2}\left(1 - \frac{\nu}{\alpha_o}\right) y_4\right|, \left|\frac{\partial \psi_4}{\partial y_2}\right| \le 0, \quad \left|\frac{\partial \psi_4}{\partial y_3}\right| \le \left|\frac{4\nu}{\alpha_o}\left(\frac{\beta}{y_1} - 1\right)\right|,$$

$$\left|\frac{\partial \psi_4}{\partial y_4}\right| \le \left|\frac{2}{y_1}\left(1 - \frac{\nu}{\alpha_o}\right)\right|$$

$\frac{\partial \psi_i}{\partial y_j}, \; i, j = 1,2,3,4$ are bounded since there exists a Lipschitz constant $K > 0$, such that

$$\left|\frac{\partial \psi_i}{\partial y_j}\right| \le K, \quad i, j = 1, 2, 3, 4 .$$

Hence $\psi_i(y_1, y_2, y_3, y_4)$, $i = 1,2,3,4$ are Lipschitz continuous and so (9) satisfying (10) is Lipschitz continuous

Proof of theorem 1: The existence of Lipschitz constant in the proof of theorem 2 implies the existence of unique solution of problem (9) which satisfies (10). And this implies the existence of unique solution of problem (6) satisfying the conditions (7).

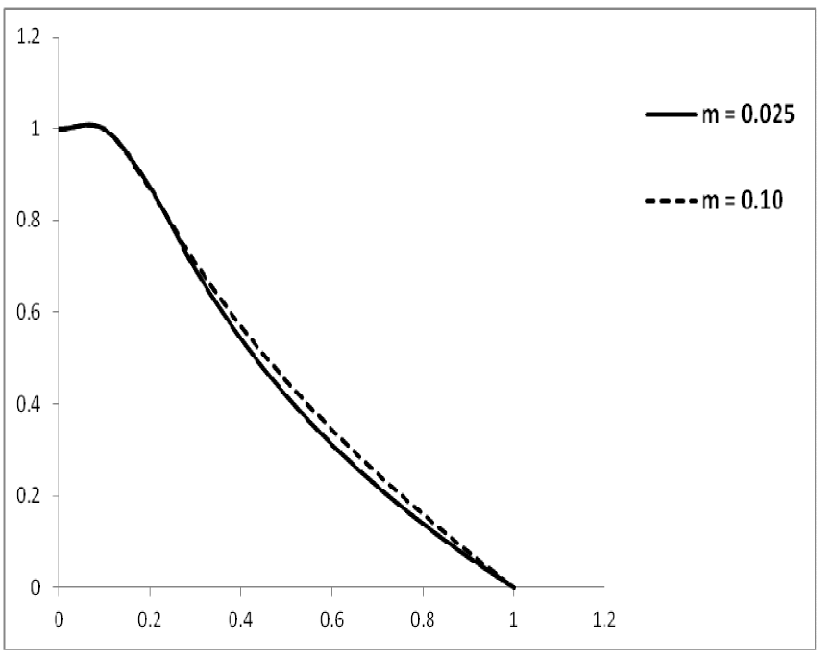


**Fig. 1. Velocity Profile for various memory parameter.**

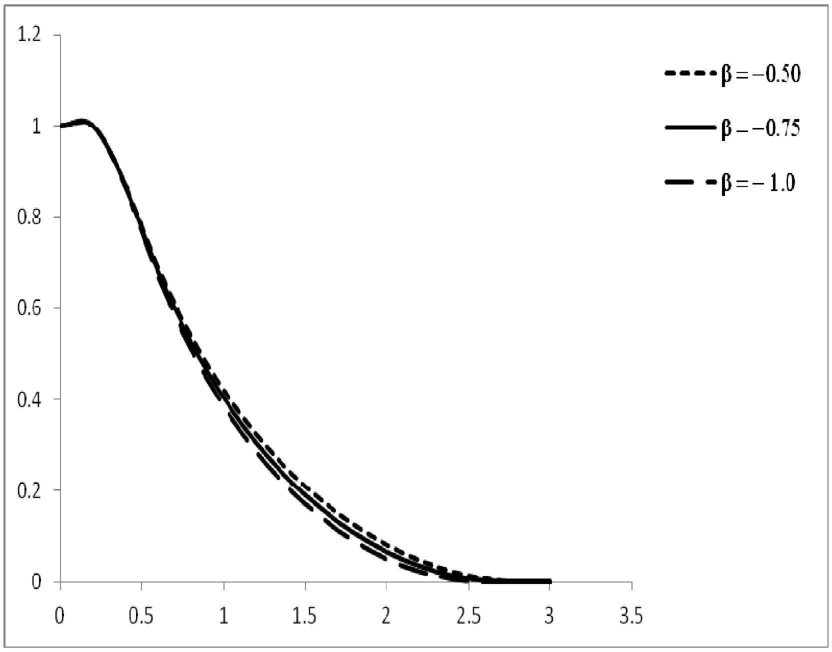


**Fig. 2. Velocity Profile for Suction parameter.**

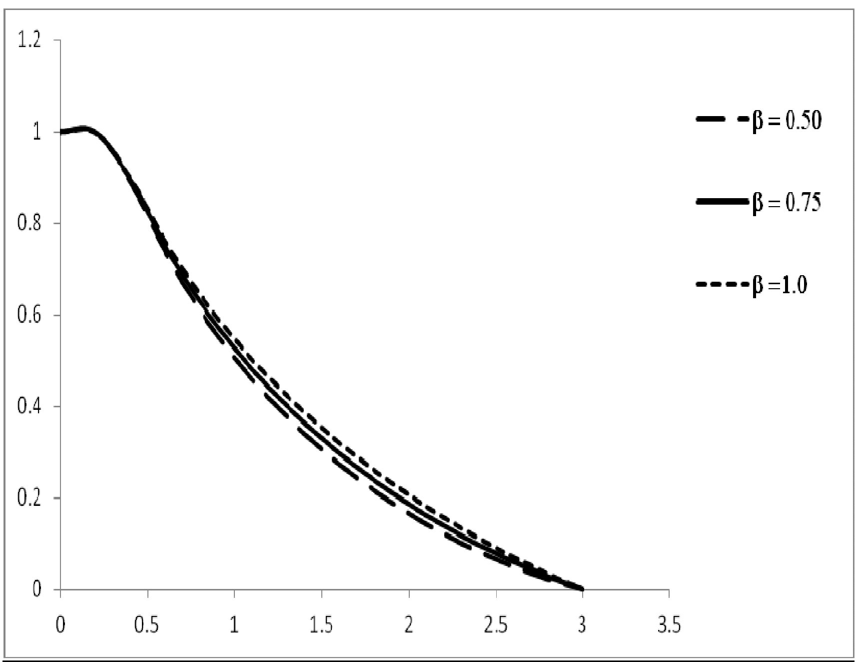


**Fig. 3. Velocity Profile for Blowing parameter.**

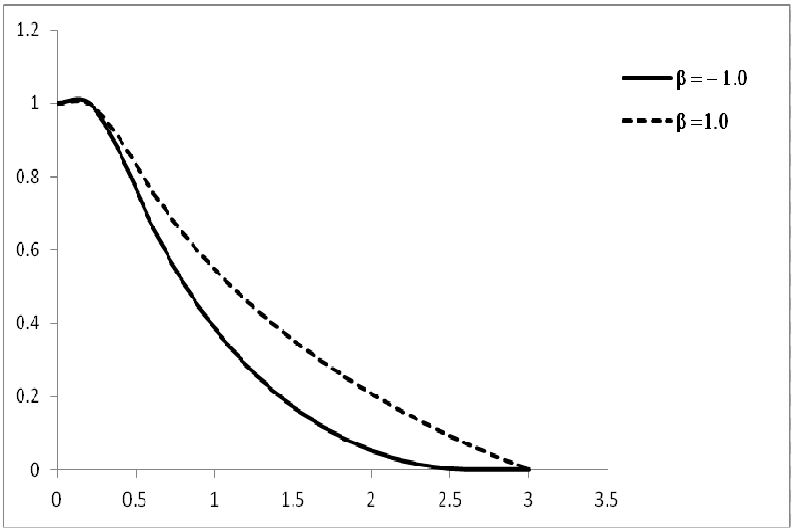


**Fig. 4. Velocity Profile for Suction and Blowing.**

TABLE I
NOMENCLATURE

| Symbol | Quantity |
| --- | --- |
| $u$ | Fluid Velocity along y-axis |
| $V$ | Fluid velocity along x-axis |
| $\nu$ | Kinematic viscosity |
| $\frac{\alpha}{\nu t}$ | Memory variable |
| $m$ | Memory Parameter |
| $\beta > 0$ | Blowing Parameter |

A. ***Case 2:*** $\beta = 0$ ***(Suction / blowing-free model )***

The equation (6) becomes

$$f''' + \frac{2}{\eta}\left(1 - \frac{\nu}{\alpha_o}\right) f'' - \frac{4\nu}{\alpha_o} f' = 0 \qquad (11)$$

satisfying (7)

Here an analytic solution of (11) subject to (7) is provided  We seek an analytical approximate solution of the form

$$f = \left(a\eta + b\eta^2 + U\right)e^{-m\eta} \qquad a, b, U > 0 \qquad (12)$$

which satisfies the boundary conditions (7)

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where $m = \sqrt{\dfrac{4\nu}{\alpha_o}}$ (13)

Then

$$f' = \left(a + 2b\eta - m\left(a\eta + 2b\eta^2 + U\right)\right)e^{-m\eta} \tag{14}$$

$$f'' = \left(-2m(a+2b\eta)+m^2\left(a\eta+b\eta^2+U\right)+2b\right)e^{-m\eta} \tag{15}$$

and

$$f''' = \left(-6mb+3m^2(a+2b\eta)-m^3\left(a\eta+b\eta^2+U\right)\right)e^{-m\eta} \tag{16}$$

The equation (14) to (16) are substituted into (12) and then simplified to give

$$\begin{aligned}
&-6mb\eta+3m^2a\eta+3m^2b\eta^2-m^3a\eta^2-m^3b\eta^3-m^3b\eta\\
&+4b-4ma-8mb\eta+2am^2\eta+2bm^2\eta^2+2m^2U\\
&-m^2b+m^3a+2m^3b\eta-\frac{m^4a}{2}\eta-\frac{m^4b}{2}\eta^2-\frac{m^4U}{2}\\
&-m^2a\eta-2m^2b\eta^2+m^3a\eta^2+m^3b\eta^3+m^3U\eta=0
\end{aligned} \tag{17}$$

$$\eta^0: \quad b - ma + \frac{m^2U}{2} = 0 \tag{18}$$

$$\eta: \quad 2\left(m^2-7\right)b - m\left(4 - \frac{m^2}{2}\right)a = 0 \tag{19}$$

$$\eta^2: \quad m^2\left(3 - \frac{m^2}{2}\right)b = 0 \tag{20}$$

We solve for $(a,b)$ in some orders of $\eta$

$$\eta^0: \quad b - ma + \frac{m^2U}{2} = 0 \tag{18}$$

$$\eta: \quad 2\left(m^2-7\right)b - m\left(4 - \frac{m^2}{2}\right)a = 0 \tag{19}$$

$$\eta^2: \quad m^2\left(3 - \frac{m^2}{2}\right)b = 0 \tag{20}$$

The equations (18) to (20) give

$$(a, b) = \left(\frac{mU}{2},\ 0\right) \tag{21}$$

Then the solution (12) becomes

$$f = \left(\sqrt{\frac{\nu}{\alpha_0}}\eta + 1\right)U\, e^{-2\sqrt{\frac{\nu}{\alpha_0}}\eta} \quad \alpha_0, \nu,\ U > 0 \tag{22}$$

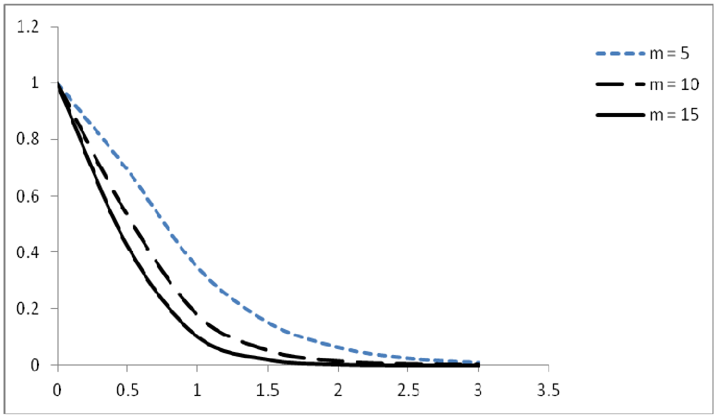


**Fig. 5. Velocity Profile for various memory parameter.**

***B. Discussion of Results and Conclusion***

The combined effects of suction/blowing and memory parameters have been studied on a Newtonian fluid flow. The criterion for the existence of a similarity solution of the problem was established.

Two cases of this model were investigated in order to understand the effect of suction/ blowing on the Raleigh-Stoke's equation. Case 1 describes a model with suction/ blowing parameter together with the memory term. in this section, theorems on the existence of unique solution were formulated and proved. This showed that the problem has a unique solution and it therefore represents a physical problem. The problem was solved numerically by using Runge-Kutta Shooting method. The pectoral representations are shown in figures 1-4.

Figure 1 shows that memory parameter has a significant effect on the fluid flow. As the memory increases, the fluid flow velocity increases. Figure 2 shows that a decrease in suction leads to a decrease in flow velocity. Figure 3 shows that blowing parameter has appreciable effect on fluid flow. As the blowing increases, the velocity increases. Figure 4 shows the difference in the model with suction parameter and another model with blowing parameter. The fluid velocity in the presence of blowing parameter is higher than that of suction parameter.

In case 2, a model without suction/ blowing parameter was investigated. An analytical solution to the resulting problem was provided. I remark that the assumed solution (12) satisfied the boundary conditions (7). The analytical solution (22) are as shown in figure 5. It therefore shows that the memory parameter has much effect on the fluid velocity. As the memory decreases, the flow velocity increases.

In conclusion, I remark that in a Newtonian fluid flow in the presence of memory

- a similarity solution exists under certain criteria.
- a unique solution is possible under some criteria and an occurrence of jump is due to numerical technique.
- combined parameters of suction/blowing and memory have appreciable effects on the flow velocity.

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