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On The Comparison of Adomian Decomposition Method and Variational Iteration Method Using a General Riccati Equation

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ABSTRACT

In this paper, Adomian Decomposition Method is compared with Variational Iteration Method using a General Riccati Equation. Both methods are applied to few examples and their results are compared to show the similarities and differences between the two methods. We proposed that the methods should be included in the Mathematical science curriculum as well as lectures note on classical mechanics and other applied physical sciences for undergraduate students

Keyword: Comparison, Adomian Decomposition Method, Variational Iteration Method & Riccati Equation.

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1. INTRODUCTION

Adomian Decomposition Method (ADM) is one of the new methods for solving initial value problem in differential equation of various kinds arising not only in the field of medicine, physical and biological science but, also in the area of economic, management and engineering (1, 2, 3, 4, 8). It is important to note that a large amount of research works has been devoted to the application ADM to wide class of linear and nonlinear and nonlinear ordinary or partial differential equation.

Variation Iteration Method (VIM) is a simple and yet powerful method for solving a wide class of nonlinear problems, first envisioned by the (4, 5). The VIM has successfully been applied to many situations such as engineering field, longwave and chemical reaction diffusion models (2, 4, 5).

2. THE BASIC OF CONCEPT OF ADM

Consider the differential equation [4]

$$Lu + Ru + Nu = g(t) \qquad (1)$$

Where L is the highest – order derivative which is assumed to be invertible R is a linear differential operator of order lesser than L, Nu represent the nonlinear terms and g is the source term.

Applying the inverse operator L^{-1} to the both sides of (1) and using the given condition, we obtain.

$$U = F - L^{-1}(Ri) - L^{-1}(Nu) \quad (2)$$

Where the function f represents the terms arising from integrating the source term 'g' using the given conditions. Adomian Decomposition Method (1, 2, 3, 4) defines the solution u(t) by the series

$$U(t) = \sum_{n=0}^{\infty} U_n(t) \quad (3)$$

Where the component $U_n(t)$ are usually determined recursively by using the relation

$$U_0 = f$$
$$U_{k+1} = L^{-1}(Ru) - L^{-1}(Nuk), \; K \geq 0 \quad (4)$$

The nonlinear operator F(u) can be decomposed into an infinite series of polynomials given by

$$F(u) = \sum_{n=0}^{\infty} A_n \quad (5)$$

Where A_n are the so-called Adomian's polynomials that can be generated for various classes of nonlinearities according to the specific algorithm developed (3, 4, 8) which yields

$$A_n = \left(\frac{1}{n!}\right)\frac{d^n}{d\lambda^n} N\left[\sum_{i=0}^{n} \lambda^i U_i\right]_{n=0} \quad (6)$$

3. ANALYSIS OF GENERAL RICCATI DIFFERENTIAL EQUATION BY USING ADM

For our construction, we shall refer to a general Riccati differential equation of the firm.

$$\frac{dy}{dt} = Q(t)\, y + R(t)y^2 + P(t), \;\; y(0) = G(t) \quad (7)$$

Where P(t), Q(t), R(t) and G(t) are scalar function.

To solve, we further assume that y(t) is sufficiently differentiable and that the solution of (7) exists and satisfies the Lipschitz condition (8). ADM usually defines an equation in operation form by considering the highest ordered derivative in the problem (WAZWAL, A.M)

In operator form, equation (7) can be written as;

$$Ly(t) = Qy(t) + R(t)y^2 + p(t) \quad (8)$$

Where the differential operator L is given as $L = \frac{d}{dt}$ (9)

There inverse operator L^{-1} is considered a one fold integral operator defined by

$L^{-1} = \int_0^t dt$ (10)

If we operator L^{-1} on the right hand side of (2) and use initial condition $y_0(0) = G(t)$, we have

$y(t) = y_0 + L^{-1} (Ry^2(t) + Qy(t) + p(t)$ (11)

Let $f(t,y) = Ry^2 (t) + Qy(t) + p(t)$ (12)

Then, equation (11) becomes

$y(t) = y_0 + L^{-1} f(t,y)$ (13)

The ADM introduce the y(t) is an infinite as

$y(t) = \sum_{n=0}^{\infty} y_n(t)$ (14)

Solution of non-linear problems.

4. THE BASIC CONCEPTS OF VIM

To illustrate the basic concept of the technique, we consider the following general differential equation

$Lu + Nu = g(t)$ (18)

Where L is a linear operator, N a nonlinear operator and g(x) is the in homogeneous term. According to variational iteration (3,4), we can construct a correction functional as follow:

$U_{n+1} (x) = U_n(t) + \int_0^t \lambda \, (LU_n(t) + N_n(t) – g(t) \, dt$ (19)

Where λ is a lagrangian multiplier, which can be identified optionally via variational iteration theory, the subscripts n denote the nth- order approximation, Un is considered variation.

5. ANALYSIS OF GENERAL RICCATI DIFFERENTIAL EQUATION FOR THE CONSTRUCTION OF A GENERAL RICCATI DIFFERENTIAL EQUATION.

$\frac{dy}{dt}$ $Q(t)y + R(t)y^2 + p(t)$ $y(0) = G(t)$ (20)

Where Q(t), R(t), P(t) and G(t) are scalar functions.

To solve equation (20) by means of He's variational iteration method, we construct a correctional function as:

$y(t) = \sum_{n=0}^{\infty} y(t)$ (13)

Where the component $y_n(t)$ will be determined recursively. Moreover, the method defined the non-linear function f(t,y) by the infinite series of the form.

$$f(t,y) = \sum_{n=0}^{\infty} A_n \qquad (14)$$

If we substitute (13) and (14) in equation (6), we obtain

$$\sum_{n=0}^{\infty} y_n(t) = \sum_{n=0}^{\infty} A_n \qquad (15)$$

The next step is to seek a way to determine the component $y_n(k)$ for which n $\geq$0. We first identify the zeroth component $y_0(t)$ by all terms that arise from the initial conditions. The remaining component is determined by using the preceding component (G. Adomian, 1994). Each term of the series (13) is given by the recurrent relation

$$y_0\ (t) = G(t)$$

$$y_{n+1} = L^{-1}\ (Ry\ (t) + Qy_n(t) + P(t)) \qquad (16)$$

We must state here that in practice all term of the series in (15) cannot be determined and the solution will be approximated by series of the form

$$Q_n(t) = \sum_{n=0}^{N-1} y_n(t) \qquad (17)$$

With (17), we obtain series solution for our (7) (A. Wazwat, 2002). The method reduces significantly the massive computation which may arise if discretisation method are used for the

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The next step is to seek a way to determine the component $y_n(t)$ for which $n \geq 0$. We first identify the zeroth component $y_0(t)$ by all terms that arise from the initial conditions. The remaining component is determined by using the preceding (G. Adomian, 1994). Each term of the series (14) is given by the recurrent relation.

$y_0(t) = G(t)$

$y_{n+1} = L^{-1} (A_n)\ n \geq 0$

$$y_{n+1} = L^{-1} [R\, y_n^2 (t) + Qy_n(t) + P(t) \qquad (20)$$

We must state here that in practice all term of the series in (14) cannot be determined and the solution will be approximated by series of the form

$$\phi_n(t) = \sum_{n=0}^{\infty} y_n(t) \qquad (21)$$

With (18), we obtain series solution for our system equation (7) (Wazwat, 2002). The method reduces significantly the massive computation which may arise if discretisation methods are used for the solution of non-linear problem.

6. THE BASIC CONCEPTS OF VIM

To illustrate the basic concepts of the VIM, we consider the following nonlinear differential equation.

$$Lu + Nu = g(t) \qquad (22)$$

Where L is a linear operator, N is a nonlinear operator and g(t) is an inhomogeneous term. According to the VIM (4,5), we can construct a correction function as follows

$$U_{n+1}(t) = U_n(t) + \int_0^t \lambda \left\{ Lu(\tau) + N\tilde{u} - g(\tau) \right\} d\tau \qquad (23)$$

Where λ is a general lagrangian multiplier, which can be identified optionally via the variational theory, the subscript n denotes the nth-order approximation, U_n is considered as a restricted variation i.e δu.

7. Analysis of general Riccati differentiation equation by Variational Iteration Method

We consider a general Riccati differentiation

$$\frac{dy}{dt} = Q(t)y + R(t)y^2 + P(t), y(10) = G(t) \qquad (24)$$

Where Q(t), R(t) and G(t) are scalar functions. To solve (21) by means of He's variational iteration method.

We construct a correctional function

$y_{n+1}(t) = y_n(t) + \int_0^t \lambda(t)\left[\frac{dy_n}{dt} - Qy_n(t) - Ry^2(t) - P(t)\right] dt$ (22)

$\delta y_{n+1}(t) = \delta y_n(t) + \delta\int_0^t \lambda(t)$ dt

$\delta y_{n+1}(t) = \delta y_n(t)$ dt

$\delta y_{n+1}(t) = (1+\lambda)\delta y_n(t) - \int_0^t \delta y_n(t)\Lambda' ds = 0$ (23)

Where $\tilde{y}_n$ is considered as restricted variations which mean

$\delta y_n = 0$. Its stationary conditions can be obtained as follow

$| + \lambda(t) = 0, \qquad \lambda'(s)|_{s=t} = 0.$ (24)

The Lagrange multiplier therefore can be identified as $\lambda(s) = -1$ and the following iteration formula is obtained.

$y_{n+1}(t) = y_n(t) - \int_0^t \left[\frac{dy_n}{d_t} - Qy_n(t) - Ry_n^2(t) - P(t)\right]$ (25)

8. NUMERICAL EXAMPLES

8.1 Example 1

We consider the system of equation

$\frac{dy}{d_t} = y^2 \qquad y(0) = 1$ (26)

Q(t) = 0, r(t) = 1, P(t) = 0 G(t) = 1

With theoretical solution given as

$y(t) = \frac{1}{1-t}$

We apply ADM operator to equation (25) to produce

$L_y = y^2$ (27)

Operating L^{-1} on both sides of (27) and use the initial condition we obtain

$y(t) = y(0) + L^{-1}(y^2)$
$y_0 = 1+t$
$y_{n+1} = 1+t+L^{-1}(y_n^2)$
$y_1 = 1+t+L^{-1}(1+2t+t^2)$
$y_1 = 1+t+t+t^2 + \frac{t^3}{3}$
$y_1 = 1+2t+t^2+ \frac{t^3}{3}$
$y_2 = 1 + 3t + 3t^2 + 2t^3 + \frac{7t^4}{6}+\frac{2t^5}{3}+\frac{t^6}{6}+\frac{t^7}{63}$

To solve equation (25) by means of the variational iteration method, we construct a correctional function.

$y_{n+1} = y_n(t) + \int_0^t \left[\frac{dy_n}{dt} + y_n^2(t)\right] dt.$

We can take the linearised solution y(t) = t + c as the initial approximation y_0 , the condition y(0) = 1 gives us c = 1 then we get $y_0 = t + 1$

$y_1 = y_0(t) + \int_0^t \left[\frac{d}{dt}y_0(t) + y_0^2(t)\right]$ dt
$y_1 = 1 + t + \int_0^0 \left[d(1+t)+(1+2+t+t^2)\right]$
$y_1 = 1+3t + t^2 + \frac{t^3}{3}$
$y_2 = 1+7t + 5t^2 + \frac{13t^3}{3}+\frac{5t^4}{3}+\frac{3t^5}{3}+\frac{t^6}{5}+\frac{t^3}{63}$

8.2 Example 2

Consider the following example

$\frac{dy}{dt} = y^2(t) + 1$

y(0) = 0 (26)

Here Q(t) = 0, R(t) = 1 P(t) = 1 and G(t) = 0. The exact solution is $y(t) = \frac{e^{2t}-1}{e^{2t}+1}$

To solve equation by means of ADM (26) becomes

$y(t) = y(0) + L^{-1}(y^2)$

$y_0 = t$

$y_{n+1} = L^{-1}\left(y_n^2\right) + L^{-1}(1)$

$y_1 = - L^{-1}(t^2) + L^{-1}(1)$

$y_1 = t \frac{-t^3}{3}$

$y_2 = L^{-1}\left(y_1^2\right) + L^{-(1)}$

$y_2 = L^{-1}\left(y_1^2\right) + L^{-(1)}$

$y_2 = L^{-1}\left(t^2 - \frac{2t^4}{3} + \frac{t^6}{9}\right) + L^{-1}(1)$

$y_2 = t = \frac{t^3}{3} + \frac{2+5}{15} - \frac{t^7}{63}$

$y_3 = t - \frac{1}{3}t^3 + \frac{2}{15}t^5 - \frac{t^7}{63} + \frac{38t^9}{2835} - \frac{134t^{11}}{51975} + \frac{4t^{13}}{12285} - \frac{t^{15}}{59535}$

To solve equation (26) by means of He' variational iteration method, we construct a correctional function.

$y_{n+1} = y_n(t) - \int_0^{t} \left[\frac{dy}{dt} + y_n^2 - 1\right] dt.$

We can take the linearised solution y(t) = t + c as the initial approximation y_0, the condition y(0) = 0 gives w c = 0. Then we get $y_0 = t$

$y_1 = y_0 - \int_0^{tt} \left[\frac{dy_0}{dt} + y_0^2 - 1\right] dt$

$y_1 = t - \int_0^{tt} \left[1 + t^2 - 1\right]$

$y_1 = t - \frac{t^3}{3}$

$y_2 = y_1 - \int_0^{tt} \left[\frac{dy_1}{dt} + y_1^2 - 1\right] dt$

$y_2 = t - \frac{t^3}{3} - \int_0^{tt} \left[\frac{-2t^4}{3} + \frac{t^6}{9}\right] dt$

$y_2 = t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63}$

$y_3 = t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63} + \frac{38t^9}{2835} - \frac{134t^{11}}{51975} + \frac{4t^{13}}{12285} - \frac{1t^{15}}{59535}$

8.3 Example 3

Consider the example $\frac{dy}{dt} = t^2 + y^2,\ y(0) = 1$ - (27)

Where Q(t) = 0, R(t) = 1, P(t) = t^2 and G(t) = 1. To solve equation (27) using ADM, we have.

y(t) = $y(0) + L^{-1}(y^2)$.

$y_0 = \frac{t^3}{3} + 1$

$y_{n+1} = y(0) + L^{-1}(t^2) + L^{-1}(y_n)$

$y_1 = \frac{t^3}{3} + 1 + \frac{t^3}{3} + \frac{t^7}{63} + \frac{2t^4}{12} + t$

$y_1 = 1 + t + \frac{2t^3}{3} + \frac{t^4}{6} + \frac{t^7}{63}$

$y_2 = 1 + 2t + t^2 + t^3 + \frac{t^4}{2} + \frac{7t^4}{30} + \frac{5t^7}{63} + \frac{t^8}{112} + \frac{t^9}{63} + \frac{4t^{11}}{2079} + \frac{t^{12}}{2268} + \frac{t^{15}}{59535}$

To solve equation (27) by using VIM, we construct a correction functional,

$y_{n+1}(t) = y_n(t) + \int_0^{tt} \left[\frac{dy_n}{dt} + t^2 + y_n^2 \right] dt$

We can take the linearised solution y(t) = $\frac{t^3}{3}$ + c as. The initial approximation y_0, the condition y(0) = 1 gives wc = 1. Then we get.

$y_1(t) = \frac{t^7}{63} + \frac{t^4}{6} + \frac{t^3}{3} + t + 1$

$y_{2(t)} = t + t^2 + \frac{4t^3}{3} + \frac{t^4}{2} + \frac{17}{15} + \frac{1t^6}{8} + \frac{1}{7} + \frac{23t^8}{504} + \frac{5t^3}{756} + \frac{2t^{11}}{693} + \frac{1t^{12}}{2268} + \frac{1t^{15}}{59535} + 1$

8.4 Example 4.

Consider this example.

$\frac{dy}{dt} = 2y(t) - {}^2(t) + 1$ y(0) = 0 (28)

Here Q(t) = 2, R(t) = -1 P(t) = 1 and G(t) = 0

The exact solution was found

$y(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2t} + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)$

We apply ADM operation to equation (28) to produce
$Ly = 2y - y^2 + 1$

Operating L^{-1} on both side of (28) and use the initial condition, we obtain

$$y_{n+1} = y_n(0) + 2L^{-1}(y_n) - L^{-1}\left(y_n^2\right) + L^{-1}(1)$$
$$y_{n+1} = 2L^{-1}(y_n) - L^{-1}\left(y_n^2\right) + L^{-1}(1)$$
$$y_0 = t$$
$$y_1 = 2L^{-1}(t) - L^{-1}(t^2) + L^{-1}(1)$$
$$y_1 = t + t^2 - \frac{t^3}{3}$$
$$y_2 = t + t^2 - \frac{t^3}{3} - \frac{2t^4}{3} - \frac{2t^5}{15} + \frac{t^6}{9} - \frac{t^7}{63}.$$

9. NUMERICAL RESULTS AND DISCUSSION

We obtained numerical solution for a General Riccati differential equation. Table 1 and 3 show the differences between ADM and VIM, because they give the different numerical values. Table 2 and Table 4 show the similarities between ADM and VIM, because they give the same numerical values.

Table 1:

T	ADM	VIM	DIFFERENCE
0	1	1	0
0.1	1.332124	1.75451	-0.4223867
0.2	1.738091	2.637666	-0.89957547
0.3	2.235195	3.683079	-1.4478843
0.4	2.845402	4.931085	-2.0856832
0.5	3.596478	6.430332	-2.83385417
0.6	4.52326	8.239536	-3.7162752
0.7	5.669079	10.42941	-4.7603283
0.8	7.087339	13.08477	-5.99743147
0.9	8.843276	16.30687	-7.4635947
1	11.01587	20.21587	-9.2

Table 2:

T	ADM	VIM
0	0	0
0.1	0.099668	0.099668
0.2	0.197375	0.197375
0.3	0.291312	0.291312
0.4	0.379947	0.379947
0.5	0.464103	0.464103
0.6	0.536983	0.536983
0.7	0.604124	0.604124
0.8	0.663301	0.663301
0.9	0.714382	0.714382
1	0.757166	0.757166

Table 3:

T	ADM	VIM	DIFFERENCE
0	1	1	0
0.1	1.211052	1.111388	0.099664271
0.2	1.448876	1.251622	0.197254209
0.3	1.721635	1.431259	0.290376387
0.4	2.039329	1.663405	0.375924083
0.5	2.414229	1.964677	0.44955181
0.6	2.861484	2.356524	0.504959925
0.7	3.400002	2.867078	0.532924008
0.8	4.053727	3.53373	0.519997678
0.9	4.853516	4.406705	0.446810904
1	5.839882	5.554005	0.285876623

Table 4:

T	ADM	VIM
0	0	0
0.1	0.109599	0.109599
0.2	0.236252	0.236252
0.3	0.375516	0.375516
0.4	0.521346	0.521346
0.5	0.666195	0.666195
0.6	0.801156	0.801156
0.7	0.91616	0.91616
0.8	1.00022	1.00022
0.9	1.041691	1.041691
1	1.028571	1.028571

9. CONCLUSION

In this paper, we applied ADM and VIM for solving a General Riccati Differential Equation. The methods are applied in a direct way without using linearization transformation, perturbation discretisation or restrictive assumption. It may be concluded that ADM and VIM are very powerful and efficient in finding numerical for a wide class of linear and non linear differential equations. They provide more realistic series of solutions that converge very rapidly in real physical problems. ADM and VIM are different, when initial condition y(0) = 1 (see Tables 1 & 3) but are the same when initial condition y(0) = 0 (see Tables 2 & 4). Therefore, we propose that the methods should be included in the Mathematical science curriculum as well as lectures note on classical mechanics and other applied physical sciences for undergraduate students.

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