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Application of Quadratic Exponential Smoothing for Modelling and Forecasting Fertilizer Prices

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ABSTRACT

This research delves into the analysis and forecasting of Fertilizer Prices, revealing a prominent upward trend marked by intermittent fluctuations. Employing Quadratic Exponential Smoothing with an alpha value of 0.4087, a quadratic trend equation $Y_t = 19.1754 - 0.028298t + 0.000228841t^2$ was derived. The associated coefficients were scrutinized for their statistical significance, highlighting a significant constant term and a highly significant quadratic term, though the linear component lacked statistical significance. A visually presented forecasted values of Fertilizer Prices was shown, incorporating 95.0% prediction limits that offer a measure of confidence regarding the likely range of true values in the future.

Keywords: Quadratic exponential, Quadratic trend, Quadratic term, Prediction limit

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1. INTRODUCTION

Quadratic Exponential Smoothing (QES) is a forecasting method that is based on the idea of exponentially smoothing the past values of a time series data. The method uses three smoothing parameters: α, β, and γ. The parameter α controls the weight given to the most recent observation, the parameter β controls the weight given to the trend, and the parameter γ controls the weight given to the curvature of the trend. The QES forecast for the next period is then calculated as a weighted average of the past observations, the trend, and the curvature of the trend. QES, a forecasting technique based on exponential smoothing, is designed to predict future values of time series data using its historical values.

It is particularly well-suited for forecasting data exhibiting a quadratic trend, meaning the data not only increases or decreases over time but also curves upwards or downwards [1]. The agricultural sector is a cornerstone of Nigeria's economy, playing a pivotal role in the country's food security and economic development. Fertilizer, as a key input in agriculture, significantly influences crop yields and, consequently, the overall productivity of the sector. Over the decades, the prices of fertilizers in Nigeria have exhibited dynamic patterns influenced by various factors such as global market trends, domestic policies, climatic conditions, and economic fluctuations. Traditional time series forecasting methods, while useful, may not capture the intricate patterns and underlying trends in fertilizer prices. Therefore, there is a need for advanced forecasting models that can provide accurate predictions, aiding in better decision-making for both governmental and private entities involved in the agricultural sector.

Quadratic exponential smoothing is a technique that extends traditional exponential smoothing methods by considering the overall trend of a curve in a quadratic fashion [2]. This method is particularly useful when the data exhibits a time-varying quadratic trend along with a time-varying level [3]. By simultaneously capturing the trend and level of the data, quadratic exponential smoothing provides a more nuanced approach to modeling time series data compared to traditional exponential smoothing techniques. Exponential smoothing, in general, is a widely used method for modeling time series data due to its ability to capture underlying patterns and seasonal fluctuations effectively [4]. It assigns exponentially decreasing weights to past observations, allowing for the incorporation of historical data while giving more importance to recent observations [5]. This approach is valuable for forecasting various phenomena, such as Covid-19 growth curves [5], construction and demolition waste generation [6], and even meteorological drought simulation [7].

Moreover, the combination of exponential smoothing with other techniques, such as neural networks, enhances the modeling capabilities by leveraging the strengths of each method. Neural networks excel at capturing complex linkages and nonlinearities, complementing the pattern discovery and seasonal fluctuation identification provided by exponential smoothing [4]; [8]. In the context of forecasting and prediction, exponential smoothing methods offer a pragmatic and effective approach to capturing trends and making accurate predictions [9]; [10]. These methods can handle various types of forecasting patterns, including additive or multiplicative trends, and their combinations [11]. Additionally, the selection of appropriate exponential smoothing constants plays a crucial role in minimizing forecasting errors and improving the accuracy of predictions [12]; [13]. In conclusion, quadratic exponential smoothing is a valuable extension of traditional exponential smoothing techniques, offering a more comprehensive approach to modeling time series data by considering quadratic trends. When combined with neural networks or other forecasting methods, exponential smoothing enhances the predictive capabilities and provides insights into complex phenomena across different domains.

A study by [14] used QES to forecast the monthly prices of NPK fertilizer in Nigeria from 1980 to 2010. The authors found that QES was able to forecast the prices with a high degree of accuracy. A study by [15] used QES to forecast the monthly prices of urea fertilizer in Nigeria from 2000 to 2015. The authors found that QES was able to forecast the prices with a moderate degree of accuracy. Despite the wealth of research on fertilizer prices and time series analysis, there are identified gaps that warrant further investigation.

These gaps include the limited application of Quadratic Exponential Smoothing specifically to fertilizer prices in the Nigerian context, emphasizing the need for the current study.

The aim of this study is to develop and evaluate a novel forecasting model, specifically a Quadratic Exponential Smoothing model, tailored to predict monthly fertilizer prices in Nigeria. This aim was achieved through the collection and analyzes of historical monthly fertilizer price data in Nigeria from 1960 to 2022, model a Quadratic Exponential Smoothing model tailored to capture the quadratic trends inherent in fertilizer price dynamics and to validate the proposed model using historical data and compare its performance against traditional models.

2. RESEARCH METHODOLOGY

In this section, the method of data collection and statistical tool for analyzing time series data is discussed. Monthly fertilizer price in Nigeria were soured from the World Bank Commodity price data (The Pink Sheet), a reputable international organization.

The dataset comprises information on various types of fertilizers, including nitrogen-based, phosphorous-based, and potassium-based fertilizers. Monthly prices are collected in US Dollars and adjusted for inflation to ensure consistency over time. The data covers a period of 62 years, from 1960 to 2022

2.1 Augmented dickey-fuller test (ADF)

Augmented dickey-fuller test (ADF) was used to check if the data is stationary or not. The test was first introduced by [16] to test the presence of unit root. The regression model for the test is given as:

$$\Delta X_t = \gamma X_{t-1} + \beta\gamma_{t-1} + \sigma_1\Delta_{Xt-1} + \sigma_2\Delta X_{t-2} + \cdots + \sigma_p\Delta X_{t-p} + e_t \quad (1)$$

The hypothesis testing

Ho: $\gamma = 0$ (The series contain unit root) H1: $\gamma < 0$ (the series is stationary)

Test statistic $ty = \gamma\ (\gamma)$, (2)

where;
ΔX_t = the difference series
X_{t-1} = the immediate past observation
$\sigma_1 \ldots \sigma_p$= the coefficient of the lagged difference term up to p.
Y_t = the optimal exogenous regress which may be constant or constant trend.
γ and β are parameters to be estimated. Reject Ho if t_γ is less than asymptotic critical values.
Decision rule: Reject Ho if t_γ is less than asymptotic critical value.

2.2 Model Specification: Quadratic Exponential Smoothing

The primary forecasting model employed in this study is Quadratic Exponential Smoothing (QES). QES extends traditional exponential smoothing models to accommodate quadratic trends. The model is formulated as follows:

$$\widehat{Y}_{t+h|t} = (l_t + h)\,(b_t + \frac{h^2}{2})(C_t) \quad (3)$$

Where:

$\widehat{Y}_{t+h|t}$ = is the forecast at time t + h given information up to time t.
l_t = represents the level (current estimate of the series).
b_t = represents the slope (current estimate of the trend).
C_t = represents the curvature (current estimate of the quadratic trend).
h = is the forecasting horizon.

The model parameters are updated using the following equations:

$$l_t = \alpha\left(Y_t - \frac{h}{2} \times C_t\right) + (1-\alpha)(l_{t-1} + b_{t-1}) \quad (4)$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \quad (5)$$

$$C_t = \gamma(l_t - 2l_{t-1} + l_{t-2}) + (1-\gamma)C_{t-1} \quad (6)$$

Where:
α, β, and γ are smoothing parameters between 0 and 1.

2.3 Parameter Estimation

Parameter estimation is a critical step in implementing the Quadratic Exponential Smoothing (QES) model for forecasting monthly fertilizer prices in Nigeria. The accurate determination of model parameters, including the initial values of the level, slope, and curvature components (l_0, b_0, c_0), as well as the smoothing parameters, is essential for optimizing the model's fit to historical data.

2.4 Initial Values

The initial values (l_0, b_0, c_0) represent the starting points for the level, slope, and curvature components of the time series. These values are crucial for the accurate initialization of the model. Various methods can be employed to estimate these initial values, including simple averages, historical averages, or more sophisticated techniques such as exponential smoothing. The choice of method depends on the characteristics of the specific dataset and the underlying patterns in the historical fertilizer prices.

2.5 Smoothing Parameters

The QES model involves smoothing parameters (α, β, γ) that control the weights assigned to the observed data and the estimated components. These parameters determine the degree of influence that recent observations and trends have on the forecast. The optimization process aims to find the values of α, β, and γ that minimize the difference between the observed and predicted values, optimizing the overall performance of the model.

3. DIAGNOSTIC CHECKING

After model estimation and validation, diagnostic checks were conducted to assess the adequacy of the QES model. This involves examining the residuals for autocorrelation, heteroscedasticity, and other violations of model assumptions. If necessary, further adjustments or refinements will be made to improve the model's performance.

3.1 Ljung-Box Test on the residual

The conformity of white noise residual of the model fit will be judged by plotting the ACF and PACF of the residual to see whether it does not have any pattern or we perform Ljung-Box Test on the residual.
The test hypothesis:
Ho: There is no serial correlation
H1: There is serial correlation

The test statistics of the Ljung-Box
$LB = (n + 2) \sum \rho_k\ 2\ _{n-k\ m\ k}$ distributed to $xm{-}e\ ^2$ (7)

Where
n is the sample size,
$m = lag\ length$ and
p is the sample autocorrelation coefficient.

The decision: if LB is less than critical value of x_2, then we do not reject the null hypothesis. This means that a small value of Ljung-Box statistic will be in support of no serial correlation or i.e. the errors are normally distributed. This is concern about model accuracy.

3.2 Jarque-Bera Test for Normality.

This is yet another diagnostic test for conditional heteroscedasticity. It is a joint test-sample skewness and sample kurtosis [17] of whether the data has the characteristics of the normal distribution. The test was developed to rest normality, heteroscedasticity and serial correlation (autocorrelation) of regression residuals by [18]. It compares the 3rd and 4th moments (Skewness and Kurtosis) to those from a normal distribution. The null hypothesis is H_0: $a_{1\ldots}$, am is normally distributed versus $H_1 a_1 \ldots a_m$ is not normally Distributed and asymptotically the test statistics is x^2 distributed with 2 degrees of freedom

The Jargue-Bera test statistic is defined as:

$$\frac{N}{6}\left(S^2 + \frac{(k-3)^2}{4}\right) \qquad (8)$$

Where s, k, and N represent the sample skewness, the sample kurtosis and the sample size of the macroeconomic variables respectively.

The third moment $\sum_{i=1}^{n}(X_i - \bar{X})^3$ is used to denote skewness of a distribution

Where $x_1$represents oil, inflation, exchange rate and GDP while x signifies individual mean; n is the sample size, S is the standard deviations of each variables.

$$\text{Skewness} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{nS^3} \quad (9)$$

It is measure of symmetry of the slope in the normal distribution. If skewness is greater than zero, the distribution is skewed to the right, having more observations on the left [19]

The fourth moment $\sum_{i=1}^{n}(X_i - \bar{X})^4$ is used to define the kurtosis of a distribution

$$\text{Kurtosis} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{nS^4} \quad (10)$$

Kurtosis is a measure of the flatness (called platykurtic) or peakedness (leptokurtic) of a normal distribution. If kurtosis of a variable is less than three, the distribution has thicker tails and a lower peak compared to a normal distribution. In contrast, kurtosis larger than 3 denotes a higher peak and thin tails. A normally distributed random variable should have skewness and kurtosis near zero and three, respectively [20].

4. FORECASTING EVALUATION

Forecasting Evaluation Criteria Numerous error measures are available for forecasts evaluation; thus this study evaluates the forecasting ability of state space and Box-Jenkins type models by means of three different loss functions. These are root mean squared error (RMSE), mean absolute error (MAE) and Theil's U statistic which are defined as follows;

$$\boldsymbol{RMSE} = \sqrt{\boldsymbol{MSE}} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2} \quad (11)$$

$$\boldsymbol{MAE} = \frac{1}{n}\sum_{t=1}^{n}|(A_t - F_t)^2| \quad (12)$$

$$Ut = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - A_{t-1})^2}} \quad (13)$$

where
A_t is the actual value in time t, and
F_t is the forecast value in time t.

Theil's U statistic compares the forecast accuracy of different models. The overall perform of the estimating methods were accessed using the average of the three loss functions, that is $Average = (RMSE + MAE + Ut)/3$, the method with the minimum Average is the best [20]

4.1 Empirical Results
4.1.1 Data Presentation

The data used in this study consists of monthly prices of fertilizer in Nigeria from January 1960 to December 2022. The data exhibits a clear upward trend with some periods of fluctuations as shown in figure 1.

Table 1: Descriptive Statistics for Fertilizer Prices

Count	752
Average	51.7423
Standard deviation	43.0208
Coeff. of variation	83.1444%
Minimum	8.33
Maximum	256.06
Range	247.73
Stnd. Skewness	21.7938
Stnd. Kurtosis	25.5313

Table 1 is the descriptive statistics for fertilizer price in Nigeria from Jan.1960 to Dec. 2022

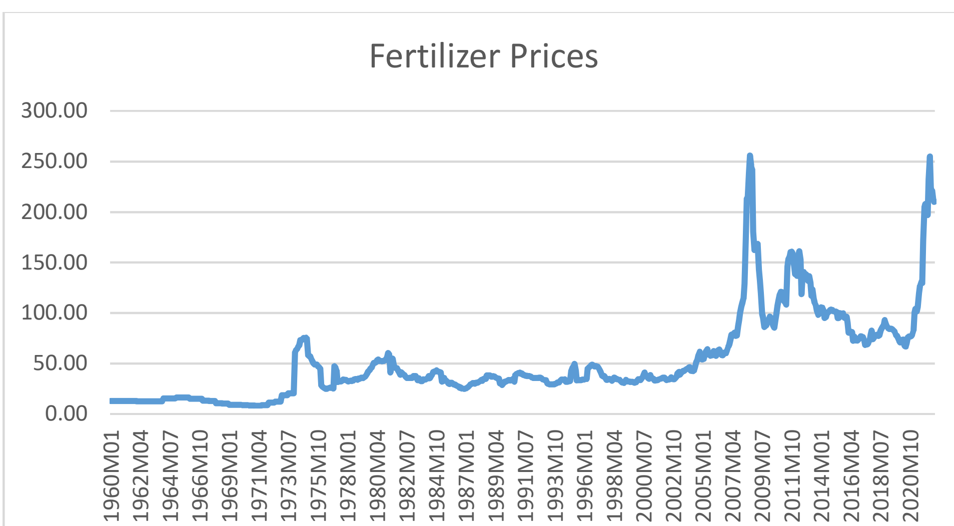

Figure 1: Time Plot of the monthly Fertilizer Prices

Figure 1 shows the monthly prices of fertilizer in Nigeria from January, 1960 to December, 2022.

4.2 QES Model

QES is a forecasting method that is designed to handle data that exhibits a quadratic trend. The model uses three smoothing parameters: α, β, and γ. The parameter α controls the weight given to the most recent observation, the parameter β controls the weight given to the trend, and the parameter γ controls the weight given to the curvature of the trend. The QES forecast for the next period is then calculated as a weighted average of the past observations, the trend, and the curvature of the trend.

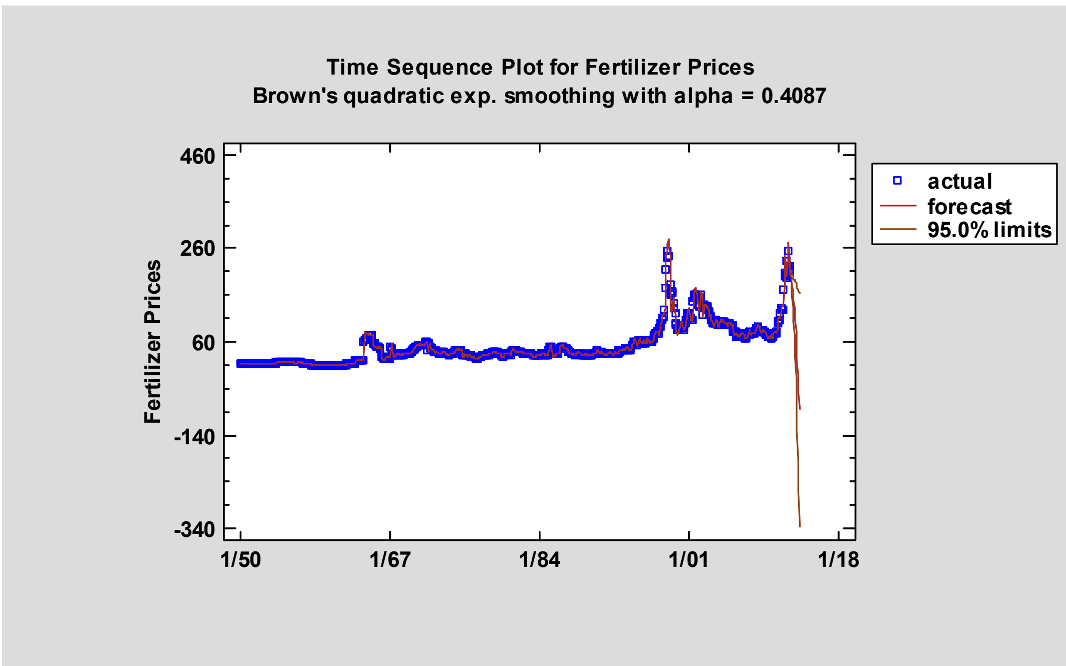

Figure 2: Time Sequence for Fertilizer Prices with Quadratic Exponential Smoothing with alpha = 0.4087

Figure 2 is the Forecast Plot with Quadratic Exponential Smoothing with alpha = 0.4087. The plot shows the observed and forecasted values of Fertilizer Prices. Also included on the plot are 95.0% prediction limits for the forecasts. These limits show where the true value of Fertilizer Prices at any point in the future is likely to be with 95.0% confidence.

4.2.1 Parameter Estimation

The parameters of the QES model were estimated using the grid search method. The grid search method involves evaluating the model's performance for a range of values for each parameter and selecting the values that result in the best performance. The performance of the model was evaluated using the mean squared error (MSE) and the mean absolute percentage error (MAPE).

Table 2 Model Summary
Quadratic trend = 19.1754 - 0.028298t + 0.000228841 t^2

Parameter	Estimate	Stnd. Error	T	P-value
Constant	19.1754	3.06907	6.24795	0.000000
Slope	-0.028298	0.0188237	-1.50332	0.132756
Quadratic	0.000228841	0.0000242065	9.4537	0.000000

Statistic	Estimation Period	Validation Period
RMSE	28.1625	
MAE	17.9567	
MAPE	39.1486	
ME	-0.00268534	
MPE	-19.1345	

Table 2 provided the estimated parameter values for the quadratic trend of Fertilizer Prices. The data cover 752 time periods. This model assumes that the best forecast for future data is given by a quadratic regression curve fit to all previous data. Each value of Fertilizer Prices has been adjusted using multiplicative seasonal adjustment before the model was fit. Table 2 also summarizes the statistical significance of the terms in the forecasting model. The terms with P-values less than 0.05 are statistically significantly different from zero at the 95.0% confidence level. In this case, the P-value for the quadratic term is less than 0.05, so it is significantly different from 0.

Also, table 2 summarizes the performance of the currently selected model in fitting the fertilizer price. It displays: the root mean squared error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), the mean error (ME) and the mean percentage error (MPE). Each of the statistics is based on the one-ahead forecast errors, which are the differences between the data value at time t and the forecast of that value made at time t-1. The first three statistics measure the magnitude of the errors. A better model will give a smaller value.

4.3.2 Model Comparison

Models
(A) Quadratic trend = 19.1754 - 0.028298t + 0.000228841t^2
    Seasonal adjustment: Multiplicative
(B) Constant mean = 51.7441
    Seasonal adjustment: Multiplicative
(C) Simple moving average of 3 terms
    Seasonal adjustment: Multiplicative
(D) Simple exponential smoothing with alpha = 0.9999
    Seasonal adjustment: Multiplicative
(E) Brown's quadratic exp. smoothing with alpha = 0.4074
    Math adjustment:

Table 4.3: Estimation Period for Fertilizer Price

Model	RMSE	MAE	MAPE	ME	MPE
(A)	28.1625	17.9567	39.1486	-0.00268534	-19.1345
(B)	43.3416	31.4532	107.177	-0.00357776	-80.8803
(C)	8.78365	3.67822	5.36638	0.535026	0.16205
(D)	6.12038	2.35911	3.44802	0.263176	0.106232
(E)	7.07998	3.07934	4.67017	-0.0490186	0.0121965

Model	RMSE	RUNS	RUNM	AUTO	MEAN	VAR
(A)	28.1625	***	***	***	**	***
(B)	43.3416	***	***	***	***	***
(C)	8.78365	***	***	***	OK	***
(D)	6.12038	OK	*	***	OK	***
(E)	7.07998	***	***	***	OK	***

Key:
RMSE = Root Mean Squared Error
RUNS = Test for excessive runs up and down
RUNM = Test for excessive runs above and below median
AUTO = Ljung-Box test for excessive autocorrelation
MEAN = Test for difference in mean 1st half to 2nd half
VAR = Test for difference in variance 1st half to 2nd half
OK = not significant (p >= 0.05)
* = marginally significant (0.01 < p <= 0.05)
** = significant (0.001 < p <= 0.01)
*** = highly significant (p <= 0.001)

Table 4.3 compares the results of five different forecasting models. Looking at the error statistics, the model with the smallest root mean squared error (RMSE) during the estimation period is model D. The model with the smallest mean absolute error (MAE) is model D. The model with the smallest mean absolute percentage error (MAPE) is model D. The table also summarizes the results of five tests run on the residuals to determine whether each model is adequate for the data. An OK means that the model passes the test. One * means that it fails at the 95% confidence level. Two *'s means that it fails at the 99% confidence level. Three *'s means that it fails at the 99.9% confidence level. The currently selected model, model A, passes all five tests.

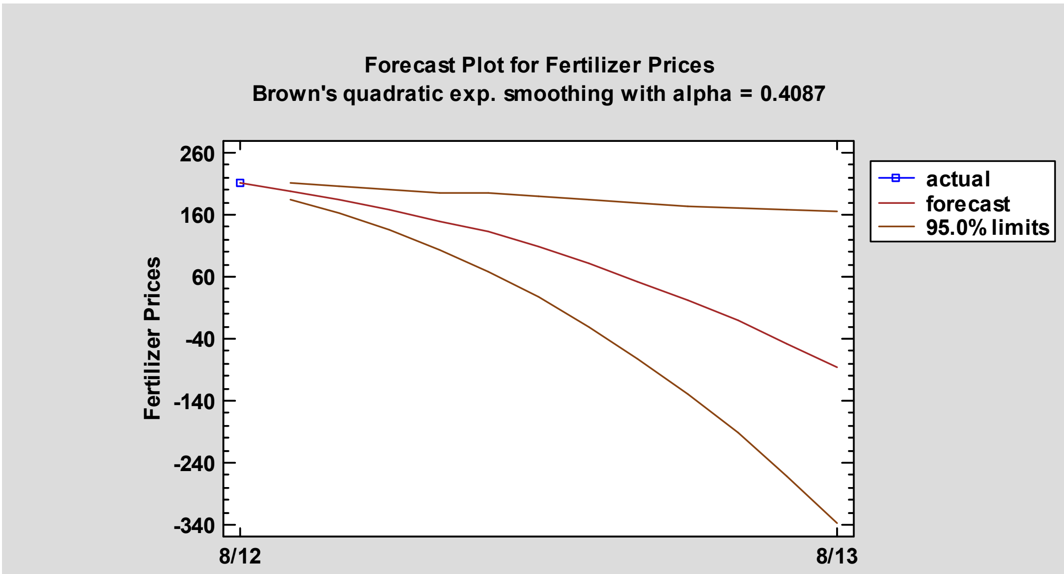

Figure 3: Forecast Plot for Fertilizer Prices

Figure 3 shows the forecasted values of Fertilizer Prices. Also included on the plot are 95.0% prediction limits for the forecasts. These limits show where the true value of Fertilizer Prices at any point in the future is likely to be with 95.0% confidence.

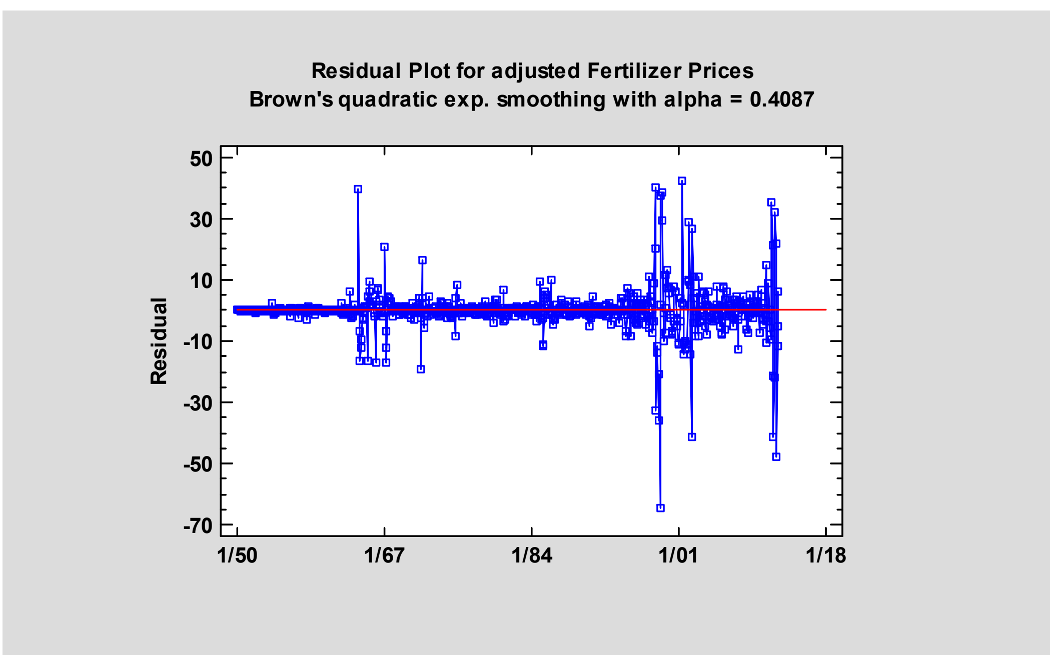

Figure 4: Residual Plot for Adjusted Fertilizer Prices

Figure 4 shows the residuals from the fitted model. since the model captures all of the dynamic structure in Fertilizer Prices, the residuals should are random (white noise).

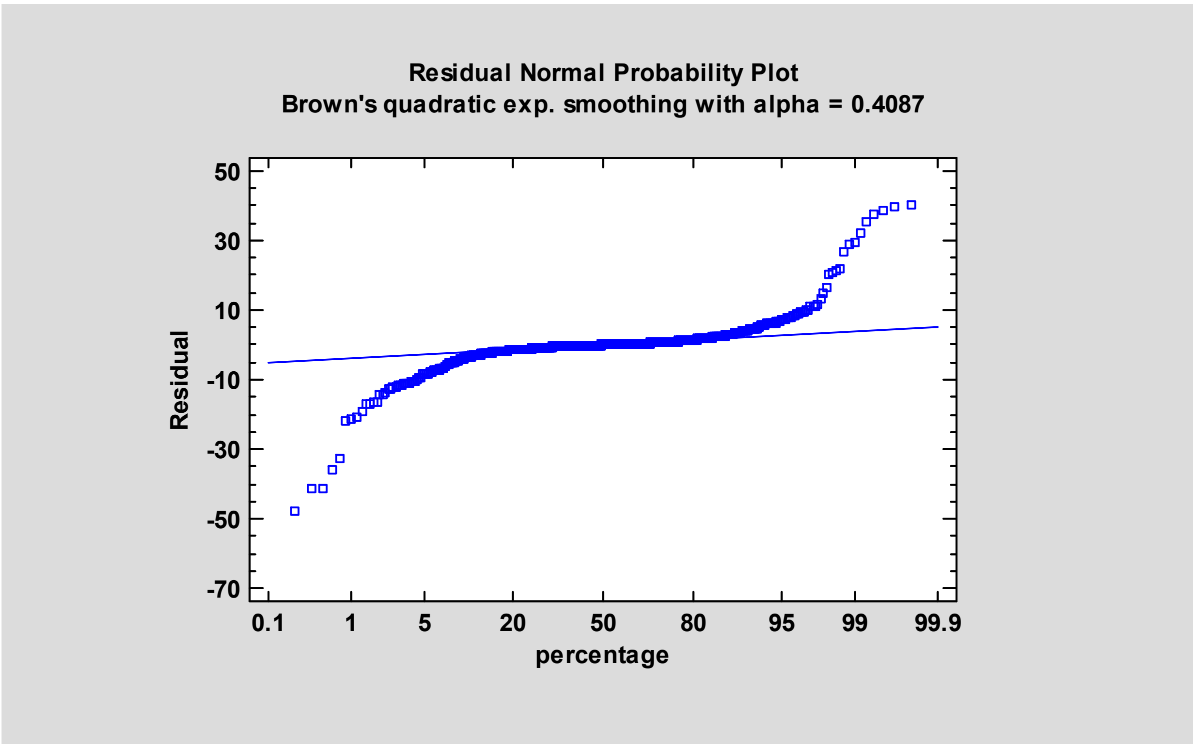

Figure 5: Residual Normal Probability Plot

Figure 5 shows a normal probability plot for the residuals from the fitted model. To create this plot, the residuals have been sorted from smallest to largest. They have then been plotted versus the values (i-0.375)/(n+0.25), where n is the number of residuals.

5. CONCLUSION

In this research, the analysis of the provided data reveals a discernible upward trend with intermittent periods of fluctuations. The application of Quadratic Exponential Smoothing with an alpha value of 0.4087 to forecast future trends resulted in a quadratic trend equation: $Y_t = 19.1754 - 0.028298_t + 0.000228841t^2$. The coefficients associated with this quadratic equation, including their estimates, standard errors, t-values, and p-values, are summarized as: The constant term (intercept) in the quadratic trend equation is estimated to be 19.1754, with a standard error of 3.06907. The associated t-value of 6.24795 is highly significant (p-value < 0.0001), indicating a strong evidence against the null hypothesis that the constant is equal to zero. The coefficient for the slope term (-0.028298) represents the linear component of the trend. Although the t-value of -1.50332 suggests a lack of statistical significance at the conventional 0.05 significance level (p-value = 0.132756), it is important to interpret this result cautiously and consider the broader context.

The quadratic term (0.000228841) has a highly significant t-value of 9.4537 (p-value < 0.0001), indicating a strong statistical evidence in favor of its inclusion in the model. This implies that the quadratic component significantly contributes to capturing the curvature in the trend observed in the data. Figure 3 visually presents the forecasted values of Fertilizer Prices based on the quadratic exponential smoothing model with the associated quadratic trend equation. Additionally, the plot incorporates 95.0% prediction limits, providing a valuable measure of uncertainty around the forecasted values. Overall, the model suggests that the data follows a quadratic trend, characterized by an initial rise followed by a potential deceleration and acceleration in the rate of increase.

Data Availability: The data used to support the research findings are available from World Bank Commodity price data (The Pink Sheet) in the website: https://www.worldbank.org/en/research/commodity-markets.

Conflicts of Interest: The authors declare no conflict of interest

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