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On Thin-Flame Effect on Temperature of a Combustion Problem.

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ABSTRACT

The paper revisits a simple model for a problem in combustion theory It also investigates the effect of flame thickness/velocity on the temperature of the reaction. Of particular interest are the criteria for the existence of unique solution of the resulting problem. The paper also provides a numerical solution by shooting method even when the reaction is assumed to be highly exothermic. The numerical results show that flame thickness and Frank-Kamenetskii parameters have appreciable effects on temperature of the reaction.

Keywords: Temperature, thin flame, Uniqueness, Shooting Method)

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1. INTRODUCTION

Research into combustion theory had attracted the interest of many fluid dynamists in the past and in recent times because of its inevitable demands in the areas of engineering and other relevant industries. It is not doubtful that the results obtained by [1-20] have added values to academic communities and the larger society. Thin-flame technique is an important tool which is specifically applicable to in-situ combustion in the recovery of oil from the earth crust. In this paper, the technique will be applied to ascertain the effect of flame velocity on the maximum temperature of the reaction and to establish that temperature field changes with time. Tam and Kiang [1] considered a simple model for combustion theory and extended the results he obtained by Tam for the case where the domain D is a sphere. They investigated for the response of the governing equation to a hot spot when $\delta_e < \delta < \delta_{cr}$. In particular, they estimated the extent and temperature of the hot spot for the problem to reach a super-critical state.

2. MATHEMATICAL EQUATIONS

Following [1], a simple model for a problem in combustion theory is governed by

$$\frac{\partial T}{\partial t} = \nabla^2 T + \delta \exp\left(\frac{\alpha T}{\alpha + T}\right) \; in \;\; D \times \{t : t > 0\} \tag{1}$$

together with the initial and boundary conditions

$$T(x,0) = h(x) \; and \;\; T = 0 \;\; on \;\; \partial D \tag{2}$$

where T, x and t are respectively the dimensionless temperature, spatial and time variables, δ a parameter and α is a constant with magnitude greater than 4.

The paper therefore considers the domain D as axial, one dimensional region where combustion processes leading to ignition occur.

Then the governing equation becomes

$$\frac{\partial T}{\partial t} = \beta \frac{\partial^2 T}{\partial x^2} + \delta \exp\left(\frac{\alpha T}{\alpha + T}\right) \tag{3}$$

satisfying $T(x,0) = h(x), \; T(\pm\infty, t) = 0$ (4)

2.1 Thin Flame Technique

Let $T(x,t) = \theta(\eta) \;\; such \;\; that \;\; \eta = x - vt.$ (5)

where v is the flame velocity.

Case 1: $\frac{1}{\alpha} \neq 0$

We obtain the equation

$$\frac{d^2\theta}{d\eta^2} + v\frac{d\theta}{d\eta} + \delta \exp\left(\frac{\alpha\theta}{\alpha + \theta}\right) = 0 \tag{6}$$

$$\theta(\pm\infty) = 0 \tag{7}$$

2.2 Existence of Unique Solution:

Following [3,4,5,7, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20], we resolve problem (6) subject to condition (7) into a system of equations and establish the criteria for the existence of unique solution.

Let $$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \eta \\ \theta \\ \theta' \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} \eta \\ \theta' \\ \theta'' \end{pmatrix} \tag{9}$$

$$\Rightarrow \begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} \eta \\ y_3 \\ -vy_3 - \delta e^{\frac{\alpha y_2}{\alpha + y_2}} \end{pmatrix} \tag{10}$$

$$\Rightarrow \begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} f_1(y_1, y_2, y_3) \\ f_2(y_1, y_2, y_3) \\ f_3(y_1, y_2, y_3) \end{pmatrix} \quad (11)$$

satisfying $\begin{pmatrix} y_1(-m) \\ y_2(-m) \\ y_3(-m) \end{pmatrix} = \begin{pmatrix} -m \\ 0 \\ G_1 \end{pmatrix}$ (12)

where $-m \le \eta \le m$ and G_1 is guessed such that $y_2(m) = 0$

Theorem:

Let D denotes the region (in four dimensional space, one dimension for η and three dimension for y_1, y_2, y_3).

Let $\left|\frac{\partial f_i}{\partial y_j}\right|$ be continuous and bounded on D. Then there is a constant $\delta > 0$ such that there exists a unique continuous vector solution $\underline{y} = (y_1, y_2, y_3)$ which satisfies equations (11) and (12).

Proof.

Considering equations (10) and (11), $\left|\frac{\partial f_i}{\partial y_j}\right|, \ i, j = 1,2,3$ are continuous in D and bounded on D. Therefore, $f(x,\eta)$ satisfies the Lipschitz condition. Hence, the problem has a unique solution.

Case 2: $\frac{1}{\alpha} \to 0$ (For highly exothermic reaction)

The equation (6) becomes

$$\frac{d^2\theta}{d\eta^2} + v\frac{d\theta}{d\eta} + \delta \exp(\theta) = 0 \quad (13)$$

satisfying the condition

$$\theta(\pm m) = 0 \quad (14)$$

We resolve equation (13) subject to condition (14) into a system of equations

Let $$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} \eta \\ \theta \\ \theta' \end{pmatrix} \qquad (15)$$

$$\Rightarrow \begin{pmatrix} z_1' \\ z_2' \\ z_3' \end{pmatrix} = \begin{pmatrix} \eta \\ \theta' \\ \theta'' \end{pmatrix} = \begin{pmatrix} \eta \\ z_3 \\ -vz_3 - \delta e^{z_2} \end{pmatrix} \qquad (16)$$

satisfying $$\begin{pmatrix} z_1(-m) \\ z_2(-m) \\ z_3(-m) \end{pmatrix} = \begin{pmatrix} -m \\ 0 \\ G_2 \end{pmatrix} \qquad (17)$$

where $-m \le \eta \le m$ and G_2 is guessed such that $z_2(m) = 0$

Then $\left|\dfrac{\partial f_i}{\partial x_j}\right|_{i,j=1,2,3}$ are continuous in D and bounded on D. Therefore, $f(\overline{X}, \eta)$ satisfies the Lipschitz condition.

Hence, problem (11) subject to condition (12), has a unique solution.

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2. NUMERICAL RESULTS

We provide the numerical solutions to problems in equations (6) and (13) subject to conditions (14) by using Runge-Kutta shooting method. It should be noted that equations (6) and (13) subject to condition (14), were respectively resolved into systems of equation (10) subject to condition (12), and equation (16) subject to condition (17). A computer programme written in Pascal language, was used to solve each of the problems.

Case 1: $\frac{1}{\alpha} \neq 0$

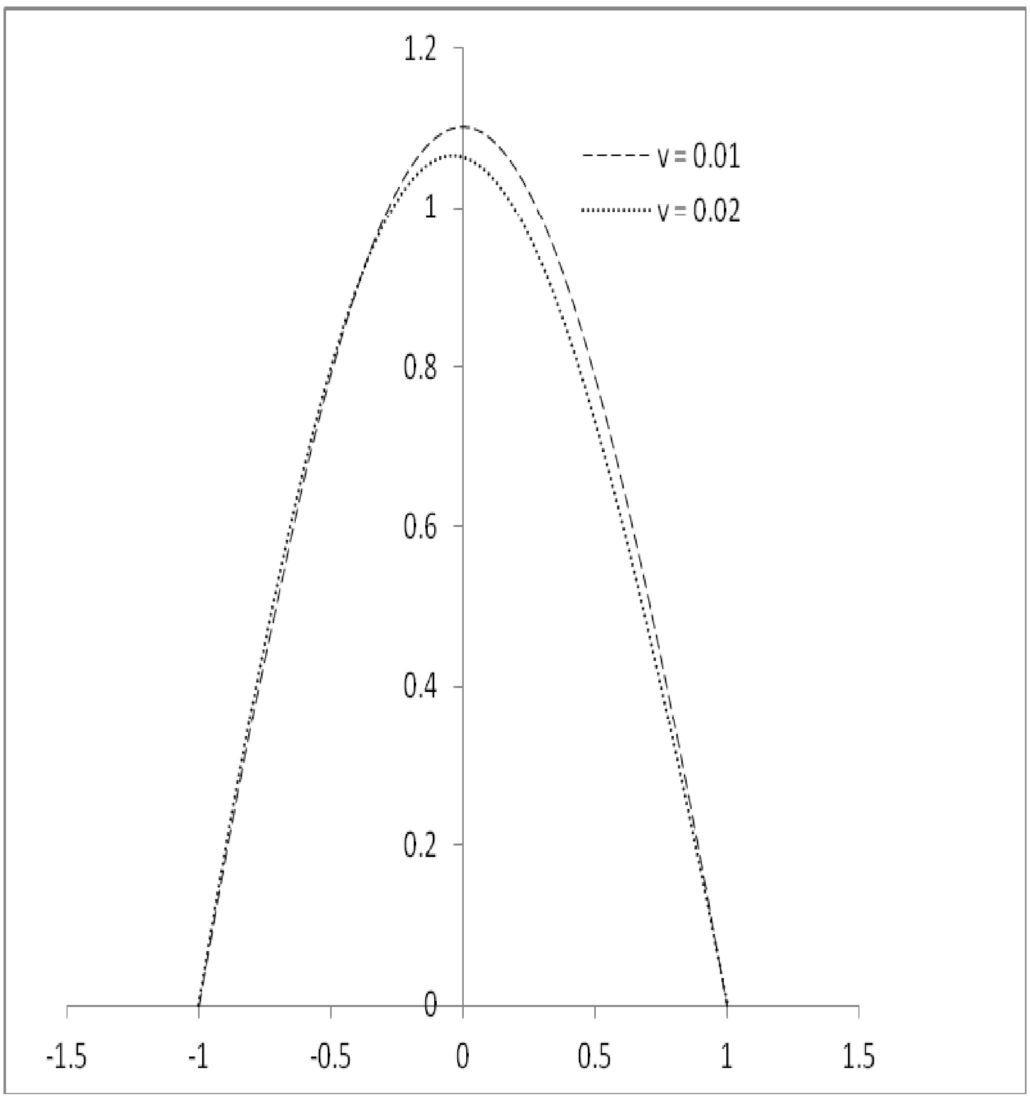

Figure 1: Temperature profile for $\frac{1}{\alpha} \neq 0,\ \alpha = 100,\ \delta = 0.878$ **for various flame thickness** v

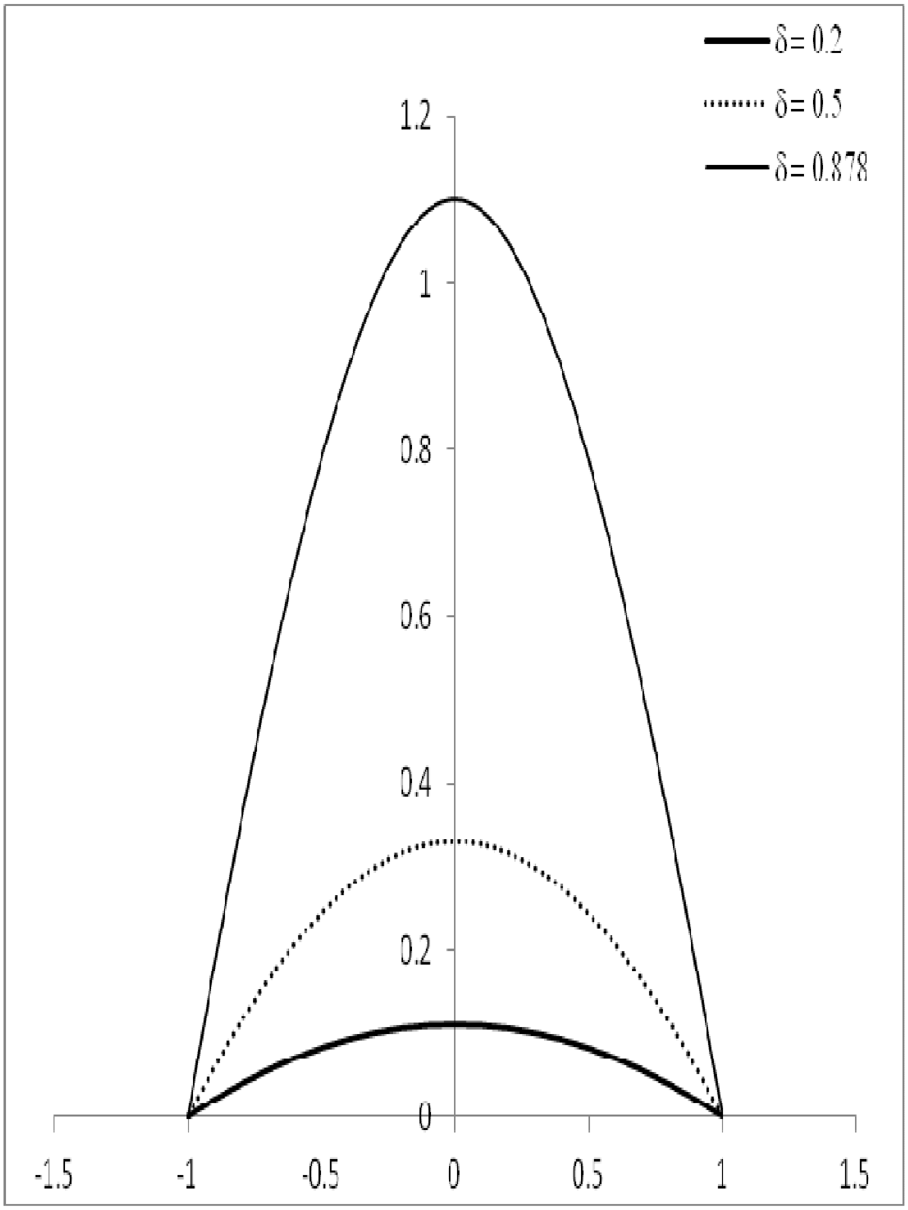

Figure 2: Temperature profile for $\frac{1}{\alpha} \neq 0,\ \alpha = 100,\ v = 0.01$ **, for various** δ

Case 2: $\frac{1}{\alpha} \to 0$

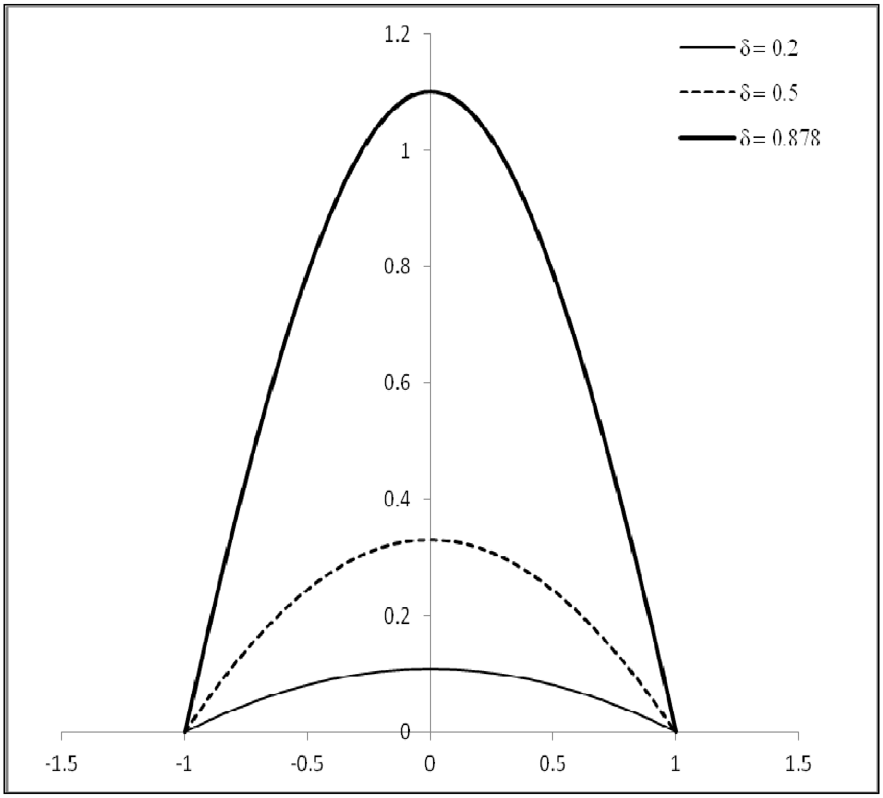

Figure 3: Temperature profile for $\frac{1}{\alpha} \to 0,\ v = 0.01$ v = and **for various** δ

3. DISCUSSION OF RESULTS AND CONCLUSION

The proof of the theorem established the fact that the problem has a unique solution and it therefore represents a physical problem. Figure 1 shows that the flame thickness v regulates the maximum temperature of the reaction. Figure 2 shows that an increase in δ increases the maximum temperature of the reaction even when $\delta_{cr} = 0.878$ Figure 3 shows that even for highly exothermic reaction, an increase in δ increases the maximum temperature of the reaction. In conclusion, flame thickness and Frank-Kamenetskii parameters have appreciable effects on temperature of the reaction. This study therefore has greater applications in many engineering and scientific processes.

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