Performance Evaluation of Hybrid Routing Rule for Call Center Operations: Using Mathematical Techniques

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ABSTRACT

Customers usually experience queue often times when they call a call center to meet their information need or make an inquiry. The queue experienced by customers at call centres can be alarming often times. Many customers are irritated by the long time spent on the queue before their calls are been answered. Call center agents are trained to handle all entry calls to a call centre but are they characterized with different performance level for the call in terms of average call handling time (AHT) and call resolution (CR). The main purpose of this paper is to evaluate the performance of hybridized routing rule Hybrid Heterogeneous call Routing Rule (HHCRR) proposed by (Mughele et al, 2017b). The method adopted is mathematical techniques by assuming 15 working hours of the day and subjected the period of time assumed into the equation. Since the proposed hybrid rule is designed to optimize, the researchers determined the viability of the rule by testing the hybrid rule mathematically. The result from the mathematical techniques deployed shows minimization of wait-time and maximization of call resolution hence optimization is achieved, validating the hybrid rule to be implementable.

Keywords: Routing Rule, Hybrid Rule, Call Center, Mathematical Techniques

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1. INTRODUCTION

A customer's experience during a service encounter consist of two parts namely: the time spent waiting for the service and the service itself. Call centres give priority to the two criteria with emphasis on one more than the other. Those that place more emphasis on time spent waiting for the service are more concerned with reducing the average time involved in handling a call while those that are concerned with the service itself aims at effective resolution of customer issues. Armony (2005) says for a call centre to reduce waiting lines with emphasis on the reduction of time spent, its best to route calls to agents who can handle customer issues the fastest, sometimes even holding a call in queue to wait for that agent than routing the call to a slower agent. This might lead to further increase in congestion, repeat calls from unreceptive issues and undue burden on some agents.

Vericourt et al. (2005), states that for a call centre to reduce waiting lines, emphasis should be on the service itself that is; call resolution. Its best to route calls to agents who resolve customer issues, sometimes holding a call in queue to wait for such agent this might also lead to increase in congestion and undue burden on some agents. After a customer has received service from a call centre agent on a particular issue, a subsequent call from that customer about the same issue is a clear sign that the issue had not been resolved during the previous service encounter, and this lack of resolution is a strong sign of customer dissatisfaction. Call center agents are trained to handle all entry calls to a call centre but they ate characterized with different performance level for the call in terms of average call handling time (AHT) and call resolution (CR). .Hence there is need to hybridize both wait-time and call resolution routing rule to enhance performance.

The main purpose of this paper is to mathematically demonstrate that the hybrid rule/algorithm is effective and can be implemented to obtain optimality when deployed for call center operation.

2. RELATED LITERATURE

Customer service call centres have obviously become a very integral part of many organisations' business operations today, inbound call centres employ millions of agents across the globe and serve as a primary customer-facing channel for many different industries. There has also been a great deal of research interest in call centre operations management, with the extensive and evolving literature thoroughly analysed (Mehrotra *et al,* 2009). Hart *et al.* (2006) provides a complete review of articles on First Call Resolution (FCR), while also pointing out the importance of measuring and using FCR. Resolving customer queries the first time around is a commonly shared goal.

Zhan and Ward (2006) noted that the challenge in call centre operation is how to determine the relevant control in the routing; that is, the decision concerning which agent should handle an arriving call when more than one agent is available. Garcia. *et a*l. (2012), noted that as time spent on queue at the call centres increases, it becomes unacceptable for customers, and this affect their satisfaction level.
Stanley *et al* (2008) posited that in a service base call centre, the two key challenges are (i. Where should a call be routed to and) (ii. Who should handle the call?) They deployed base case FIFO approach for the simulation to analyse performance-based routing strategies in call centres. Their work shows the potential for significant improvements in call centre performance especially Average Speed to Answer (ASA). This was achieved by using rules based on historic performance data such as Average call Handling Time (AHT) and first call Resolution (FCR) rates.

Garcia. *et a*l. (2012), noted that as time spent on queue at the call centres increases, it becomes unacceptable for customers, and this affect their satisfaction level. A study was conducted using Univariate Analysis of Variance (ANOVA) to determine customer's perception of their wait experience at call centres. Their result showed that though the time spent on the queue waiting can lead to customer's dissatisfaction, nevertheless, it is not as important as the agent's ability. More so, the concept of routing rules to be deployed for efficient call resolution rate was not emphasised.

Dabrowski (2013) observed that the key performance indicators to measure call centre metrics performance such as average speed of answer, cost per call, agent utilization rate, first contract resolution rate, customer satisfaction and aggregate call centre performance are not effectively maximised. He used CallLogic system to improve the fundamental call routing logic of the Northeast Utilities call centres. Although the findings from the CallLogic system led to discoveries and ideas on how to improve the fundamental call routing logic of the Northeast Utilities call centres, the CallLogic project achieved high success in the average call handling time. The study only made mention of call Resolution rate and its impact on operational success.

The quality of service accessibility and customer waiting time are dominant performance measures (Vericourt and Zhou 2005b). Hence capacity planning and call routing software system strive to minimize cost while achieving self imposed service level constraints, though considering low average time waiting in queue, these approach do not consider the quality of service rendered to customers (Vericourt and Zhou 2005b). Low quality of service has significant impact on the call centre operations; this operational impact of service failure is often ignored by call centre capacity planning and call routing management system. Their work was motivated by the fact that a major European telecommunications service provider discovered that customers needed to talk to more than three different agents before their problems are resolved.

Read (2002), also observed that when using routing rules that emphases on reducing queues, calls are quickly routed to agents, without considering the root of the problem being fixed, to avoid a call back of such customers on the same problem. Garcia et al (2012) also noted that call centre managers and decision makers tends to only look for information that simply confirms existing beliefs and often disregard all other information, the authors believes that these will enable such call centre operators implement a convenient routing rule even if it is not the optimal rule.

Jouine et al (2007), their work considered two basic multiclass call center models, with and without reneging the challenge / problem is that customers at times encounter delay upon arrival at a call center. The first method used was to estimate virtual delays that will be used within the announcement step. Their second model, they develop a call center incorporating reneging, The model takes into consideration change in customer behavior that may occur when delay signals or information is communicated to the customers in order to estimate virtual delays of new arrivals. To improve customers satisfaction and reduce congestion, call center have in recent times started experimenting by informing arriving customers about anticipated delay, this is noted in (Armony and Maglaras 2004), When customers are waiting on the queue, they have no means of estimating the queue lengths or progress rate, hence there is tendency for increase in axiety during the waiting period.

Basically, Jouine et al (2007) from the result presented were able to model customer's reactions to delay announcements in such a setting with priorities and to provide an analysis for this case. Their work also introduces the need for announcing delays when a new customer finds the queue empty.
Adan et al. (2013), in a related work, considered a system based on assumption that the system is overloaded and a such all server are always busy and a fraction of the customers are forced to abandonment. They deployed FCFS and skilled based routing rule methodology. The authors also emphasized that to optimize performance, determining the right level of cross-training seems more important than laying more emphasis on the choice of routing rule.

Adan et al, (2013), specifies the system as follows;

Customers are of types C = { C1- - - - -, C1 }, servers are of types S = {S1 - - - - -,SJ}, and a bipartite gragh G of compatible matches between C,S. Graph G specifies the level of cross-training; it has arc (i,j) £ G if server type Sj has the skills to serve the customer type Ci. An additional assumption follows:

Customers of type Ci arrive in independent Poisson stream or distribution of rate λ_i, and have patience distribution F_i which is absolutely continuous. There exist $n\hat{J}$ servers of type $\acute{C}_i$ by a server of type Sj has a random duration distributed as Gij, with rate $\mu_{ij} = 1/\mu_{ij}$. The notation subscript I for customers of type Ci and subscript j for servers of type Sj. Given that data λi, Fi, G, nj, Gij, under FCFS policy, include waiting times abandonment rates, routing flows between customer types and server types, and workload of each server type.

Selvi and Sathya (2012) observed that the Erlang B model is a formular for blocking, a probability derived from Erlang distribution. The Erlang B describes an unsuccessful call, when all servers are busy and the call is neither queued nor retired but loss completely. It is assumed that calls attempts arrive following a passion process (Selvi and Sathy, 2012; Osahenvem and Odiase 2016; Sathen 2010; and Aldor-Norman et al, 2010), so calls are independent. More also, it is assumed that message length (holding times) are exponentially distributed as depicted in (Markovian system) and this is generally applied under general holding time distributions. Usually, Erlangs are dimensionless quantities as average call arrival rate λ, which is multiplied by average call length, h. Blocking occurs when there are new calls waiting to be served and all servers are already busy. The Erlang B formular assumes that blocked traffic is immediately cleared.

Dabrowski (2013) observed that the key performance indicators to measure call centre metrics performance such as average speed of answer, cost per call, agent utilization rate. Mehrotra et al (2012), developed a framework for the optimisation of routing rules using first-come-first serve (FCFS) among wait-time routing rule and Probability Routing (PR), among call resolution routing rule to develop a hybrid rule. Mughele and Chiemeke (2016) evaluated the three call resolution oriented routing rules discussed by Mehrotra et al 2012. A similar study was conducted by Mughele et al (2017a) where a comparative analysis of waiting time routing rules for queue reduction in call center. Mughele et al, (2017c), deployed a simulation model that can be used for the optimisation of a call center. They deployed discrete event driven simulation to optimise the routing rule among call resolution orienting rule which is indicated to be SOR (Mughele and Chiemeke 2016).

In a related study conducted by (Mughele and chiemeke 2017), the authors specifically addressed the problem associated with the utilization of call center system in terms of service rate, waiting time of agent in an agent group. The number of call type i handled by a call center is determined by the maximal service rate of the agents in agent group j. The techniques they deployed was graph theory analysis to enhance optimisation

Mughele et al (2017b) developed a framework as a single routing rule that is able to solve Min/Max problem simultaneously in call center. This paper is an extension of (Mughele et al 2017b). This work is set to test and determine if the hybrid rule (HHCRR) proposed by Mughele et al (2017b), is efficient and implementable by mathematically evaluating the hybrid rule to determine its efficiency.

3. METHODOLOGY

In this work, we analyze various call routing rules for determining which calls should be handled by which call centre agents. The system design for the study used entity relationship diagram, while the use-case diagram was used to depict the different actors within a call center system. The study also compared the algorithms of some existing routing rules to determine the optimal. Four of the routing rules were wait-time routing rules and three were call resolution routing rule as obtained from (Mughele and Chiemeke, 2016) and (Mughele et al 2017a). The algorithms were tested with date collected from a Call Center of a Telecommunication Organisation in Nigeria. The optimal routing rule for wait-time and call resolution routing rules were SSTF and SQR respectively (Mughele and chiemeke, 2017). The algorithm for the two routing rules is hybridised, and tested with data from the call center to determine its performance, the hybrid was called Hybrid Heterogeneous Call Routing (CR) Rule (HHCRR) (Mughele et al 2017b). The performance of the algorithm was determined mathematically using data obtained from the call center. The mathematical procedure adopted was dry running of the hybrid algorithm by assuming 15 working hours of the day,

As adapted from Mehrotra *et al.* (2012), the benchmark routing rule will be the First-Come-First-Served/Longest-Wait (FCFS/LW) rule, because this is the routing rule deployed in majority of the call centres. The following model consist of the algorithm for the hybrid routing rule, which is made up the optimal of the wait-time routing rule (SSTF) (Mughele and Chiemeke 2016) and CR rate routing rule (SQR) (Mughele et al 2017a) expressed using the IF CASE STATEMENT.

Declaration

Start

Let $Q_i(t)$ represents the number of type i customers waiting for service at time t and
Let $f_j(t)$ be the number of available agents of type j who are free at time t,
Where $0 \leq f_j(t) \leq n_j$, for all j, t.
Let Multiple call types be indexed by i = 1, 2 ...I and
Let Multiple agent groups be indexed by j = 1, 2 ...J.
Calls of type i arrive at a rate of $\wedge_i$.
There are n_j agents in group j, with $n_j \varepsilon Z^+$
Each agent in group j serves call type i with rate μ_{ij}
/Here we allow agents to be trained to handle only a subset of all the call types/
If agent group j is not capable of handling call type I then $\mu_{ij} = 0$
When $\mu_{ij} > 0$ we say there is a "match" between call type i and agent group
In addition, we assume independent of past history each agent of group j has a resolution probability for each call of type i of $p_{ij} \varepsilon [0, 1]$.
/First Come First Serve/ Longest Waiting (FCFS/LW)/
    If($(0 \leq i \wedge_i)$ AND ($f_j(t) > 0$)
      There is a "match" between call type i and agent group
    Else
Agent group j is not capable of handling call type I
$\mu_{ij} = 0$
/Fastest Call First Rule (FCF)/
    If ($i \wedge_i$)
      If ($f_j(t) > 0$)
      Then
    $i = \text{argmax}_{i:Qi(t)>0}\{\mu_{ij} \mid \mu_{ij} > 0\}$;

Else
j = $\text{argmax}_{j:fj(t)>0}\{\mu ij\ |\mu ij > 0\}$;
/Shortest Service Time First (SSTF)/
If($n_j > 0$)

Then
J= $\text{argmax}_{i:Qi(t)>0}\{\mu_{ij} - \max_{k\neq j}\mu_{ik}\ |\mu_{ij} > 0\}$

Else
j = $\text{argmax}_{j:fj(t)>0}\{\mu_{ij} - \max_{k\neq j}\mu_{ik}\ |\mu_{ij} > 0\}$
/Highest Service Time First (HSTF)/
If($n_j > 0$)

i = $\text{argmax}_{i:Qi(t)>0}\{p_{ij}\mu_{ij}\ |\mu_{ij} > 0\}$;
Else
j = $\text{argmax}_{j:fj(t)>0}\{p_{ij}\mu_{ij}\ |\mu_{ij} > 0\}$

/Shortest Queue Routing (SQR)/
If($n_j > 0$)
i=$\text{argmax}_{i:Qi(t)>0}\{p_{ij}\mu_{ij} - \max_{k\neq j}p_{ik}\mu_{ik}\ |\mu_{ij} > 0\}$
Else
j = $\text{argmax}_{j:fj\ (t)>0}\{p_{ij}\mu_{ij} - \max_{k\neq j}p_{ik}\mu_{ik}\ |\mu ij > 0\}$
/Probabilistic Routing (PR)/
If($n_j > 0$)
i= $\text{argmax}_{i:Qi(t)>0}\{p_{ij}\ |\mu_{ij} > 0\}$
Else
j = $\text{argmax}_{j:fj(t)>0}\{p_{ij}\ |\mu_{ij} > 0\}$
/Relative Resolution Probability Routing (RRPR)/
If($n_j > 0$)
i=$\text{argmax}_{i:Qi(t)>0}\{p_{ij} - \max_{k\neq j}p_{ik}|\mu_{ij} > 0\}$
Else
j = $\text{argmax}_{j:fj(t)>0}\{p_{ij} - \max_{k\neq j}p_{ik}|\mu_{ij} > 0\}$
/Shortest Service Time First (SSTF)/
If($n_j > 0$)
Then
J= $\text{argmax}_{i:Qi(t)>0}\{\mu_{ij} - \max_{k\neq j}\mu_{ik}\ |\mu_{ij} > 0\}$

Else
j = $\text{argmax}_{j:fj(t)>0}\{\mu_{ij} - \max_{k\neq j}\mu_{ik}\ |\mu_{ij} > 0\}$
/Shortest Queue Routing (SQR)/
If($n_j > 0$)
i=$\text{argmax}_{i:Qi(t)>0}\{p_{ij}\mu_{ij} - \max_{k\neq j}p_{ik}\mu_{ik}\ |\mu_{ij} > 0\}$
Else
j = $\text{argmax}_{j:fj\ (t)>0}\{p_{ij}\mu_{ij} - \max_{k\neq j}p_{ik}\mu_{ik}\ |\mu ij > 0\}$
Stop.

Table 1 contains the operational variables deployed for the equations in this study.

Table 1: Operationalisation of research variables

Variable	Description of variables
$time\ periods\ t_i$	time period per day :7am to 9pm for all agents that is 15 hours per day
Call type i	Multiple call types such that i = 1, 2 ...I where I is 8 in our model
Agent j	Multiple agent groups such that j = 1, 2 ...J. where J is 35 in our model
$C_{j,t}$	Cost of an agent of type j having/working in time t
p_i	represent the proportion of call type i from the total new arrival that goes into the various call type i queue
$Q_i(t)$	number of type i call waiting for service at time t
$f_j(t)$	number of available agents of group j who are free at time t, where $0 \leq f_j(t) \leq n_j$, for all j, t.
λ_i	arrive rate of calls of type i
λ_T	The total arrival rate
n_j	no of agents in group j, such that $n_j \varepsilon Z^+$
X_{ij}	proportion of calls type i routed to agent group j
$X_{ij,t}$	proportion of calls type i routed to agent group j at time t
$y_{ij,t}$	No of agents in agent group j that handles call type i at time t
μ_{ij}	service rate of Agent group j for call of type i
μ_z	service rate of Agent group j for call of type i
β_i	arrival of unresolved calls of call type i who call back
β_{ij}	total arrival rate of agent group j for call type i who call back.
θ_{ij}	resolution probability of agent group j of call type i
ρ_j	total utilization of agent group j
$\Gamma_{i-}\Gamma_{i+}$	the lower and upper bound such that each call type i must be served at total utilization between bounds
ρ	proportion of time each server is busy

4.4 Proposed Hybrid Routing Rule

Here we focus on a rule that combines the optimal rule for call resolution and waiting time routing rules. As identified in previous analysis, Shortest Service Time First (SSTF) was the most optimal for Waiting-Time Routing Rules and Shortest Queue Routing (SQR) was the most optimal for Resolution Probabilistic Routing Rules.

For our hybrid algorithm, the following are assumed

1. A call of a particular type that arrives when agents of multiple matching groups are free will be routed to a matching agent group (j) that has the relatives Shortest Service Time and shortest queue for that call type.
2. Let t_i represent the time period per day 7am to 9pm i.e. 15 hours per day
3. Let Multiple call types be indexed by i = 1, 2 ...I where I is 8 in our model
4. Let Multiple agent groups be indexed by j = 1, 2 ...J. where J is 35 in our model
5. Let p_i represent the proportion of call type i from the total new arrival that goes into the various call type i queue. This is computed using

$$p_i = \frac{Total\ Calls\ of\ type\ i}{total\ Calls} \dots\dots\dots\dots\dots\dots. Equation\ 1$$

6. Let $Q_i(t)$ represents the number of type i call waiting for service at time t
7. Let $f_j(t)$ be the number of available agents of group j who are free at time t, where $0 \le f_j(t) \le n_j$, for all j, t.
8. Let λ_i. represents arrival rate of calls of type i such that $\lambda_i = t_i\lambda$
9. Let n_j represent no of agents in group j, with $n_j \in Z^+$
10. Let Xij represent proportion of calls type i routed to agent group j
11. Let μ_{ij} represent service rate of Agent group j for call of type i. This is computed by

$$u_{ij} = \frac{1}{AverateHandlingTime_j} \dots\dots\dots\dots\dots Equation\ 2$$

12. Let μ_z represents the total service rate of call type i computed using equation

$$\mu_z = \sum_{j=1}^{j} n_j u_{ij} \left(\frac{\frac{\lambda_{ij}}{u_{ij}}}{\sum_{i'} \frac{\lambda_{i'j}}{u_{i'j}}} \right) \dots\dots\dots\dots\dots\dots\dots. Equation\ 3$$

13. Let β_i accounts for arrival of unresolved calls of call type i who call back. This is computed using equation

$$\beta_i = \frac{\lambda_i}{1 - \sum_j (1 - \theta ij) x_{ij}} \dots\dots\dots\dots\dots\dots\dots. Equation\ 4$$

14. Let β_{ij} represents total rate of available agent group j for call type i who call back.

$$\beta_{ij} = \frac{\beta_i x_{ij}}{n_j} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. Equation\ 5$$

15. Let θ_{ij} represent the resolution probability of agent group j of call type i. This is the proportion of calls type i resolved by agent group j with total arrival rate of 1- β_i
16. Let ρ_j represent total utilization of agent group j. This is computed by equation

$$\rho_j = \sum_i \frac{\beta_i x_{ij}}{\mu_{ij}} \Big/ n_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Equation 6}$$

17. Γ_{i-} and Γ_{i+} represent the lower and upper bound such that each call type i must be served at total utilization between bounds.

$$\Gamma_i = \frac{\beta_i}{\mu_z}$$ …………………………………… Equations 7

4.5 Mathematical Procedure to Test Run Hybrid Routing Rule (HHCRR):
Here we assume a Call Center with 15hours of working period of the day. Inserting the time into the algorithm, this will enable us optimize mathematically, by reducing time spent on the queue and at the same time enhance resolution rate.

$$t_i = \frac{period}{day} (we\ assume\ 15hrs)\ working\ hourin\ a\ day$$

$$t_i = 15$$

$$t_2 = 15 + 1(incramental\ step\ for\ each\ itertion)$$

OR

$$t_i = 15$$

$$then\ t_1 = 15$$

$$t_2 = t_1 + 1$$

Where

$$i = 1,2,3,4,5,6,7,8$$

$$I = i$$

$$t_i = 15$$

For every customer call, and agent attended to at the allocated daily working hours of 15 hours is illustrated as,

$t_i = 15/x$, where x signify customers time for each call received by the agent.

And x ranges from $x_{1,\ldots\ldots\ldots,}\ x_n$

$$t_2 = \frac{15}{x_2}, then\ x_2 = 15/t_2$$

$$j = 1, 2, 3, 4, \ldots 35$$

$$\mu ij = {}^{1}\!/_{Average\ handling\ time}$$

$$\mu_{ixj} = {}^{1}\!/_{\epsilon j} \qquad j = 35$$

$T_j = 35$ Where Tj define agents for each call

$$Tj = {}^{35}\!/_{y}, \qquad define\ per\ call\ for\ each\ respondent\ from\ custormer\ care\ agent$$

$$t_2 = {}^{35}\!/_{y2}$$

$$\mu_{ixj} = Avg = {}^{Total\ call}\!/_{Total\ agent}$$

$$\rho_j = \frac{\sum_i \frac{\beta_i x_{ij}}{\mu_{ij}}}{n_j}$$

$let\ P_i = 60$

The matrix table 1: consist of data from data set

i	i^i	j'	U_{ij}	X_{ij}	μ_j	i	j
1	1	2	3	3	3	1	4
2	½	3	6	6	6	3	2
3	$1/3$	4	12	12	12	2	5
≡	≡	≡	≡	≡	≡	8	34
γ	$\sum 1/i$	35	$\sum U_{ij}$	$\sum X_{ij}$	$\sum \mu_{ij}$		

$$U_z = \left(\sum_{j=1}^{i} \eta_j U_{ij}\right)\left(\frac{\frac{\lambda_{ij}}{U_{ij}}}{\sum_{i1} \frac{\lambda_{ij}}{U_{ij}}}\right)$$

$i = 1$ $\qquad\qquad$ $1,2,\ldots 35$

$i = 2$

$$\sum_{4=1}^{4} \eta_{iU_{ij}} = (2(3) + 3(6) + 4(12))$$

$$= 6 + 18 + 48$$

$$\sum \eta_i U_{ij} = 72$$

$where \qquad i = 1,\ldots,8$

$$\lambda_i = Li\lambda$$

$$\lambda_i = 1$$

$$\lambda_1 = \left(15/x_1\right) = 15\ (1) = 15$$

$$\lambda_2 = 15 = 2 \times \frac{15}{-} = 7.5$$

$$U_z = 72\left(\frac{3/3}{3/\left(1/\left(1/3\right)\right)}\right)3\left(1/3\right)$$

$$72\left(\frac{3/3}{3\times 3}\right) = 72\left(1/9\right)$$

$$U_z = 72/9$$

$$\beta_i = \frac{\lambda_i}{1-\sum_i (1-\theta)x_{ij}}$$

$$\lambda_i = t_1\lambda = \qquad i = 1\ldots,8$$

$$\lambda_1 = t_1\lambda_1^0$$

$$\lambda = 1$$

$$\lambda_1 = \lambda$$

$$\lambda_2 = \lambda_1$$

$$(t_2 = t_1 + 1)$$

$$\lambda_1 = 15(1)$$

$$\lambda_1 = 15$$

$$\lambda_2 = t_2 \lambda_1$$

$$\lambda_2 = (t_1 + 1)15 \min$$

$$\lambda_2 = (15 + 1)(15)$$

$$\lambda_2 = 16(15)\ldots\ldots\ldots\lambda_n$$

$\theta =$ Prepare the resolution rate

$\theta_i =$ Resolution of i l = 1, 2, … , 8

$\theta_{ij} =$ Resolution of θ_{xj}

$$j = 1,\ldots\ldots.,35$$

$$i = 1,\ldots\ldots,8$$

$$\theta_i = \frac{1}{D}$$

Probability of the resolution rate must be less than one (1)

$\lambda_1 = 50$ $\qquad\qquad$ $\sum(1 - t) = 0.95$

$$\beta = \frac{50}{11}$$

$$\beta_i = \frac{50}{1 - (-0.9)}$$

$$\frac{50}{1 - 0.9}$$

$$\frac{10 - 1}{10} = \frac{9}{10} = 50 \Big/ \tfrac{9}{10}$$

$$\beta_1 == 50 \Big/ \tfrac{9}{10}$$

Table 2: From matrix table

	C_{ij}	2	3	4	5
1	1	3	5	2	8
2	4	5	1	7	6
3	8	9	1	9	(10)

$$\theta \quad {}_{3 \times 5 = 10} \quad = \frac{1}{10}$$

$$\beta_1 = \frac{500}{9} \qquad \frac{1}{2} = 0.2$$

$$\beta_2 = 120$$

Probability of resolution rate must be less than one

$$\theta_{ij} = \frac{1}{3} = 0.333$$

$$\sum_j \left(1 - \theta_{ij}\right) x_{ij}$$

$$j = 1 \quad or \quad y = 35$$

Using the parameters from the extracted table 2 above, the following is computed

$$\Sigma_j (1 - \theta_{ij}) X_{ij}$$

$$\Sigma_1 (1 - \theta_{2x1}) X_{2x1}$$

$$\Sigma_1 (1 - 0.333)(0.33)$$

$$n_j = J = agent$$

$$where\ J = 1, 2, \ldots, 35$$

$$\beta_1 = \frac{\lambda_1}{1 - (1 - 0.333)(0.33)} = \frac{50}{1 - (0.2211)} = \frac{50}{0.7789}$$

$$\beta_1 = 64.2$$

$$\beta_2 = \frac{\lambda_2}{1 - \Sigma_3 (1 - \theta_{i,j}) x_{ij}}$$

$$where\ \lambda_2 = \lambda_1 + 1 = 50 + 1 = 51$$

$$\beta_2 = \frac{51}{1 - (1 - \theta_{2x3})(x_{2x3})} = \frac{51}{1 - (1 - 1)\left(\frac{1}{6}\right)}$$

$$\beta_2 = \frac{51}{1 - 0} = \frac{51}{1} = 51$$

This justifies that resolution rate is increasing at a consistent order

$$\beta_8 = \frac{X_8}{1 - \Sigma(1 - \theta_{ij})x_{ij})}$$

$$\beta_{ij} = \frac{\beta_i x_{ij}}{n_i} = \beta_{1,2} = \frac{642(0.33)}{35} = 64.2\ x\ \frac{0.33}{35} = 0.7$$

$$P_j = \frac{\left(\Sigma_i \frac{\beta_i x_{ij}}{\mu_{ij}}\right)}{n_j}\ i = 1, \dots, 8; \quad j = 2$$

$$P_2 = \frac{\left(\Sigma_1 \frac{\beta_1 x_{1,2}}{\mu_{1,2}}\right)}{n_2} = \frac{\left(\frac{642}{3}\right)}{12} = 10.7$$

$$P_2 = 10.7$$

$$r_i = \frac{\beta_i}{i\mu_2}$$

$$r_1 = \frac{\beta_1}{\mu_2}$$

$$\beta_1 = 64.2$$

$$\mu_2 = \frac{72}{9}$$

$$r_1 = \frac{64.2}{\frac{72}{9}} = \frac{64.2\ x\ 9}{72} = 8.025$$

4. DISCUSSION OF RESULT

The result from the mathematical techniques shows a consistent reduction of the values of i from 15minutes to 7.5minutes this value will continue to reduce as computation is conducted further. This value implies that wait-time of calls in the queue keep reducing consistently. While the resolution rates on the other hand increases consistently, rating from 50 to 51 resolution rate. The simultaneous decrease and increase of the values for wait-time and call resolution validates the fact that optimization is achieved with the hybrid routing rule. The hybrid algorithm can further be implemented and designed to improve and enhance call center operations and also increase customer satisfaction and brand loyalty. The hybrid rule if implemented and deployed will be able to resolve both the challenge of wait-time on queue and effective call resolution by proffering low wait-time and enhanced call resolution rate

5. CONCLUSION

The result from the mathematical techniques of the hybrid algorithm shows that optimization was achieved mathematically, by assuming 15hour working hours of the day. The value obtained from mathematically equation with the assumed values proves that the proposed hybrid routing rule can be designed, developed and implemented**.** This will enhance call center operations, optimization and improves customer satisfaction.

6. AREA FOR FURTHER RESEARCH

This paper only considered the performance of the hybrid framework to enhance call handling for call center management by mathematically evaluating and testing the hybrid rule proposed by (Mughele et al, 2017b). Due to limitation of time and resources some key areas in the domain of discourse were not considered such as;

i. The role of Customer Relationship Management (CRM) in enhancing the efficiency of call handling in call center operations
ii. Exploring the domain of knowledge based system to increase the efficiency of call center management

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