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Application of ADM and DTM on the Free Vibration of Uniform Shear Beam with Free Ends

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ABSTRACT

This study deals with the numerical solutions of the first natural frequency of uniform shear beams with constant shear distortion and constant stiffness reading on Winkler Foundation. Adomian Decomposition Method was used to solve for the first natural frequency at free-ends. The results were tabulated and graphical result for the mode shapes was presented.

Keywords: Differential transform method, Adomian Decomposition Method, Shear Beam, Winkler Foundation.

Aims Research Journal Reference Format:
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1. INTRODUCTION

Beams are structures that are used in our daily lives, from building construction to carpentry works etc. Shear Beam is often derived from Timoshenko Beam which is an improvement of Euler-Bernoulli beam. [13] Considered the free vibration analysis of Uniform shear beam on Winkler Foundation with varying constant distortion and constant stiffness resting on Winkler Foundation. [3] analysed beam bending problem on three-parameter elastic foundation. [1] Worked on the Free Vibration of Beam on Variable Winkler Elastic Foundation by Using Differential Transform method. [2] Provided a solution to Timoshenko Beam Column in The Generalized End Condition and Non-classical mode of Vibration of Shear Beams. [4] Used DTM to solve the Analysis of Vibration of an Elastic Beam Supported on Elastic Soil. Our main objective in this work is to present a comparison between ADM and DTM for the solution of Shear Beam at free ends. [14] Presented an algorithm for Adomian Decomposition Method which we used his ideas to solve a uniform Shear beam at free ends. [5] Provided Formulas for Frequency and Mode Shape.

This work gave an accurate solution to the first natural frequency of uniform shear beam at free ends.

2 SHEAR BEAM MODEL ON A ONE-PARAMETER

Elastic Foundation

If a shear beam resting on a one parameter elastic (Winkler) foundation, the governing differential equation for the system without damping effect and if the effect of rotatory inertial is neglected and only the effect of shear distortion on the dynamic deflection of beam is considered and its subjected to a static axial force can be written in a Cartesian coordinate system *{x, y}* as

$$m\frac{\partial y^2(x,t)}{\partial t^2} - S\Big(\frac{\partial y^2(x,t)}{\partial x^2} - \frac{\partial\phi(x,t)}{\partial x}\Big) - P\frac{\partial^2 y(x,t)}{\partial x^2} + Ky(x,t) = 0$$
$$EI\frac{\partial^2\phi(x,t0}{\partial x^2} + S\Big(\frac{\partial y(x,t)}{\partial x} - \phi(x,t)\Big) = 0$$
$$0 < x < L \qquad (1)$$

Where $m = \rho A$ is the mass of the beam per unit length, k is the stiffness of the foundation per unit length, P is the axial force (positive and negative signs represented tension and compression), E is Young's Modulus of elasticity, I is the second moment of inertial, $y(x, t)$ is the vertical displacement of the beam, $\varphi(x, t)$ is the rotation of the beam, and $S = kiGA$ is the shear distortion of the beam where Ki is the effective shear area and G is the shear modulus of the beam. The boundary condition of the beam we are considering is given below:

For Free-Free beam the end conditions are;

$$\frac{\partial\phi}{\partial x} = 0, S\Big(\frac{\partial y}{\partial x} - \phi\Big) = 0 \qquad at \;\; x = 0, L \qquad (2)$$

3. FREE VIBRATION ANALYSIS.

Now free vibration analysis of the uniform shear beam on a one parameter elastic (Winkler) foundation is discussed as follows: The solution is separated due to its variables as given in the following form to formulate the analysis of the presented problem.

$$Y(x,t) = H(x)e^{i\omega t},$$
$$\phi(x,t) = G(x)e^{i\omega t} \qquad (3)$$

Where ω is the circular frequency for the vibration Substituting equation (3) into the governing equation, the equation of motion becomes as follows:

For the first equation in equation in the governing equation we have,

$$mi^2\omega^2 H(x)e^{i\omega t} - S\Big[\frac{d^2}{dx^2}H(x)e^{i\omega t} - \frac{d}{dx}G(x)e^{i?t}\Big] - p\frac{d^2}{dx^2}H(x)e^{i\omega t} + kH(x)e^{i\omega t} = 0 \qquad (4)$$

Which can be further simplified as;

$$(k - m\omega^2)H(x) - S\Big[\frac{d^2}{dx^2}H(x) - \frac{d}{dx}G(x)\Big] - p\frac{d^2}{dx^2}H(x) = 0 \qquad (5)$$

Also for the second equation in equation in the governing equation we have,

$$EI\frac{d^2}{dx^2}G(x)e^{i\omega t} + S\Big[\frac{d}{dx}H(x)e^{i\omega t} - G(x)e^{i\omega t}\Big] = 0 \qquad (6)$$

Equation (6) can be further simplified as;

$$EI\frac{d^2}{dx^2}G(x) + S\Big[\frac{d}{dx}H(x) - G(x)\Big] = 0 \qquad (7)$$

Without loss of generality the following dimensionless quantities can be introduced;

$$X = \frac{x}{L}, \quad G(X) = \frac{G(x)}{L}, \quad H(X) = \frac{H(x)}{L} \tag{8}$$

Inserting equation (8) into equation (5) we have,

$$S\frac{d^2}{d(LX)^2} + P\frac{d^2LH(X)}{d(LX)^2} - S\frac{dLG(X)}{d(LX)} - (k - m\omega^2)LH(X) = 0 \tag{9}$$

We further simplify equation (9) to become

$$\frac{d^2}{dX^2}H(X) - \lambda\frac{d}{dX}G(X) - (\mu - \gamma\omega^2)H(X) = 0 \tag{10}$$

Where

$$\lambda = \frac{SL}{(S+P)}, \quad \mu = \frac{L^2k}{(S+P)}, \gamma = \frac{L^2m}{(S+P)} \tag{11}$$

Inserting equation (8) into equation (7) we have,

$$EI\frac{d^2}{d(LX)^2}LG(X) + S\left[\frac{d}{d(LX)}LH(X) - LG(X)\right] = 0 \tag{12}$$

We further simplify equation (12) to become,

$$\frac{d^2}{d(X)^2}G(X) + \alpha\frac{d}{dX}H(X) - \beta G(X) = 0 \tag{13}$$

Where

$$\alpha = \frac{SL}{EI}, \beta = \frac{SL^2}{EI}$$

The boundary condition in view of equation (6) now becomes;

For Free-Free end;

$$S\left[\frac{d}{dX}H(X) - G(X)\right] = 0, \quad \frac{d}{dX}G(X) = 0 \quad at \quad X = 0, L \tag{14}$$

Table 1 Properties of a one- parameter shear beam-column on an Elastic (*Winkler) foundation*

Young modulus of area moment of inertia EI	363.35Knm^2
Cross-Sectional area A	0.0097389m^2
Stiffness of the foundation k	77.17MPa
Shear distortion S	Varying values: 10MN, 15MN, 20MN and 40MN
Length L	1m
Mass m	297.5kg/m
Axial force P	0
Density	7830kg/m^3

4. NUMERICAL TECHNIQUES

A. Adomian Decomposition Method

The Adomian decomposition method, proposed by Adomian initially with the aims to solve frontier physical problem, has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, biology and chemical reactions etc. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly. It is well known that the key of the method is to decompose the nonlinear term in the equations into a peculiar series of polynomials $\sum_{n=1}^{\infty} A_n$, where A_n are the so-called Adomian polynomials. Adomian formally introduced formulas that can generate Adomian polynomials for all forms of nonlinearity. Recently, a great deal of interests has been focused to develop a practical method for the calculation of Adomian polynomials An. However, the methods developed by [6–12] also require a huge size of calculations. [14] Established a promising algorithm that can be easily programmed in Maple, and be used to calculate Adomian polynomials for nonlinear terms in the differential equations.

Let us first recall the basic principles of the Adomian decomposition methods for solving differential equations. Consider the general equation Fu ¼ g, where F represents a general nonlinear differential operator involving both linear and nonlinear terms, the linear term is decomposed into L þ R, where L is easily invertible and R is the remainder of the linear operator. For convenience, L may be taken as the highest order derivate. Thus the equation may be written as

$$Lu + Ru + Nu = g, \qquad (15)$$

Where Nu represents the nonlinear terms. Solving Lu from (15), we have

$$Lu = g - Ru - Nu.$$

Because L is invertible, the equivalent expression is

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu. \qquad (16)$$

If L is a second-order operator, for example, L^{-1} is a twofold integration operator and $L^{-1}Lu = u - u(0) - tu'(0)$, then Eq.(18) for u yields,

$$u = a + bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu. \qquad (17)$$

Therefore, u can be presented as a series

$$u = \sum_{n=0}^{\infty} u_n, \qquad (18)$$

with u_0 identified as $a + bt + L^{-1}g$, and u_n $(n > 0)$ is to be determined. The nonlinear term Nu will be decomposed by the infinite series of Adomian polynomials

$$Nu = \sum_{n=0}^{\infty} A_n, \quad (19)$$

where A_n's are obtained by writing

$$v(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n, \quad (20)$$

$$N(v(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n. \quad (21)$$

Here λ is a parameter introduced for convenience. From(20) and (21) we deduce

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N(v(\lambda))\right]_{\lambda=0}, \quad n = 0, 1, \dots \quad (22)$$

Now, substituting (18) and (19) into (17) we obtain

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R \sum_{n=0}^{\infty} u_n - L^{-1} \sum_{n=0}^{\infty} A_n.$$

Consequently, we can write

$$u_0 = a + bt + L^{-1}g,$$
$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0,$$
$$\vdots$$
$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n.$$

All of u_n are calculable, and $u = \sum_{n=0}^{\infty} u_n$. Since the series converges and does so very rapidly, the n-term partial sum $\varphi_n = \sum_{i=0}^{n-1} u_i$ can serve as a practical solution.

B. Differential Transform Method

We used the differential transform method (DTM) which is a numerical method based on Taylors expansion. This method constructs an analytical solution in form of a polynomial. Unlike the traditional high order Taylor's series method which requires a lot of symbolic computations, the differential transform method is an analytical solution in the form of a polynomial. But it is different from Taylor series method that requires computation of the high order derivatives. The differential transform method is an iterative procedure that is described by the transformed equations of original functions for solution of differential equations. Also, the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transform of the original functions and the solution of this algebraic equations give the desired solution of the problem. Consider the functions *w*(*x*) which is analytic in a domain *D* and *x* = *x*0 represent any point in *D*. the function is represented by a power series whose centre is located at *x*0. The differential transform of the function w(x) is given as;

$$W(k) = \frac{1}{k!}\left[\frac{d^k w(x)}{dx^k}\right](x = x_0) \quad (23)$$

Where w(x) is the original function and W(k) is the transformed function.

The inverse transformation is defined as:

$$W(x) = \sum_{k=0}^{\infty} (x - x_0)^k W(k) \quad (24)$$

Combining equations (23) and (24) gives

$$W(x) = \sum_{k=0}^{\infty} \frac{(x - x_0)}{k!} \left[\frac{d^k w(x)}{dx^k}\right]_{x=x_0} \qquad (25)$$

Table 2: The fundamental operations of DTM

Original function	Transformed function
$w(x) = g(x) \neq h(x)$	$W(k) = G(k) \pm H(k)$
$W(x) = \lambda g(x)$	$W(k) = \lambda G(k)$
$w(x) = \frac{\partial g(x)}{\partial x}$	$W(k) = (k+1)G(k+1)$
$w(x) = \frac{\partial^m g(x)}{\partial x^m} g(x)$	$W(k) = (k+1)(k+2)\cdots(k+m)G(k+m)$
$w(x) = 1$	$W(k) = \partial(k)$
$w(x) = g(x)h(x)$	$W(k) = \sum_{m=0}^{k} H(m)G(k-m)$

Table 3: Theorems for differential transform method for boundary conditions.

$x=0$		$x=1$	
Original B.C	Transformed B.C	Original B.C	Transformed B.C
$W(0) = 0$	$W(0) = 0$	$W(1) = 0$	$\sum_{k=0}^{\infty} W(k) = 0$
$\frac{dW(0)}{dx} = 0$	$W(1) = 0$	$\frac{dW(1)}{dx} = 0$	$\sum_{k=0}^{\infty} KW(k) = 0$
$\frac{d^2(0)}{dx} = 0$	$W(2) = 0$	$\frac{d^2W(1)}{dx^2}$	$\sum_{k=0}^{\infty} k(k-1)W(k) = 0$
$\frac{d^3(0)}{dx} = 0$	$W(3) = 0$	$\frac{d^3W(1)}{dx^3} = 0$	$\sum_{k=0}^{\infty}(k-1)(k-2)W(k) = 0$

5. ADM FORMATION

Using the algorithm derived by Wenhai Chen et. Al (2004) equation (1) becomes

> $ICs := [0, [0, A], [0, B]]:$

> $Eqs := [[(u, v) \rightarrow 0.11 \cdot diff(u, t) - 0.11 \cdot v, (u, v) \rightarrow 0, t \rightarrow 0], [(u, v) \rightarrow -diff(v, t) - (1.92925 - \gamma 1) \cdot u, (u, v) \rightarrow 0, t \rightarrow 0]] : n := 3:$

> $result1 := Adomian(ICs, Eqs, n);$

$$result1 := [A\,t - 0.05500000000\,A\,t^2 + 0.01833333333\,B\,t^3 + 0.002016666667\,A\,t^3 + 0.004079166668\,B\,t^4 + 0.0009166666670\,(1.929250000 - 1.\,\gamma 1)\,A\,t^5 - 0.00005545833335\,A\,t^4 + 0.0008269250004\,B\,t^5 + 0.0001191666667\,(1.929250000 - 1.\,\gamma 1)\,A\,t^6 + 0.000002400793650\,(1.929250000 - 1.\,\gamma 1)\,B\,t^7, B\,t + 0.5000000000\,B\,t^2 + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,A\,t^3 + 0.0009166666664\,(1.929250000 - 1.\,\gamma 1)\,B\,t^5 + 0.03708333335\,(1.929250000 - 1.\,\gamma 1)\,A\,t^4 + 0.1666666667\,B\,t^3 + 0.00002182539683\,(1.929250000 - 1.\,\gamma 1)^2\,A\,t^7 + 0.0002887500000\,(1.929250000 - 1.\,\gamma 1)\,B\,t^6 + 0.007517500004\,(1.929250000 - 1.\,\gamma 1)\,A\,t^5 + 0.04166666668\,B\,t^4]$$

> $K := B\,t + A + 0.0009166666670\,(1.929250000 - 1.\,\gamma 1)\,B\,t^5 + 0.004583333332\,(1.929250000 - 1.\,\gamma 1)\,A\,t^4 + 0.0001359722223\,(1.929250000 - 1.\,\gamma 1)\,B\,t^6 + 0.0008158333336\,(1.929250000 - 1.\,\gamma 1)\,A\,t^5;$

$$K := B\,t + A + 0.0009166666670\,(1.929250000 - 1.\,\gamma 1)\,B\,t^5 + 0.004583333332\,(1.929250000 - 1.\,\gamma 1)\,A\,t^4 + 0.0001359722223\,(1.929250000 - 1.\,\gamma 1)\,B\,t^6 + 0.0008158333336\,(1.929250000 - 1.\,\gamma 1)\,A\,t^5$$

> **solve({ }, [gamma1]);**

$$[[\gamma 1 = (0.0002500000000\,(1.049297639\,10^{13}\,B\,t^6 + 6.295785835\,10^{13}\,A\,t^5 + 7.073916669\,10^{13}\,B\,t^5 + 3.536958332\,10^{14}\,A\,t^4 + 4.000000000\,10^{16}\,B\,t + 4.000000000\,10^{16}\,A)) / (t^4\,(1.359722223\,10^9\,B\,t^2 + 8.158333336\,10^9\,A\,t + 9.166666670\,10^9\,B\,t + 4.583333332\,10^{10}\,A))]]$$

> $G := B + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,B\,t^3 + 0.5000000000\,(1.929250000 - 1.\,\gamma 1)\,A\,t^2 + 0.04166666668\,(1.929250000 - 1.\,\gamma 1)\,B\,t^4 + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,A\,t^3 + 0.00002182539683\,(1.929250000 - 1.\,\gamma 1)^2\,B\,t^7 + 0.0001527777777\,(1.929250000 - 1.\,\gamma 1)^2\,A\,t^6 + 0.008333333336\,(1.929250000 - 1.\,\gamma 1)\,B\,t^5 + 0.04166666668\,(1.929250000 - 1.\,\gamma 1)\,A\,t^4$

$$G := B + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,B\,t^3 + 0.5000000000\,(1.929250000 - 1.\,\gamma 1)\,A\,t^2 + 0.04166666668\,(1.929250000 - 1.\,\gamma 1)\,B\,t^4 + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,A\,t^3 + 0.00002182539683\,(1.929250000 - 1.\,\gamma 1)^2\,B\,t^7 + 0.0001527777777\,(1.929250000 - 1.\,\gamma 1)^2\,A\,t^6 + 0.008333333336\,(1.929250000 - 1.\,\gamma 1)\,B\,t^5 + 0.04166666668\,(1.929250000 - 1.\,\gamma 1)\,A\,t^4$$

> diff(, t);

$$0.5000000001\,(1.929250000 - 1.\,\gamma 1)\,B\,t^2 + 1.000000000\,(1.929250000 - 1.\,\gamma 1)\,A\,t + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,B\,t^3 + 0.5000000001\,(1.929250000 - 1.\,\gamma 1)\,A\,t^2 + 0.0001527777778\,(1.929250000 - 1.\,\gamma 1)^2\,B\,t^6 + 0.0009166666662\,(1.929250000 - 1.\,\gamma 1)^2\,A\,t^5 + 0.04166666668\,(1.929250000 - 1.\,\gamma 1)\,B\,t^4 + 0.1666666667\,(1.929250000 - 1.\,\gamma 1)\,A\,t^3$$

> solve({ }, [gamma1]);

$$\Big[[\gamma 1 = 1.929250000], \Big[\gamma 1 = \frac{1}{t^4\,(7.63888889\,10^{8}\,B\,t + 4.583333331\,10^{9}\,A)}\,(0.0002500000000\,(5.894930556\,10^{12}\,B\,t^5 + 3.536958332\,10^{13}\,A\,t^4 + 8.333333336\,10^{14}\,B\,t^3 + 3.333333334\,10^{15}\,A\,t^2 + 3.333333334\,10^{15}\,B\,t^2 + 1.000000000\,10^{16}\,A\,t + 1.000000000\,10^{16}\,B\,t + 2.000000000\,10^{16}\,A))\Big]\Big]$$

>

The above presentation is for S=40 and it was used for S=10,S=15 and S=20 as presented in table 4.

7. DTM FORMATION

Taking the differential transformations of equation (10) of the previous chapter, we have

$$(k+1)(k+2)\bar{H}(k+2) - \lambda[(k+1)\bar{G}(k+1)] - \mu\bar{H}(k) = 0 \qquad (26)$$

We got the below recurrence relation

$$\bar{H}(k+2) = \frac{\lambda[(k+1)\bar{G}(k+1)] + (\mu - \gamma\omega^2)H\bar{H}(k)}{(k+1)(k+2)} \qquad (27)$$

Also taking the differential transformation of equation (13) of the previous chapter, we have

$$(k+1)(k+2)\bar{G}(k+2) + \alpha[(k+1)\bar{H}(k+1)] - \bar{G}(k) = 0 \qquad (28)$$

We got the below recurrence relation

$$\bar{G}(k+2) = \frac{\beta\bar{G}(k) - \alpha[(K+1)\bar{H}(k+1)]}{(k+1)(k+2)} \qquad (29)$$

Several iterations are carried out during the analysis procedure and the boundary condition for the case is written by using the solution for displacement of the beam. The boundary condition produces an equation containing two unknowns due to the initial approximation. This boundary condition in non-dimensionless form is;

Free-Free end

$$\bar{G}(1) = 0, \quad \bar{H}(1) - \bar{G}(1) = 0 \quad \text{at} \quad \bar{X} = 0 \qquad (30)$$

$$\sum_{k=0}^{\infty} k\bar{G}(k) = 0, \quad \sum_{k=0}^{\infty} k\bar{H}(k) - \sum_{k=0}^{\infty} \bar{G}(k) = 0 \quad \text{at} \quad \bar{X} = 1 \qquad (31)$$

7. NUMERICAL RESULTS

A number of case studies are carried out with respect to parameter S that lead to a variation of Shear distortion of the beam with the aid of mathematical computational software (MAPLE 18). The results are in tables 4 below, and mode shapes of the cases are also represented graphically.

Table 4 Free Vibration Frequencies for Clamped-Clamped End resting on Winkler Foundation with varying Shear distortion at constant stiffness of the foundation(K=77.17MPa)

S	10	15	20	40
ω_1(DTM)	7.71700	5.14466	3.85849	1.92925
ω_1(ADM)	7.71700	5.14466	3.85849	1.92925

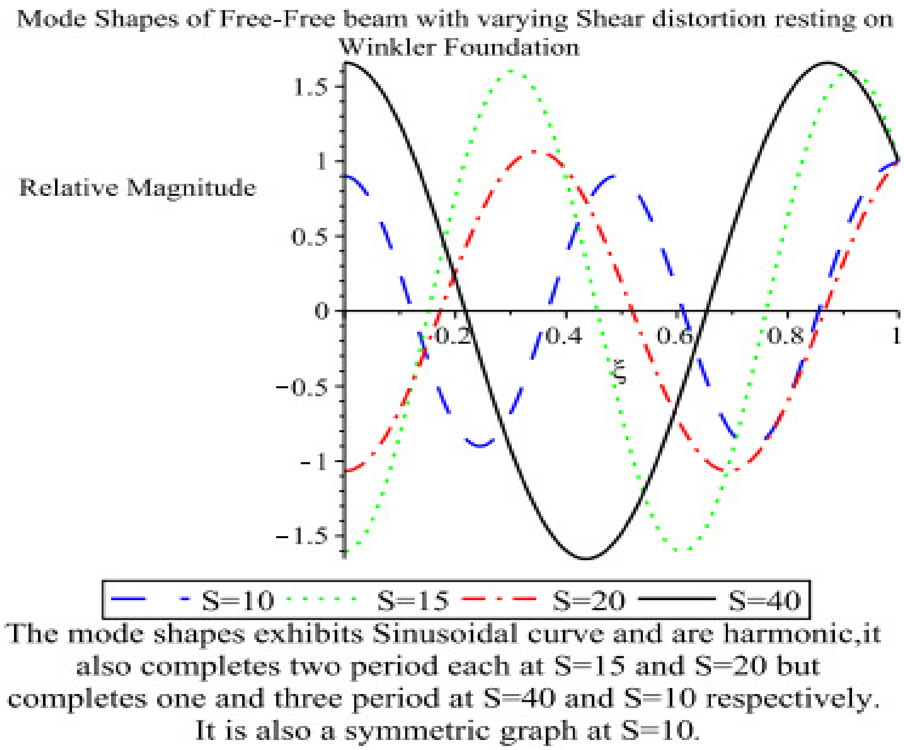

Fig 1 : Mode Shapes of Free-Free beam with varying Shear distortion resting on Winkler Foundation

8. SUMMARY AND CONCLUSION

In the study, both DTM and ADM were used and it was discovered that the result gotten was virtually the same. All steps were very straight forward. Maple program was also very helpful to solve the problem. The algorithm presented by [14] was very useful and applicable, very good and accurate agreement is observed. The reason for using both methods is to validate our results and to be sure that the result we got when we used DTM is accurate.

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