Advances in Mathematical & Computational Sciences Journal,
Vol. 6 No. 2, June, 2018

Article Citation Format
Obanla O. James, Ilo Hammed O. & Hassan Fatai A. (2018). Leslie Matrix Model and Female Population In Nigeria. Journal of Digital Innovations & Contemp Res. In Sc., Eng & Tech. Vol. 5, No. 4. Pp 1-10

Article Progress Time Stamps

Article Type: Research Article
Manuscript Received: 15th May, 2018
Review Type: Blind
Final Acceptance:: 17th September, 2018
DOI Prefix: 10.22624

Leslie Matrix Model and Female Population In Nigeria

***Obanla O. James, Ilo Hammed O. & Hassan Fatai A.**
Department of Statistics
Ogun State Institute of Technology
Igbesa, Ogun State, Nigeria
E-mail: winning4ril@gmail.com

ABSTRACT

In this paper, we verify the accuracy of the Leslie matrix model, which is a discrete age-structured method that uses fertility coefficients and survival ratios, as factors for projecting female population. Based on available Nigerian census data for the year 2006, Leslie matrix was built to project female population in Nigeria for fifteen years starting from the year 2006 to 2021. Graphical descriptive statistics were run using Microsoft Excel and SPSS. The results revealed that time specific growth rates (λ_1, λ_2, λ_3) for 2011, 2016 and 2021 are 14.72%, 14.50 %, 13.60% increase, respectively. To test the accuracy of this method, we compared the projected population data for the year 2011 with the estimated population data by National Population Commission for female population in Nigeria. In conclusion, we recommend that Leslie matrix model is suitable for projecting middle age group population.

Key words: Leslie matrices, fertility coefficient, population projection.

1. INTRODUCTION

The need for population projection is essential to national planning and development. Government, private sectors, even to researchers, projection of human population is of prime importance. Population projection is a tool for computations of future projection size and characteristics of population based on certain assumptions about future trends in fertility, mortality and migration. Importance of projection is key for housing, employment, estimating the future production of goods and services, future school enrolments, enable planning of investments in school buildings, teacher training and production of educational material, food, power, water, transport, incidence and prevalence of various diseases and planning for the number of hospitals, hospital beds and specialized facilities as well as training programs for medical specialists.

In view of the importance and significance of census data, this research work was motivated not only by the failure of Nigeria government in giving attention to decennial conduct of census but also in her failure to conduct credible population census exercise. These are attributed to tribal and political interest of individual in power (government). Census taking began at least 5800 years ago in Egypt, Babylonia, China, Palestine, and Rome (Halacy,1980). However, because of the exorbitant cost of conducting census by Governments, the place of population projection came in into play, where Mathematicians, statisticians, demographers have published several works on population projection techniques. In addition, certain complex characteristics like age, sex, are considered in the demographic studies. It is of strong observation that social researchers of kinds have special interest in the age and sex structure or components of a population, which is attributed to social relationships within a community which are considerably affected by the relative numbers of males and females and the relative numbers at each age (Frank Hobbs, 2004).

Moreso, South and Trent (1988) wrote on cross-national analysis of the effect of sex composition on women's roles, the population growth approximatively correlates to the fertility rates of females in the population. So, female population and the growth within this subgroup, can be used to project population growth. However, this research work is aimed at; making age specific projection of Nigeria female population for 2021, to check the accuracy of Leslie Matrix for projecting the Nigerian female population and to study the population growth rate.

The rest of this research work is structured as follows: Sections 2, 3, and 4 are review on methods adopted in the building of Leslie matrix model, description of data and results. Discussion of results follows while conclusion came last on section 5.

2. METHODS

2.1 Leslie Matrix

When it comes to the study of the growth of a given population it is of great importance having in account the survival and reproduction rates of the individuals of that population. However, these characteristics differ according to the age of the individuals, their body size or any other individual characteristics influencing survival and fertility. The Leslie matrix model was invented by P. H. Leslie and describes the growth of the female portion of a human or animal population. In this model, the population is divided into groups based on age classes of equal duration. It is a discrete and age structured model. The purpose is to project the population from time t to time $t+1$, in years or some other time unit, assuming that the unit of time is the same as the age class width (*projection interval*). A model with projection interval of one year will differ from one that projects from month to month or decade to decade. Supposing that the individuals of the population are classified into k age classes, the *population projection matrix*, often referred as a *Leslie matrix*, is:

$$L = \begin{bmatrix} R_1 & R_2 & R_3 & \dots & R_{k-1} & R_k \\ S_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & S_2 & 0 & \dots & 0 & 0 \\ & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & S_{k-1} & S_k \end{bmatrix}$$

Where R_i is the reproduction rate and S_i is the survival rate, of age class i, for $i=1,\dots, k$.

In some models the last age class is assumed to be removed from the population after a time unit, so the entry, S_k , is 0 in the matrix *L*. If the population at time *t*, distributed into the *k* classes is:

$$X^{(t)} = \begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_k^{(t)} \end{bmatrix}$$ where $x_i^{(t)}$ is the population, at time *t*, in age class *i*=1,..., *k* ,
then the population at time *t*+1, in age class *i*=1,..., *k* will be:

$$x_1^{(t+1)} = R_1 x_1^{(t)} + R_2 x_2^{(t)} + \cdots + R_k x_k^{(t)} ,$$
that is, the population in age class one must have originated from reproduction, and not be survivors of any other age class, between times *t* and *t*+1.

$x_{i+1}^{(t+1)} = S_i \, x_i^{(t)}$, for *i*=1,..., *k*-1,
that is the population in age class *i*+1 will be the survivors of class *i*, between times *t* and *t*+1.

Using matrix notation the previous equations can be written as $^{(t+1)} = L(t)$.

2.1.1 Survival Probabilities

The survival probability depends on the age of the individual from age x to x + 1, and is approximated by

$$P_i = \frac{l(i+1)}{l(i)} \qquad (1)$$

where the age is assumed to be known (Kot, 2001), if the age is not known (Caswell, 2001), then l(x) can be approximated by taking its average within each age class over the interval i − 1 6 x 6 i. Thus,

$$P_i \approx \frac{l(i) + l(i+1)}{l(i-1) + l(i)}. \qquad (2)$$

2.1.2 Fertilities

Fertility depends on the distribution of births and deaths in the age class, given by,

$$F_i = P_i m_{i+1}, \qquad (3)$$

Fi is the number of offspring born in the following year, multiplied by the survival probability

2.2 Age Specific Growth Rate

Population growth rate is simply defined as the summary parameter of trends in population density or abundance. It reveals whether density and abundance are increasing, stable or decreasing, and how quick they are changing as year comes by. This describes the per capita rate of growth of a population, either as the factor by which population size increases per year, conventionally given the symbol λ (= N_{t+1}/N_t) (4)

λ is referred to variously as 'finite growth rate', 'finite rate of increase', 'net reproductive rate' or 'population multiplication rate'. In the simplest population model all individuals in the population are assumed equivalent, with the same death rates and birth rates, and there is no migration in or out of the population, so exponential growth occurs.

3. RESULTS

3.1 Data Description:

Data used herein are on the Federal Republic of Nigeria which has an area of 923,769 square kilometers (made up of 909,890 square kilometers of land area and 13,879 square kilometers of water area), which is located between 3° and 14° East Longitude and 4° and 14° North Latitude. The longest distance from East to West is about 767 kilometers, and from North to South 1,605 kilometers. In addition, these data are secondary, which are obtained from Nigeria Demographic and Health Survey 2008, publication.

The base population data were obtained as a result of population and housing census of 2006, which was carried out from the 21st to 25th March, 2006. The life table from which the survivor ratio was computed was published by World Health Organization.

In the construction of Leslie Matrix Model, from the under listed indices, the components of the Matrix was built, they are:

A life table of female in Nigeria which was obtained from 2006 World Health Organization Report. The importance of this table was to compute survival ratio which is a probability value and placed immediately below the leading diagonal values in the matrix. Age Specific Fertility Rate was obtained from adjusted Age-Sex Specific population distribution of Nigeria for Female (2006). This however was required in order to compute fertility coefficient. Nigeria female population of 2006 was used as the base population upon which population projections were made for 2011, 2016 and 2021. Sex ratio at birth = female birth divided by all birth

3.2 Results

Projections for the year 2011, 2016 and 2021 were made by using the Leslie matrix constructed from fertility and survival rates in Table 2 versus the data provided for female population as obtained in the 2006 National Population census as shown in table 3. For better comparison of actual (National Population commission) versus projected values, Figure 2 shows how both base population and projected population decrease as age increases.

Though, the actual 2011 population data were reported with 65+ as the highest age group where the projected population were reported with 85+ as the highest age group, however it is observed that the data have greater deviation at lower ages (0-4, 5-9, 10-14) and higher ages (60-64, 65+) when presenting the percentage error per age group. In comparison, ages 15-19, 20-24, 25-29, 40-44 and 50-54 have close values for both actual and projected population.

Table 2: Component of Leslie Matrix Model

AGE	FERTILITY COEFFICIENT	SURVIVAL RATIO
0-4		
5-9		0.943976959
10-14		0.977549852
15-19	0.265504367	0.981923348
20-24	0.493706468	0.977788861
25-29	0.581476507	0.970026888
30-34	0.528814484	0.959488526
35-39	0.353274406	0.949122341
40-44	0.190899834	0.942448617
45-49	0.096547043	0.941143868
50-54		0.935198124
55-59		0.919048689
60-64		0.885948215
65-69		0.82527354
70-74		0.729144495
75-79		0.591047733
80-84		0.417896608
85+		0.277085219

\

TABLE 1:LESLIE MATRIX MODEL

	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85+
0-4	0	0	0	0.265504	0.493706	0.581477	0.528814484	0.353274	0.1909	0.096547	0	0	0	0	0	0	0	0
5-9	0.943977	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10-14	0	0.97755	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15-19	0	0	0.981923	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20-24	0	0	0	0.977789	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25-29	0	0	0	0	0.970027	0	0	0	0	0	0	0	0	0	0	0	0	0
30-34	0	0	0	0	0	0.959489	0	0	0	0	0	0	0	0	0	0	0	0
35-39	0	0	0	0	0	0	0.949122341	0	0	0	0	0	0	0	0	0	0	0
40-44	0	0	0	0	0	0	0	0.942449	0	0	0	0	0	0	0	0	0	0
45-49	0	0	0	0	0	0	0	0	0.941144	0	0	0	0	0	0	0	0	0
50-54	0	0	0	0	0	0	0	0	0	0.935198	0	0	0	0	0	0	0	0
55-59	0	0	0	0	0	0	0	0	0	0	0.919049	0	0	0	0	0	0	0
60-64	0	0	0	0	0	0	0	0	0	0		0.885948	0	0	0	0	0	0
65-69	0	0	0	0	0	0	0	0	0	0	0	0	0.825274	0	0	0	0	0
70-74	0	0	0	0	0	0	0	0	0	0	0	0	0	0.729144	0	0	0	0
75-79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.591048	0	0	0
80-84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.417897	0	0
85+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.277085	0

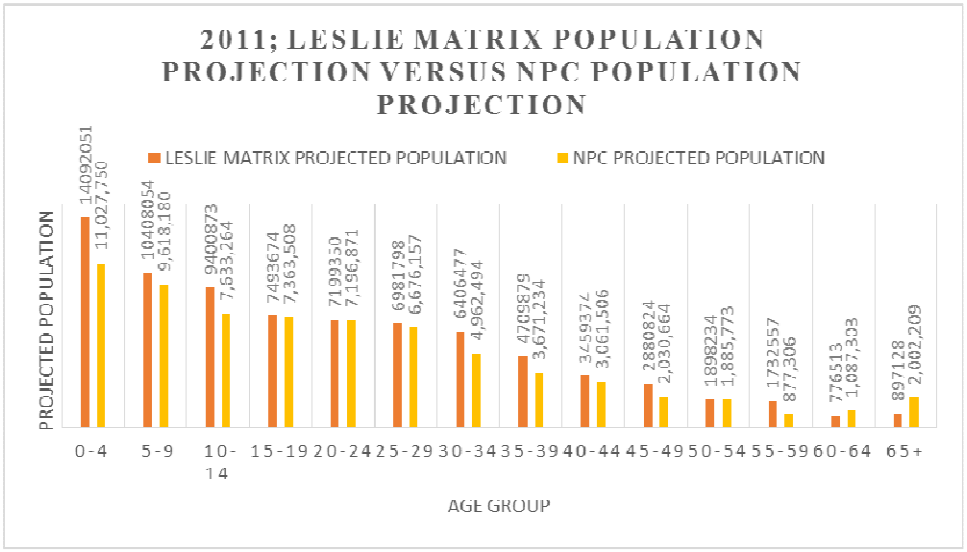


Figure 1

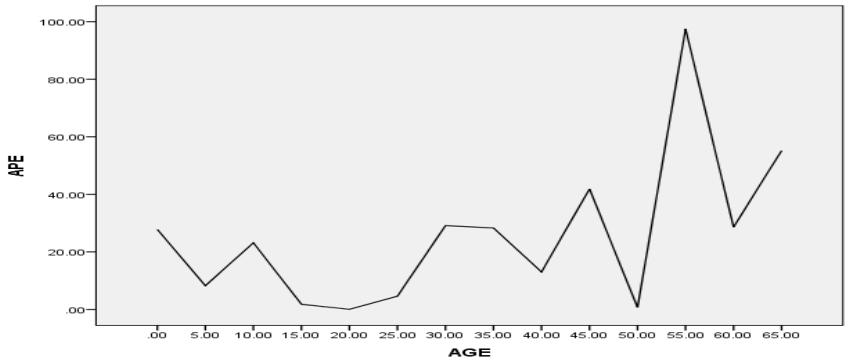


Figure 2: Projection error between actual vs. projected population values.

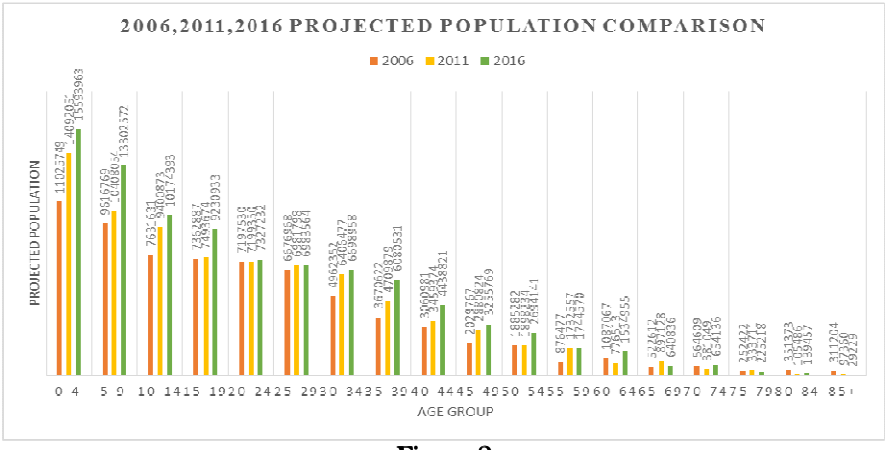
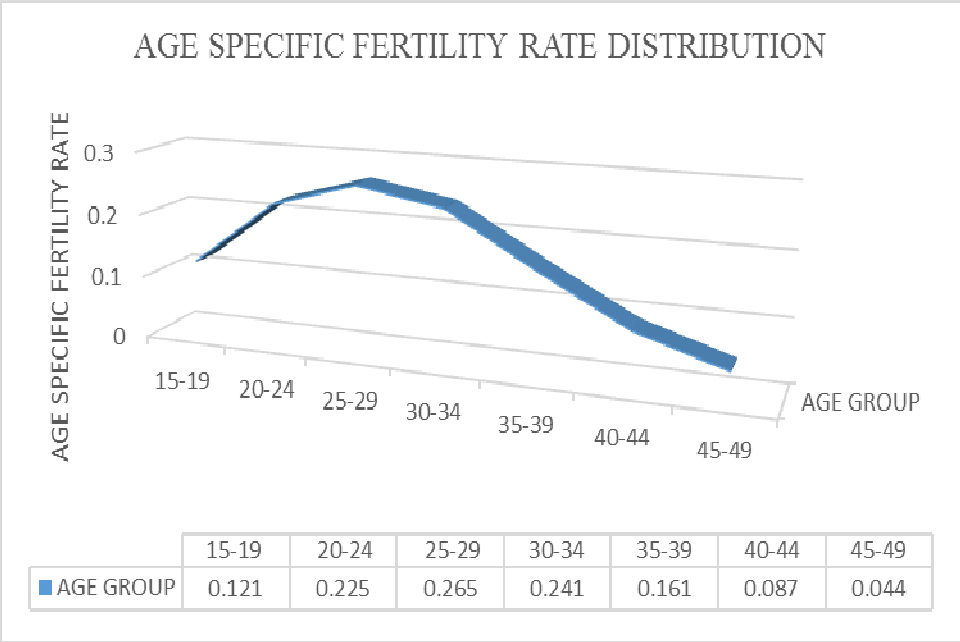


Figure 2



	15-19	20-24	25-29	30-34	35-39	40-44	45-49
AGE GROUP	0.121	0.225	0.265	0.241	0.161	0.087	0.044

Figure 3

4. DISCUSSION

Based on our results, Leslie matrix projections are not reliable for the age groups 0-10 and age groups over 65+ years old, but seemed reliable for the rest of the age groups. This claim is supported by our percent error calculations, which show maximum error in projection to be 27%, whereas at extremes it exceeds well above 50%. The projected female population trend above shows increasing population growth rate as years come by. More so, close examination of population age groups reveals steady increase. The total population size for Nigeria female in 2006 was 69,086,302. The total projected population size for Nigeria female in 2011 as projected was 79,254,392.

The time specific growth rate, λ_1, is computed as the total population in 2011 divided by the total population in 2006. However, λ_1 = 79,254,392/69,086,302 = 1.1472. Similarly, total projected population size for Nigeria female in 2016 was 90,749,278. More so, the time specific growth rate, λ_2, is computed as the total population in 2016 divided by the total population in 2011 (90,749,278/ 79,254,392 = 1.1450) which revealed that there is an increase in the population by (1.1450) x 100% = 14.50% increase. Where the time specific growth rate λ_3 for 2021 is computed as (103,117,411 / 90,749,278 = 1.136 x 100% =13.60% increase

Table 3: 2006- 2021 Leslie Matrix Projected Population and NPC Projected Population

Age Group	Population For Nigerian Female 2006	Projected Population (2011)	Actual Populaion BY NPC (2011)	Projected Population (2016)	Projected Population(2021)
0-4	11025749	14092051	11,027,750	15593963	16981435
5-9	9616769	10408054	9,618,180	13302572	14720342
10-14	7631631	9400873	7,633,264	10174393	13003929
15-19	7362887	7493674	7,363,508	9230933	9990470
20-24	7197530	7199350	7,196,871	7327232	9026837
25-29	6676968	6981798	6,676,157	6983564	7107613
30-34	4962352	6406477	4,962,494	6698958	6700653
35-39	3670622	4709879	3,671,234	6080531	6358131
40-44	3060981	3459374	3,061,506	4438821	5730590
45-49	2029767	2880824	2,030,664	3255769	4177570
50-54	1885282	1898234	1,885,773	2694141	3044789
55-59	876477	1732557	877,306	1744570	2476048
60-64	1087067	776513	1,087,303	1534955	1545598
65-69	522612	897128	2,002,209	640836	1266758
70-74	564609	381049		654136	467262
75-79	252422	333711		225218	386626
80-84	351373	105486		139457	94118
85+	311204	97360		29229	38641
TOTAL	**69,086,302**	**79,254,392**	**81,397,007**	**90,749,278**	**103,117,411**

5. CONCLUSION

This work has proposed Leslie matrix model for the population projection of Nigeria female,
It is observed that little or no application of this model is published to the Nigerian context, so making it unique. Data used to build up this matrix model were obtained from National Population Commission and World Health Organization. Graphical descriptive statistics were run using Microsoft Excel and Statistical Package for Social Science (SPSS) 21. However, the comparison analysis shows that Leslie Matrix projected population are most suitable for the population projection of the rest of the age groups except ages 0-10 and 65+.The time specific growth rate, λ_i for 2011 showed 14.72% increase and 14.50 % increase for 2016, while 2021 would experience 13.60% increase as time specific growth rate.

REFERENCES

1. Ana P.G and Margarida R.D (2015) Some Matrix Models. European Scientific Journal/special/ edition issn: 1857 – 7881 (print) e - issn 1857- 7431
2. Annual Abstract of Statistics (2012). Federal Republic of Nigeria. National Bureau of Statistics
3. Caswell H. (2001). Matrix Population Models (2nd edition). Sinauer Associates.
4. Dan Halacy (1980). Census 190 Years of Counting. Elsevier/nelson Books
5. Frank Hobbs (2004).Age and Sex Composition. Paducah, Kentucky
6. Kot M. (2001). Elements of Mathematical Ecology. Cambridge University Press.
7. Leon S. J. (2010). Linear Algebra with Applications. 8th edition, Prentice Hall.
8. Leslie, P. H.(1945). On the Use of Matrices in Certain Population Mathematics. Biometrika, **33**(3), 183-212
9. Life Expectancy at Birth, Total (Years) (2016, May 17). The World Bank. World Bank.
10. National Population Commission (NPC) (2009). [Nigeria] and ICF Macro
11. Nigeria Demographic and Health Survey (2008). Abuja, Nigeria: National Population Commission and ICF Macro.
12. Richard M. Sibly and Jim Hone (2002) Population growth rate and its determinants: An overview. Availablefrom:https://www.researchgate.net/publication/11065962_Population_growth_rate_and_its_determinants_An_overview [accessed Sep 28 2018]
13. Scott J. South and Katherine Trent (1988).Sex Ratios and Women's Roles: A Cross-National Analysis, American Journal of Sociology 93.
14. Statistical Report of Women and Men in Nigeria 2015 (2016, November). National Bureau of Statistics.
15. World Health Organization (Global Health Observatory Data): Life Tables by Country. (2015, Feb.). Retrieved on April, 10 from http://aps.who./data