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On the Numerical Simulation of the Effect of Saturation Terms on the Susceptible Individual in Susceptible-Exposed Infected-Recovered- Susceptible (SEIRS) Epidemic Model.

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ABSTRACT

Abstract: In this research work, a computational numerical approach, Variational Iteration Method (VIM), to the solution of an epidemic model (Kolawole, 2015) is presented. The result shows that as the saturation term for the susceptible individual increases, the susceptible and recovered individual increase drastically and at a point the disease dies out.

Keywords: VIM, saturation terms, epidemic model, Lagrange multiplier.

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1. INTRODUCTION

Mathematical models can be categorized, based on the described diseases, population and environment, as linear, non-linear, autonomous or non-autonomous model. Kunniya and Nakata (2012) studied the long-term behavior of non-autonomous SEIRS epidemic model where $m_1 = m_2 = 0$. They obtained new sufficient conditions for the permanence (uniform persistence) and extinction of infectious population of the model.

In this paper , the work done by Kunniya and Nakata was extended to include non-linear incidence rate to investigate the effect of saturation term m_1 for the susceptible individual variational iteration method proposed by He (1998 & 1999).

Differential equations are widely used to describe real life problems including modeling of HIV (Vergu et al, 2005 & Xiaohua, 2007). All the referenced authors (1-9) have used different numerical methods to solve different types of differential equations in attempt to search for better, accurate, efficient and elegant method for the solution.

Variational Iteration Method has been shown to solve a large class of linear and nonlinear problems with approximation converging to exact solution rapidly.In this work, we present VIM for the modeling of the effect of saturation terms on the susceptible individual in (SEIRS) Epidemic Model.

The idea of variational calculus was proposed by Inokuti et al (1978) and later modified by He (1998 & 1999) into a variational iteration method. This method and its modification has been applied to different types of differential equations (Olayiwola, 2014a, 2014b).

2. MATHEMATICAL MODEL OF SEIRS EPIDEMIC

In this section, the system of differential equations that described the model is presented as follows:

$$\left.\begin{aligned}
\frac{dS}{dt} &= N - \frac{\beta SI}{1+m_1S+m_2I} - \mu S + \delta R \\
\frac{dE}{dt} &= \frac{\beta SI}{1+m_1S+m_2I} - (\mu+\xi)E \\
\frac{dI}{dt} &= \xi E - (\mu+\gamma)I \\
\frac{dR}{dt} &= \gamma I - (\mu+\delta)R
\end{aligned}\right] \qquad (1)$$

Where:
S(t) = susceptible individual
E(t) = exposed individual
I(t) = infected individual
R(t) = recovered individual
N = birth rate
$\beta =$ Disease transmission coefficient
$\mu =$ Mortality or death rate
$\xi =$ Recovery rate
$\gamma =$ Rate of losing immunity
$m_1 =$ Saturation term for susceptible individual
$m_2 =$ Saturation term for infected individual
$\dfrac{1}{1+m_1S+m_2I} =$ Incidence rate inclusive the saturation terms m_1 and m_2

3. THE VARIATIONAL ITERATION METHOD

The idea of Variational calculus can be traced to Inokuti et al (1978) and later, He (1998 & 1999) modified it and presented a Variational Iteration Method that has been proved elegant in the solution of different types of differential equations.
According to the Variational Iteration Method, we consider the differential equation.

$$L(u)+N(u)=g(s). \qquad (2)$$

Where L is a linear operator, N is a non-linear operator, and $g(s)$ is an inhomogeneous term. A correction functional to (1) can be constructed as :

$$U_{n+1}(s)=U_n(s)+\int_0^s \lambda[LU_n(\tau)+N\breve{u}_n(\tau)-g(\tau)]d\tau \qquad (3)$$

Where λ is a general Lagrange multiplier which can be identified optimally by variational calculus and $\tilde{u}_n(\tau)$ is known as the restricted variation i.e. $\delta\tilde{u}_n(\tau)=0.$

4. VARIATIONAL ITERATION METHOD FOR THE SOLUTION OF THE EFFECT OF SATURATION TERMS ON THE SEIRS EPIDEMIC MODEL

In this section, the VIM will be used to study the effect of saturation terms in the susceptible individual in SEIRS epidemic model. To investigate the effect of m_1 , we proceed as follows:

Applying equation (3) in (1) we obtained the following system of correctional functional:

$$S_{n+1}(t)=S_n(t)+\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2 I_n(\tau)}-\mu\bar{S}_n(\tau)+\partial\bar{R}_n(\tau)\right]d\tau$$

$$E_{n+1}(t)=E_n(t)+\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$

$$I_{n+1}(t)=I_n(t)+\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi\bar{E}_n(\tau)+(\mu+\gamma)\bar{I}_n(\tau)\right]d\tau \qquad (4)$$

$$R_{n+1}(t)=R_n(t)+\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma\bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

Where $\lambda_1, \lambda_2, \lambda_3,$ and λ_4 are general Lagrange Multiplier, $\tilde{S}_n, \tilde{E}_n, \tilde{I}_n,$ and $\tilde{R}_n$ denote restricted variation i.e. $\delta\tilde{S}_n=\delta\tilde{E}_n=\delta\tilde{I}_n=\delta\tilde{R}_n=0$.
The stationary values that corresponds to the correctional functionals are:

$$\partial S_{n+1}(t)=\partial S_n(t)+\partial\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}-\mu \bar{S}_n(\tau)+\partial \bar{R}_n(\tau)\right]d\tau$$

$$\partial E_{n+1}(t)=\partial E_n(t)+\partial\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$

(5)

$$\partial I_{n+1}(t)=\partial I_n(t)+\partial\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi \bar{E}_n(\tau)+(\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$

$$\partial R_{n+1}(t)=\partial R_n(t)+\partial\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma \bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

Equation (5) gives $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -1$.

With $\lambda_i, i = 1 \ldots 4$, we obtained the following iterative scheme:

$$S_{n+1}(t)=S_n(t)-\int_0^t \left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta S_n(\tau)\bar{I}_n(\tau)}{1+m_1 S_n(\tau)+m_2 I_n(\tau)}-\mu S_n(\tau)+\partial R_n(\tau)\right]d\tau$$

$$E_{n+1}(t)=E_n(t)-\int_0^t \left[\frac{dE_n(\tau)}{dt}-\frac{\beta S_n(\tau)I_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)E_n(\tau)\right]d\tau$$

(6)

$$I_{n+1}(t)=I_n(t)-\int_0^t \left[\frac{dI_n(\tau)}{dt}-\xi E_n(\tau)+(\mu+\gamma)I_n(\tau)\right]d\tau$$

$$R_{n+1}(t)=R_n(t)-\int_0^t \left[\frac{dR_n(\tau)}{dt}-\gamma I_n(\tau)+(\mu+\partial)R_n(\tau)\right]d\tau$$

Using the following initial and computational values;

$$S_\circ(t)=18, E_\circ(t)=16, I_\circ(t)=6, R_\circ(t)=3, \beta=0.19, \mu=0.3, N=50,$$
$$\gamma=0.1, \partial=0.05, \xi=0.25, m_2=0$$

The following results can be readily obtained by Maple 18 when $m_1 = 0.3$.

$$\begin{aligned} S_0(t) &= 18 \\ E_0(t) &= 16 \\ I_0(t) &= 6 \\ R_0(t) &= 3 \end{aligned} \tag{7}$$

$$\begin{aligned} S_1(t) &= 18 + 429.2300000t \\ E_1(t) &= 16 - 74.32000000t \\ I_1(t) &= 6 + 1.600000000t \\ R_1(t) &= 3 - 0.4500000000t \end{aligned} \tag{8}$$

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$$\begin{aligned} S_4(t) = &-0.6215147875(t + 2.2202481230432243)(t + 0.0388076874995379 74)(t - 2.2964985869154106) \\ &(t^2 + 51.47499037316561t + 668.6608537345531)(t^2 + 1.1551437027786224t + 0.8294102063660227) \\ &(t^2 - 0.45743292997766793t + 0.46047965470900754)(t^2 - 1.383102162852016t + 0.5731256690711399) \end{aligned}$$

$$\begin{aligned} E_4(t) = &-0.6215147875(t + 2.2180024180373383)(t + 0.1706732780199015)(t - 0.1099176533604067 5) \\ &(t - 0.8263188050976197)(t - 2.2945767970023496)(t^2 + 51.47504449133282t + 668.6623122515947) \\ &(t^2 + 1.0214648031473688t + 0.825338937498461)(t^2 - 0.9022155283351456t + 0.5912676376233885) \end{aligned} \tag{9}$$

$$\begin{aligned} I_4(t) = &-0.4727266693(t + 28.02938955014713)(t + 2.3646301415046564)(t - 2.5520909663187843) \\ &t^2 + 0.5462931004251304t + 0.17089565872435158)(t^2 - 1.3157803048456138t + 0.4390733773648209) \end{aligned}$$

$$\begin{aligned} R_4(t) = &-0.05436913336(t + 27.893714307769276)(t + 1.3407809235994779)(t - 1.0918068666487761) \\ &(t^2 - 0.02131232814t + 1.351322017) \end{aligned}$$

5. RESULTS AND DISCUSSION

In Figure 1, the graph reveals that the susceptible individuals are not either increasing or decreasing since the saturation terms have no effect while in Figure 2 the result reveals the asymptotic stability of the disease free equilibrium since the exposed and infected approaches zero. Figures 3 and 4 show perfect asymptotic stability of the disease free equilibrium because the exposed and infected individuals die out rapidly as m_1 increases.

The simulation result reveals the stable and unstable nature of disease free equilibrium i.e. at unstable nature. These results show the asymptotic stability nature of disease free equilibrium. Hence, the saturated term for susceptible individual plays a vital role in disease eradication.

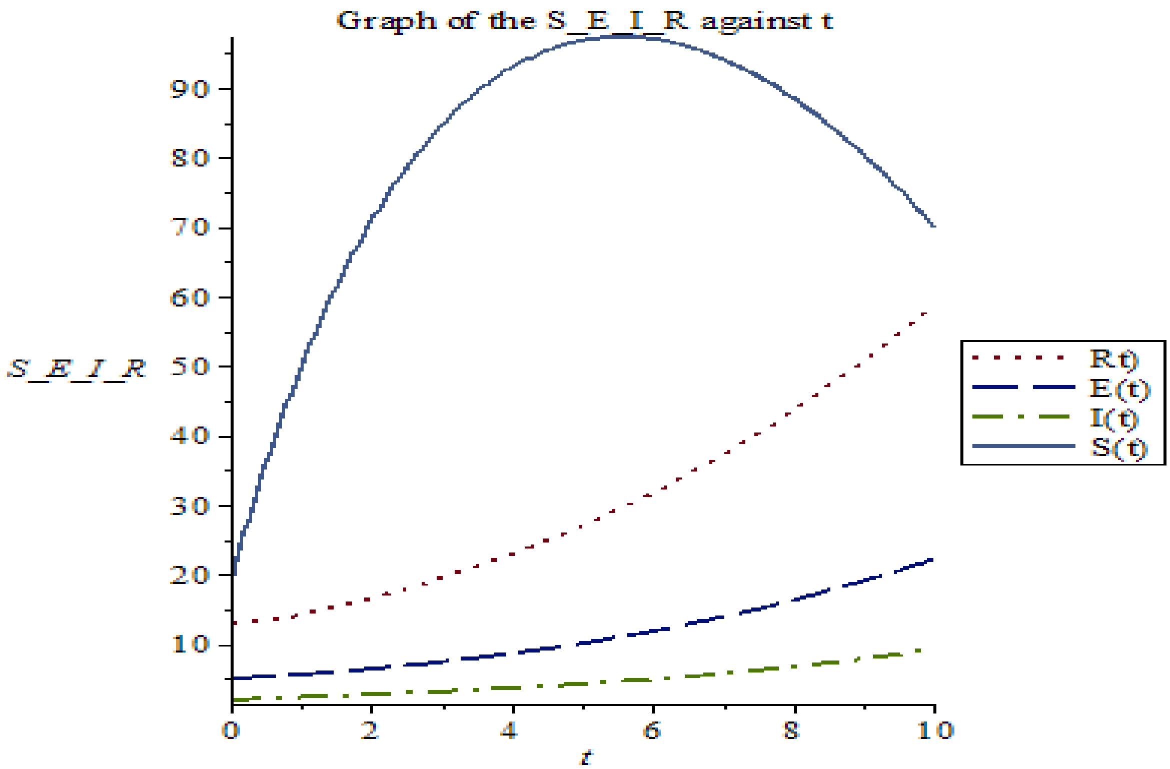

Fig. 1. Graph of simulated result when $m_1 = m_2 = 0$

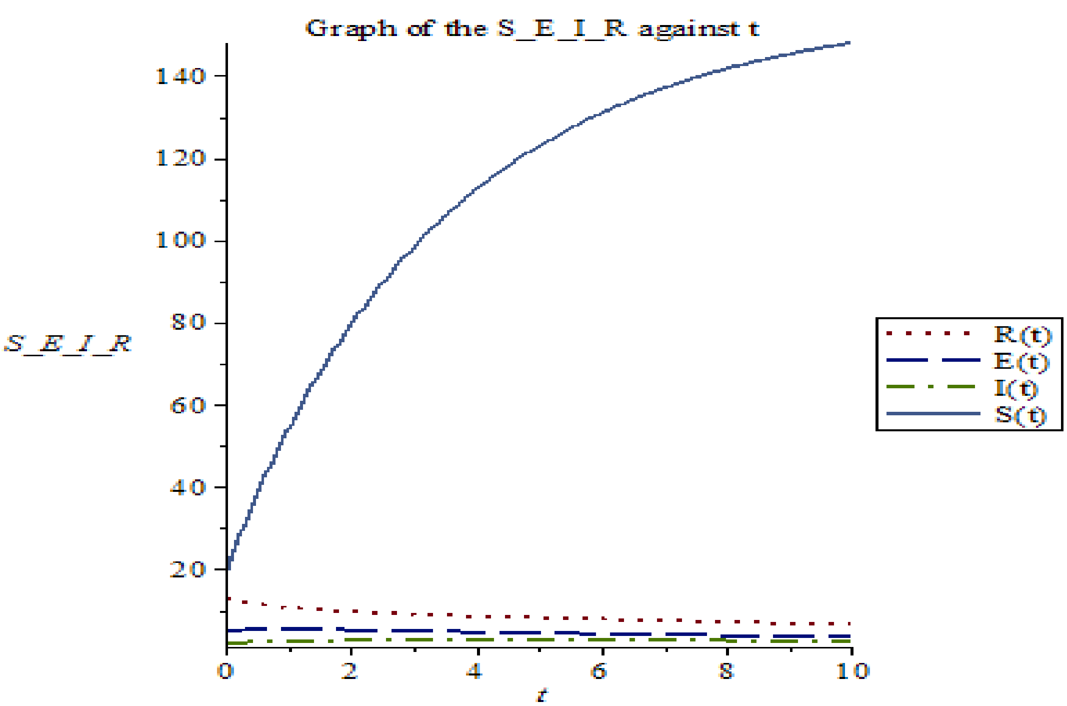

Fig. 2. Graph of simulated result when $m_1 = 0.3$

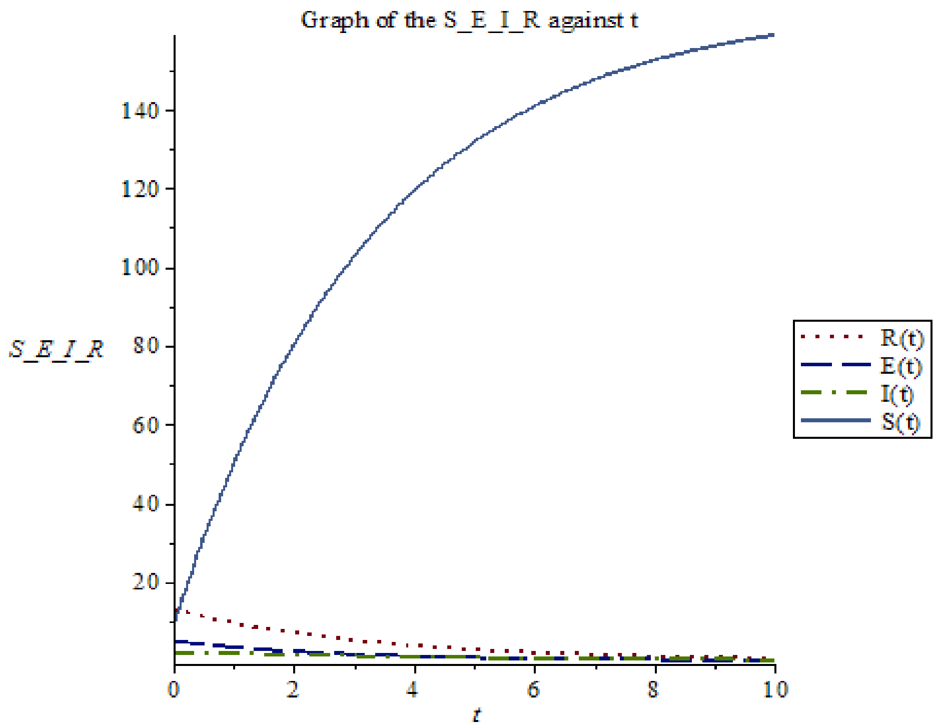

Fig. 3. Graph of simulated result when $m_1 = 0.6$

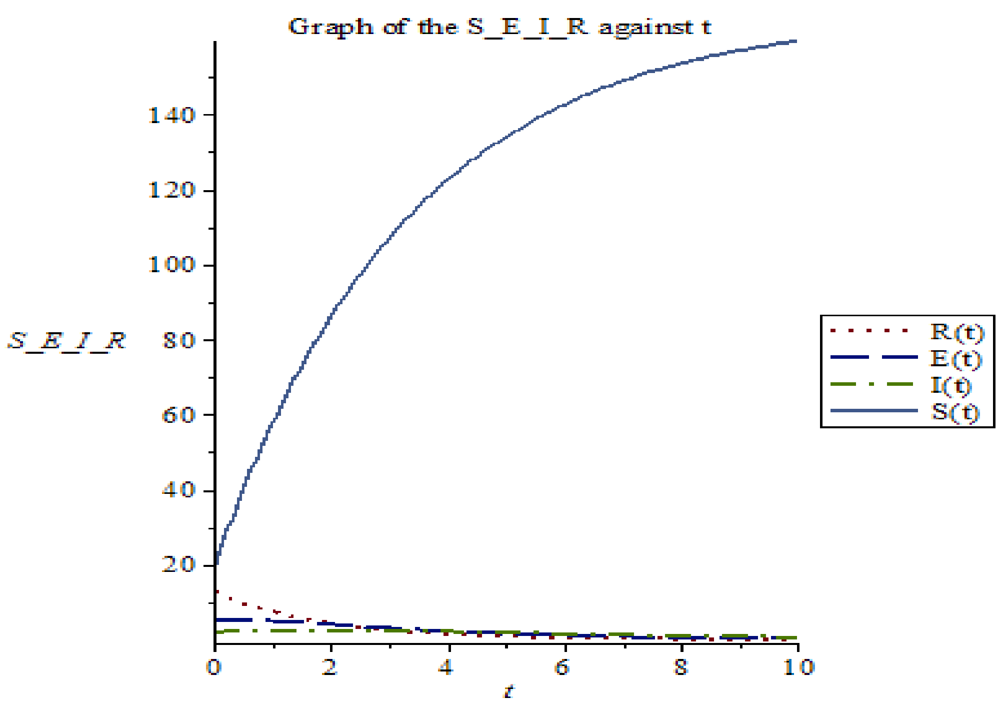

Fig. 4. Graph of simulated result when $m_1 = 0.9$

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