Unsteady MHD Slip Boundary Layer Flow of a Nanofluid Along a Stretching Sheet

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ABSTRACT

In this paper, we have examined the boundary layer flow of a nanofluid containing gyrostatic micro-organisms along a linearly stretching sheet in the presence of a time dependent magnetic field. The governing partial differential equations for continuity, momentum, energy, concentration of nanoparticles, and motile micro-organisms density are converted into a system of the ordinary differential equations via a set of similarity transformations. These equations are numerically solved using the fourth order ordinary differential equation solver known as bvp4c in MATLAB software. The values of the shear stress at the surface which is represented by the Skin friction are compared with numerical and analytical values reported by other researchers are presented in a table and are in good agreement.

Keywords: Slip; MHD; Nanofluid; Nusselt Number; Bioconvection.

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1. INTRODUCTION

Unsteady Magneto Hydrodynamic (MHD) boundary layer flow of an electrically conducting incompressible fluid with a convective surface boundary condition is frequently encountered in many industrial and technological applications such as extrusion of plastics, the manufacture of Rayon and Nylon, the cooling of reactors, purification of crude oil, textile industry, polymer technology, and metallurgy. As a result, the simultaneous occurrence of buoyancy and magnetic field forces on fluid flow has been investigated by many researchers. MHD (Magneto-Hydrodynamics) is the science of the motion of electrically conducting fluids (Nanofluids) under the influence of applied magnetic forces (Bondareva et al., 2015).

Bio convection refers to a macroscopic convection motion of Nano fluid caused by the density gradient induced by collective swimming of motile microorganisms (Kuznetsov, 2011).

These self-propelled motile microorganisms enhance the density of the base fluid by swimming in a particular direction, thus causing bio convection. Kuznetsov (2011) analyzed on Nano fluids containing gyrotactic microorganisms and reaffirm that the resultant large-scale motion of fluid caused by self-propelled motile microorganisms increases mixing and prevent nanoparticle agglomeration in fluids.

Heat and mass transfer in the boundary-layer flow of unsteady viscous Nano fluid along a vertical stretching sheet in the presence of magnetic field, thermal radiation, heat generation, and chemical reaction are presented by Eshetu and Shankar (2014). The authors found out that the velocity, temperature, and concentration profiles of the unsteady flow are less. than the corresponding parts of the steady state flow scenario. Kuznetsov and Nield (2010) studied analytically the free convective boundary-layer flow of a nanofluid past a vertical plate. Their results showed that the reduced Nusselt number is a decreasing function of each of buoyancy force parameter, Brownian motion parameter and thermophoresis parameter.

According to Aziz (2010), the concept of no-slip condition at the boundary layer is no longer valid for fluid flows in micro electromechanical systems and must be replaced by slip condition. Many researchers studied the effect of linear momentum and nonlinear slip on the MHD boundary layer flow with heat/mass transfer of free/forced/combined convection past different geometries. In spite of the importance of MHD related studies on boundary layer flow problems, the possibility of fluid exhibiting apparent slip phenomenon on the solid surface has received little attention.

This reveals that more work still needs to be done on unsteady boundary layer flow of Nano fluid under slip condition. Thus, the present study investigates the combined effects of velocity slip, temperature slip, bio convection Lewis number, and magnetic field and unsteadiness parameters on boundary layer flow with heat and mass transfer characteristics of a nanofluid containing oxystatic microorganisms over a stretching sheet.

2. MATHEMATICAL FORMULATION

The flow is governed by the continuity equation, the momentum equation, energy equation, concentration and the conservation equations which are listed respectively below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B B_0^2 u}{\rho} \tag{1}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left[\frac{\partial u}{\partial y}\right]^2+\tau\left[D_b\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_t}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{2}$$

(3)

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left[\frac{\partial^2 T}{\partial y^2}\right] \tag{4}$$

$$\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)\right]=D_n\frac{\partial^2 n}{\partial y^2} \tag{5}$$

Where
t is time, u and v are the velocity components, x and y are the Cartesian coordinates
T is the temperature, DT is the thermophoresis diffusion coefficient
BD is the Brownian diffusion coefficient, Ø is the nanoparticle volume fraction
n is the density of motile microorganisms, υ is the kinematic viscosity
ρ is the density of the fluid
σB is the fluid electrical conductivity, B0 is the strength of magnetic field, k is the thermal conductivity, cp is the specific heat at constant pressure, Dn is the diffusivity of microorganisms, b is the chemo taxis constant, and WC is maximum cell swimming speed.

The initial and boundary conditions are taking as:

$$t=0,\ u=u_w(x,t),\ T=T_\infty,\ v=0,\ C=C_\infty,\ n=n_\infty\ everywhere$$

$$t\geq 0,\ u=0,\ v=0,\ T=T_\infty,\ C=C_\infty,\ n=n_\infty\ \ all\ \ at\ \ x=0$$

$$y=0,\ u=u_w+N\frac{\partial u}{\partial y},\ T=T_w(x,t)+D\frac{\partial u}{\partial y},\ C=C_w,\ n=n_w$$

$$u=0,\ v=0,\ T\to T_\infty,\ C\to C_\infty,\ n\to n_\infty,\ y\to\infty$$

Where

$$u_w(x,t)=\frac{ax}{(1-\lambda tt)},\ T_w(x,t)=T_\infty+\frac{bx}{(1-\lambda tt)},\ B(t)=\frac{B_0}{(1-\lambda tt)^{\frac{1}{2}}}$$

We introduce the stream function ψ x,) such that:

$$u=\frac{\partial\psi}{\partial y},\ v=\frac{\partial\psi}{\partial x}$$

The following similarity) such that transformations are used in equations (1) – (5) and the boundary conditions

$$\eta=\sqrt{\frac{a}{\upsilon(1-\lambda tt)}}\,y,\ \psi=\sqrt{\frac{a\upsilon}{1-\lambda tt}}\,xf(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\ \chi(\eta)=\frac{n-n_\infty}{n_w-n_\infty}$$

The above equations are partial differential equations and cannot be solved numerically just like that, it has to be transformed into ordinary differential equations by similarity transformation. Thus, similarity transformations are the transformations by which an n-independent variable partial differential system can be converted to a system with n − 1 independent variables. After successful transformations of all the equations along with the boundary conditions, the following equations are obtained.

$$\text{Momentum}\, f'''+ff''-f'^2-Mf'-A\left(f'-\frac{\eta}{2}f''\right)=0$$

Energy$\theta'' + ¿(f''^2) + PrNb\,❑'\theta' + \Pr Nt\,\theta'^2 + Pr\,\theta'\left(f - A\frac{\eta}{2}\right) = 0$

Nanoparticle concentration

$❑'' + \frac{Nt}{Nb}\theta'' + ❑' \leq \left(f - A\frac{\eta}{2}\right) = 0$

Conservation for microorganism

$\chi'' + \chi' Lb\left(f - \frac{\eta}{2}\right) - Pe[\chi' ❑' + (\chi + \Omega)❑''] = 0$

Boundary conditions

$f(0) = 0, f'(0) = 1 + \alpha f'', \theta(0) = 1 + \gamma\theta', (0) = 1, \chi(0) = 1$

all at $\eta = 0$.

$f(\infty) = f'(\infty) = 0, \theta(\infty) = 0, (\infty) = 0, \chi(\infty) = 0\; as\; \eta \to \infty$

3. NUMERICAL SIMULATIONS

After successful transformations, the set of resulting nonlinear differential equations is solved using Runge –Kutta fourth order algorithm along with shooting technique. A Matlab special in built function for solving first order boundary value problems using fourth order method called Bvp4c has been employed for the purpose of simulation in this work. All the equations were firstly converted into a set of first order differential equations and were then carefully coded into matlab in the differential equation function. The boundary conditions were also coded into boundary value function. Then, about nine initial guesses were made and the solutions were obtained across a mesh of ten equally spaced points with a tolerance of 10-7 and a time step of approximately 0.3 as the case maybe. The results were then validated by comparing them with published literature values.

4. RESULTS AND DISCUSSION

Having numerically solved the set of nonlinear ordinary differential equations resulting from the similarity transformations the different pertinent parameters of interest such as velocity and temperature slip, magnetic, bio convection Lewis number and unsteadiness parameter on the heat and mass transfer properties of the flow are fully explored. Also, parameters of interests such as Skin friction, Nusselt number, etc were obtained and presented in tables.

In order to test for the validity and accuracy of the solution, the values of the shear stress at the surface which is represented by the Skin friction are compared with numerical and analytical values reported by Aminreza et al (2012) and Sahoo and Do (2010) in Table 4.1 This table shows the numerical solution obtained by the present algorithm and the exact solution reported by Aminreza et al (2012) and Sahoo and Do (2010) are in good agreement.

Similarly, Table 4. 2 compare results for the reduced Nusselt number (-θ' (0) ¿.

Table 4.1: Comparison of results for the skin friction coefficient when A = Nt = Nb = 0. 1,Ec = Pe = M =Lb == 0, Le = Pr = 10.

1.0β		Aminreza et al (2012)	Sahoo and Do (2010)	Present result
2.0	0	1.000000	1.001154	1.001396
	0.1	0.872082	0.871447	0.873440
	0.2	0.776377	0.774933	0.777700
	0.3	0.701548	0.699738	0.702840
	0.5	0.591195	0.589195	0.592430
	1.0	0.430160	0.428450	0.431279
	2.0	0.283980	0.282893	0.284937

Table 4.2: Comparison of results for the reduced Nusselt number when Le=Pr=10, no slip condition with varying Nt and Nb.

Nb	Nt	Aminreza et al (2012)	Khan and Pop (2010)	Present result
0.1	0.1	0.9523768	0.9524	0.9522609
0.1	0.2	0.6931743	0.6932	0.6931502
0.1	0.3	0.5200790	0.5201	0.5201161
0.1	0.4	0.4025808	0.4026	0.4026594
0.1	0.5	0.3210543	0.3211	0.3211623
0.2	0.1	0.5055814	0.5056	0.5055271
0.3	0.1	0.2521560	0.2522	0.2521323
0.4	0.1	0.1194059	0.1194	0.1193960
0.5	0.1	0.0542534	0.0543	0.0542494

5. CONCLUSIONS

In this paper, we have examined the boundary layer flow of a nanofluid containing gyrotactic micro-organisms along a linearly stretching sheet in the presence of a time dependent magnetic field. These equations are numerically solved using the fourth order ordinary differential equation solver known as bvp4c in MATLAB software. The results show the following conclusions: The local skin friction coefficient increases with the simultaneous increase of the magnetic and unsteadiness parameter. The Local Nusselt number is decreasing with a simultaneous increase of magnetic and unsteadiness parameter.

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