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Comparison of Vector Autoregressive Model (VAR) And Bayesian Vector Autoregressive Model (BVAR) Models for Modelling Economic Growth

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ABSTRACT

The study attempted to develop forecasting model for inflation as well as IPP growth in a multivariate time series Bayesian framework, known as Bayesian Vector Autoregressive (BVAR) model. The main advantage of using this model is the incorporation of prior information which may boost the forecasting performance of the model. The results revealed that the diagnostics results of the models are appear to be satisfactory and out of sample percentage root mean square error (PRMSE) for WPI for four quarters is 1.4932 percent, whereas, for IIP, it is 4.2508 percent. Further, for selecting the a suitable values for lambda and theta, we have tried various combination for these parameters between 0 to 1 and based on PRMSE, we found that lambda=0.3 and theta=0.9 are suitable values for BVAR(2). Therefore, BVAR(2) with lambda=0.3 and theta=0.9 was fitted. From the results, it can be observed that, out of sample PRMSE has been reduced while using BVAR in both the cases i.e. for WPI as well as IIP.Based on the comparison of forecasting performance of VAR and BVAR model, measured in terms of out-of-sample percentage root mean square error, it was found that BVAR model performed better than VAR model in case of inflation as well as IPP growth forecast

Keywords: Vector Autoregressive Model (VAR), Bayesian Vector, Modelling, Economic Growth

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1. INTRODUCTION

The search for better forecasting techniques to get reliable forecast is always vital. The frequently used forecasting models are univariate time series models like autoregressive model, moving average model, autoregressive moving average model, and multivariate time series model. The merit of using multivariate time series model is along with incorporating past information of the target variable, it allows to incorporate inter-temporal interdependence of other variables for improving the forecasting performance. The commonly used multivariate time series model is vector autoregressive (VAR) model but the major setback of this model is the problem of over-parameterisation. By the nature of the model, it requires to estimate large number of parameters which leads to large standard error. So, if some restriction can be imposed on the parameters then the performance of the model should be improved. The facility of imposing restrictions is available in Bayesian Statistics by the way of prior information on parameters or coefficients.

As, the name itself says about prior information, it is the information about the parameters which come before the experiment by the way of other experiments and personal belief of the forecaster, etc., and then assigning probability distribution to each coefficients of the model. This Bayesian VAR (BVAR) approach provides more accurate forecasts (Litterman (2000). BVAR is also superior to VAR since it is robust to the choice of national variables, even when misspecified national variables are included (Olatayo and Oniyide, 2006).Hence, a modified VAR restricting certain parameters is sometimes preferred. In general, the prior being used for BVAR is Minnesota prior or Litterman's prior proposed by Litterman in 2000. Some important studies Bayesian VAR is used are (Litterman, 2000), (Kadiyala and Sune, 1997).

In this study, we used a Bayesian Vector Autoregressive (BVAR) model to analyze and discuss Nigeria Economy by allowing possibility of interactions between the important macroeconomic variables. The variables used for this study are industrial output growth and inflation rate.

1.1 Objectives of the Study

i. Reviewing the theory Bayesian Vector Autoregressive (BVAR).
ii. Modeling Nigerian economic growth using BVAR and VAR
iii. Comparison of the estimate of BVAR and VAR using meta diagnostic tools.
iv. Forecasting Nigerian economic growth using BVAR and VAR.

1.2 Source of data

The data used for this study were obtained from the Statistical Bulletin of the Central Bank of Nigeria (CBN), various issues.

2. LITERATURE REVIEW

The use of VAR models has been recommended by Sims (2000) as an efficient alternative to verify causal relationships in economic variables and to forecast their evolution. On the theoretical level, this approach has its foundation in the work of Wold (1938), Box and Jenkins (2006). Given the vector of variables, the classical VAR model explains each variable by its own past values and the past values of all other variables by a well-defined relation. For macroeconomic forecasting, VAR has become a standard tool. VARs produce dynamic forecasts that are consistent across equations and forecast horizons. The issue which has entailed for a long time the controversy between the supporters and detractors of the Bayesian procedure is the estimation of the parameters of a model, either by using the statistical inference techniques or, on the contrary, by taking into account the previous knowledge of the economic system. The application of this procedure implies that a priori probability has to be chosen and it can only be applied to models with a finite number of parameters. Yet, since most of the macroeconomic variables are from stochastic tendencies, the specification of their distribution turns out to be necessary.

Usually, the hypothesis of normality for the coefficients is adopted since, in most cases, the underlying economic theory has little influence on the distribution of errors. In the field of multivariate modeling, Litterman (2000) suggested the use of the Bayesian procedure as an efficient way of avoiding some of the problems posed by Sims VAR models. The over-parametrisation is mainly the cause of these problems. Indeed, even if the reduced-size systems are involved, too many parameters have to be considered, which turns out to be non-significant after applying the hypothesis tests. Thus, it is necessary to put forward that the out-of-sample forecasts obtained by means of a standard VAR model depend a lot on the number of lags, even though the values observed and calculated are very close on the estimation period. In order to bypass these difficulties, Litterman (2000) introduces some a-priori knowledge in the formulation of his model by means of a distribution of probabilities.

The primary focus of monetary policy, both in Nigeria and elsewhere, has traditionally been the maintenance of a low and stable rate of aggregate price inflation along with sustainable economic growth. The underlying justification for this objective is the widespread consensus supported by numerous economic studies that inflation is costly so far as it undermines real, wealth-enhancing economic activity. If anything, this consensus is probably stronger today than it ever has been in the past. Indeed, it could be argued that much of the improvement in Indian living standards which has taken place over the last two decades would not have been achieved without the establishment of a credible low inflation environment. This study focuses mainly on BVAR models. Over the past twenty years, the BVAR approach has gained widespread acceptance as a practical tool to provide reasonably accurate macroeconomic forecasts when compared to conventional macroeconomic models or alternative time series approaches.

3. METHODOLOGY

The vector autoregression (VAR) is an econometric model used to capture the linear interdependencies among multiple time series. VAR models generalize the univariate autoregressive model (AR model) by allowing for more than one evolving variable. All variables in a VAR are treated symmetrically in a structural sense (although the estimated quantitative response coefficients will not in general be the same); each variable has an equation explaining its evolution based on its own lags and the lags of the other model variables. VAR modeling does not require as much knowledge about the forces influencing a variable as do structural models with simultaneous equations: The only prior knowledge required is a list of variables which can be hypothesized to affect each other intertemporally.

3.1 Definition

A VAR model describes the evolution of a set of *k* variables (called *endogenous variables*) over the same sample period (t = 1, ..., T) as a linear function of only their past values. The variables are collected in a $k \times 1$ vectory_t, which has as the i^{th} element, $y_{i,t}$, the observation at time "t" of the i^{th} variable. For example, if the i^{th} variable is GDP, then $y_{i,t}$ is the value of GDP at time t.

A *p-th order VAR*, denoted **VAR(*p*)**, is

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t, \quad (1)$$

where the *l*-periods back observation y_{t-l} is called the *l*-th ***lag*** of *y*, *c* is a $k \times 1$ vector of constants (intercepts), A_i is a time-invariant $k \times k$matrix and e_t is a $k \times 1$ vector of error terms satisfying

$E(e_t) = 0$— every error term has mean zero;
$E(e_t e_t') = \Omega$— the contemporaneous covariance matrix of error terms is Ω (a $k \times k$positive-semi definite matrix);
$E(e_t e_{t-k}') = 0$ for any non-zero k — there is no correlation across time; in particular, no serial correlation in individual error terms.

A *p*th-order VAR is also called a **VAR with *p* lags**. The process of choosing the maximum lag *p* in the VAR model requires special attention because inference is dependent on correctness of the selected lag order.
Rewriting the VAR in equation (1) as a system of multivariate regressions yields

$$Y = XB + U \quad (2)$$

Where $Y = (Y_1, Y_2, \dots, Y_T)'$ is a $T \times n$ matrix where T is the number of observed time periods, $X = (X_1, X_2, \dots, X_T)'$ with $X_t = (Y_{t-1}', Y_{t-2}', \dots, Y_{t-p}')'$ is a $T \times k$ matrix where $k = np + 1$,$B = (A_1, A_2, \dots, A_T, c)'$ is a $k \times n$ matrix containing all parameters, and $U = (u_1, u_2, \dots, u_T)'$ is a $T \times n$ matrix of the error terms.

The normal inverted Wishart prior has the form:

$$vec(\mathrm{B}|\Psi) \sim N(vec(\mathrm{B}_0), \Psi \otimes \Omega_0 \text{ and } \Psi \sim IW(S_0, \sigma_0) \quad (3)$$

where the prior parameters $\mathrm{B}_0, \Omega_0 S_0, \sigma_0$, are defined such that the prior expectation and variance of B coincide with the Minnesota prior expectations and variances for the autoregressive matrices $A_1, A_2, \ldots, A_p$:

$$E\left[(A_k)_{ij}\right] = \begin{cases} \delta_i, j = i, k = 1 \\ 0, otherwise \end{cases} \quad (4)$$

$$V\left[(A_k)_{ij}\right] = \begin{cases} \frac{\lambda^2}{k^2}, j = i \\ v\frac{\lambda^2}{k^2}\frac{\sigma_i^2}{\sigma_j^2}, otherwise \end{cases} \quad (5)$$

and the expectation of Ψ is equal to the fixed residual covariance matrix of the Minnesota prior $\Psi = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$. The matrices $A_1, A_2, \ldots, A_p$ are assumed to be independent and normally distributed and for the intercepts c an uninformative (so-called "diffuse") prior is assumed. Following BGR (2010), it was imposed on a random walk prior, i.e. $\delta_i = 1$, for all nonstationary variables, i.e. that the prior mean of the variables is characterized by random walk with drift $Y_t = c + Y_{t-1} + u_t$, and a white noise prior, i.e. $\delta_i = 0$, for all stationary variables.

The so-called hyperparameter λ controls the overall tightness of the prior distribution around the random walk and therefore represents the confidence in the prior distribution with respect to the information contained in the data. If the hyperparameter is set to $\lambda = 0$ the posterior is equivalent to the prior, whereas if $\lambda = \infty$ the posterior expectation is equivalent to an OLS estimate. The strategy for choosing the tightness of the priors is explained below. The parameter $1/k^2$ is the rate at which the prior variance decreases with increasing lag length and reflects the prior belief that more recent lags provide more reliable information than more distant ones. In order to have a prior which can be implemented simply, the normal inverted Wishart prior has to be based on the assumption $v = 1$, i.e. that the variation of a given variable is equally explainable by its own lags and lags of other variables. This condition prohibits the prior from treating lags of the dependent variable differently from lags of other variables (apart from the scale) and is in a sense the price for being able to relax the strict covariance matrix assumption of the Minnesota prior.

4.. RESULTS

4.1 Stationary

The Augmented Dickey Fuller test is used for testing stationarity at the level and at first difference. The graph in Appendix I shows instability or volatility of the headline inflation overtime. See Appendix II for the table showing Unit Roots. Based on the results of the test statistics, it is observed that the variables are stationary at first difference.

4.2 Selection of Order of VAR

For the purpose of selecting order of VAR, the Minimum Information Criteria as well as Univariate Model White Noise Diagnostics are being used and based on these criteria, the order of VAR is found to be two. (See Appendix III)
Based on these results, it can be observed that the diagnostics results of this model are appears to be satisfactory. And out of sample percentage root mean square error (PRMSE) for WPI for four quarters is 1.4932 percent, whereas, for IIP, it is 4.2508 percent.

4.3 Selection of values of lambda and theta in Litterman prior

Here, since the VAR model is developed at difference of the variables, therefore, the absolute value of the parameters would be less than one and hence mean of prior distribution is taken as zero, whereas, the degree of closeness of parameters to the prior mean can be controlled by suitable values of lambda and theta. Further, for selecting the a suitable values for lambda and theta, we have tried various combination for these parameters between 0 to 1 and based on PRMSE, we found that lambda=0.3 and theta=0.9 are suitable values for BVAR(2). Therefore, BVAR(2) with lambda=0.3 and theta=0.9 was fitted. From the results, given in the table 4.1, it can be observed that, out of sample PRMSE has been reduced while using BVAR in both the cases i.e. for WPI as well as IIP.

Table 4.1 : Comparison of the Models

Model	Out of Sample PRMSE	
	WPI	IIP
VAR(2)	1.4932	4.2508
BVAR(2)	1.4400	3.6055

The models fitted are given below:

Model I:

$$dlwpi = 0.00961 + 0.00144S_{1t} - 0.00197S_{2t} + 0.00613S_{3t} + 0.22639dlwpi_{t-1} + 0.05563 + 0.08854dlml_{t-1} - 0.16897dlwpi_{t-2} + 0.05573dliip_{t-2} + 0.11449dlml_{t-2}$$

Model II:

$$dliip = 0.02409 + 0.00772S_{1t} + 0.03394S_{2t} - 0.09845S_{3t} - 0.41799dlwpi_{t-1} + 0.20349dliip_{t-1} + 0.13dlml_{t-1} - 0.00292dlwpi_{t-2} - 0.06808dliip_{t-2} - 0.15537dlml_{t-2}$$

Model III:

$$dlml = 0.0049 + 0.03024S_{1t} + 0.05074S_{2t} + 0.0399S_{3t} - 0.1708dlwpi_{t-1} + 0.07945dliip_{t-1} - 0.2761dlml_{t-1} - 0.0642dlwpi_{t-2} - 0.14172dliip_{t-2} + 0.11877dlml_{t-2}$$

Model Forecast

Table 1: The forecast for 2014:1-2014:4

Qrts	Forecast	95% Confidence Interval		Actual
		Lower	Upper	
1	1118.7	-293.8	1243.5	
2	1231.5	-243.9	1425.1	
3	1321.9	-252.2	1338.2	
4	1536.1	-324.2	1622.3	

5. CONCLUSION

The objective of the study was to obtain a BVAR model. Based on the results, it can be observed that the diagnostics results of the models are appear to be satisfactory and out of sample percentage root mean square error (PRMSE) for WPI for four quarters is 1.4932 percent, whereas, for IIP, it is 4.2508 percent. Further, for selecting the a suitable values for lambda and theta, we have tried various combination for these parameters between 0 to 1 and based on PRMSE, we found that lambda=0.3 and theta=0.9 are suitable values for BVAR(2). Therefore, BVAR(2) with lambda=0.3 and theta=0.9 was fitted. From the results, it can be observed that, out of sample PRMSE has been reduced while using BVAR in both the cases i.e. for WPI as well as IIP.

In this study, with the objective of getting better forecast of inflation as well as IIP growth, quarterly data on WPI, IIP and M1 since first quarter of 2004-95 to fourth quarter of 2007-08 were used and we developed a VAR as well as Bayesian VAR (BVAR) model for forecasting the target variables. Further, based on the comparison of performance of these two models, it is found that the forecasting performance, measured in terms of out-of-sample percentage root mean square error of VAR model being used for forecasting inflation as well as IIP growth, has improved by applying Bayesian technique. It is recommended that the Bayesian VAR modeling should be encouraged, since it is more efficient than the VAR, and moreso, that it provides better forecast and more robust to the choice of variables even when misspecified variables are included.

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