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The 5th and 7th Higher Order of Hybrid Block Simpson's Method for Stiff Ordinary Differential Equations

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ABSTRACT

A self-starting Simpson's type block hybrid method (BHM) consisting of very closely accurate members each of order p=q+2 as a block is constructed. We got he higher order members of each by increasing the number k in the multi-step collocation (MC) used to derive the k-step continuous formula ($5 \le k \le 7$) through the aid of MAPLE software program. This paper presents and identify a continuous hybrid block schemes (CHBS) through the addition of one off-mesh collocation points in the MC. The (CHBS) is evaluated along with it's first derivative where necessary to give a hybrid block schemes for a simultaneous application to the ordinary differential equations (ODEs}. Some test problems to confirm the reliability of these scheme of experiments were carried out.

Keywords: Continuous hybrid block schemes (CHBS), Multi-step collocation (MC), hybrid block schemes, the k-step continuous formula ($5 \le k \le 8$), ODEs.

1. INTRODUCTION

Accuracy, Stability and Efficiency are the three conflicting basic aims in the design of schemes to solve ordinary differential equations. In [1,2,3] we identify a continuous hybrid block scheme (CHBS) through the addition of one or more off-mesh collocation points in the multi-step collocation (MC) as represented by (2.1.8). The (CHBS) is evaluated at some distinct points involving mesh and off-mesh points along with its first derivative, where necessary, to give multiple hybrid block schemes for the treatment of stiff ordinary differential equations. This paper is classified into sections. Section 1.0 is definitions of terms, we restate the MC procedure involving off-mesh collocation points for each k and we analyze on its convergence analysis obtained in a block form in section 2.0. By obtaining the order and error constants in a block form, the stability regions are also plotted. Section 3.0 is the numerical implementation of the block hybrid schemes on stiff (ODEs) and we give conclusion in section 4.0.

Definition 1.1 Linear Multi-Step Method
A k-step linear multi-step (lmm) is of the form

$$\sum_{j=0}^{k}\alpha_j y_{n+j} = h\sum_{j=0}^{k}\beta_j f_{n+j} \tag{1.1}$$

Where

$$f_{n+j} = f(x_{n+j}, y_{n+j}), \qquad y_{n+j} = y(x_{n+j})$$

α_j and β_j are constants and satisfy the constraints

$$\alpha_k \neq 0, \alpha_0^2 + \beta_0^2 > 0$$

(1.1) is explicit if $\beta_k = 0$ and implicit if $\beta_k \neq 0$

2. DERIVATION TECHNIQUES OF MC.

Let us consider the first order system of ODEs

$$y^1 = f(x,y), \qquad a < x < b, \quad y,f \in \Re^s \tag{2.1}$$

where y satisfies a given set of s associated conditions, which are either all initial, all boundary or mixed conditions.

The idea of the k – step MC, following [4], is to find a polynomial U of the form

$$U(x) = \sum_{j=0}^{t-1}\varphi_j(x)y_{n+j} + h\sum_{j=0}^{m-1}\phi_j(x)f\left(x_j, u(\bar{x}_j)\right), x_n \le x \le x_{n+x} \rightleftarrows \tag{2.2}$$

Where t denotes the number of interpolation points $x_{n+i}, i = 0,1,..,t-1,$ and m denote the number of distinct collocation points $\overline{x_i} \in [x_n, \ x_{n+k}]$, $i = 0,1,---, m-1$ *the points* $\overline{x_i}$ are chosen from the step x_{n+i} as well as one or more off step points.

The following assumptions are made;

1. Although the step size can be variable, for simplicity in our presentation of the analysis in this paper, we assume it is constant $h = x_{n+1} - x_n$, $\quad N = \dfrac{b-a}{h}$ with the steps given by $\{x_n / x_n = a + nh, \ n = 0,1,---,N\}$,

2. That (2.1) has a unique solution and the coefficients $\phi_j(x)$, $\varphi_j(x)$ in (2.2) can be represented by polynomials of the form

$$\varphi_j(x) = \sum_{i=0}^{t+m-1} \varphi_{j,i+1} x^i, j\varepsilon\{0,1,---,t-1\} \tag{2.3}$$

$$h\phi_j(x) = \sum_{i=0}^{t+m-1} \phi_{j,i+1} x^i j\varepsilon\{0,1,---,m-1\} \tag{2.4}$$

with constant coefficients $\phi_{j,i+1}$, $h\varphi_{j,i+1}$ to be determined using the interpolation and collocation conditions:

$$u(x_{n+i}) = y_{n+i}\ , i\varepsilon\ \{0,1,---,t-1\} \tag{2.5}$$

$$u^1(\overline{x}_i) = f\left(\overline{x}_i, u(\overline{x}_j)\right), j\varepsilon\{0,1,---,m-1\} \tag{2.6}$$

With this assumptions we obtain an MC polynomial, following [4, 5], in the form

$$u(x) = \sum_{i=0}^{t+m-1} a_i x^i, a^i = \sum_{j=0}^{t-1} c_{i+1,j+1} + \sum_{j=0}^{m-1} c_{i+1,j+t+1} f_{n+j} \tag{2.7}$$

Where $x_n \le x \le x_{n+k}$ and c_{ij} $i, j\varepsilon\{1,2,---,t+m\}$ are constants given by the elements of the inverse matrix $C = D^{-1}$. The MC matrix D is a nonsingular (m+1) square matrix of the type

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & - & - & - & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & - & - & - & x_{n+1}^{t+m-1} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & - & - & - & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\overline{x_0} & - & - & - & (t+m-1)\overline{x_0^{t+m-1}} \\ 0 & 1 & 2\overline{x_1} & - & - & - & (t+m-1)\overline{x_1^{t+m-1}} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 0 & 1 & 2\overline{x_{m-1}} & - & - & - & (t+m-1)\overline{x_{m-1}^{t+m-1}} \end{bmatrix} \tag{2.8}$$

2.1 Five steps Block Hybrid Simpson's Method with one off-step point.

The parameters required for equation (2.8) are k=5, t=1, m= k+2; $\left(x_n, x_{n+1}\right)$;

$$\left(\bar{x}_0 = x_n \,,\, \bar{x}_1 = x_{n+1} \,,\, \bar{x}_2 = x_{n+2} \,,\, \bar{x}_3 = x_{n+3} \,,\, \bar{x}_4 = x_{n+4} \,,\, \bar{x}_{\frac{9}{2}} = x_{n+\frac{9}{2}} \,,\, \bar{x}_5 = x_{n+5}\right)$$

Hence the matrix (2.8) takes the following shape.

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 \\ 0 & 1 & 2x_{n+\frac{9}{2}} & 3x^2_{n+\frac{9}{2}} & 4x^3_{n+\frac{9}{2}} & 5x^4_{n+\frac{9}{2}} & 6x^5_{n+\frac{9}{2}} & 7x^6_{n+\frac{9}{2}} \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 \end{bmatrix} \tag{2.9}$$

Using the maple software environment to evaluate (2.9) at the grid points

$x = x_{n+1}\,, x = x_{n+2}\,, x = x_{n+3}\,, x = x_{n+4}\,,\ x = x_{n+\frac{9}{2}},\ x = x_{n+5}$

We obtain the six discrete schemes, namely,

$$y_{n+1} = y_n + h\left[\frac{28199}{90720}f_n + \frac{78553}{70560}f_{n+1} - \frac{4519}{5040}f_{n+2} + \frac{13691}{15120}f_{n+3} - \frac{9841}{10080}f_{n+4} + \frac{13808}{19845}f_{n+\frac{9}{2}} - \frac{1537}{10080}f_{n+5}\right]$$

$$y_{n+2} = y_n + h\left[\frac{169}{567}f_n + \frac{6691}{4410}f_{n+1} - \frac{5}{63}f_{n+2} + \frac{517}{945}f_{n+3} - \frac{206}{315}f_{n+4} + \frac{9472}{19845}f_{n+\frac{9}{2}} - \frac{67}{630}f_{n+5}\right]$$

$$y_{n+3} = y_n + h\left[\frac{1013}{3360}f_n + \frac{11601}{7840}f_{n+1} + \frac{45}{112}f_{n+2} + \frac{689}{560}f_{n+3} - \frac{1017}{1120}f_{n+4} + \frac{464}{735}f_{n+\frac{9}{2}} - \frac{153}{1120}f_{n+5}\right]$$

$$y_{n+4} = y_n + h\left[\frac{170}{567}f_n + \frac{3296}{2205}f_{n+1} + \frac{104}{315}f_{n+2} + \frac{1664}{945}f_{n+3} - \frac{63}{315}f_{n+4} + \frac{8192}{19845}f_{n+\frac{9}{2}} - \frac{32}{315}f_{n+5}\right]$$

$$y_{n+\frac{9}{2}} = y_n + h\left[\frac{2151}{7168}f_n + \frac{374463}{250880}f_{n+1} + \frac{1215}{3584}f_{n+2} + \frac{31077}{17920}f_{n+3} + \frac{3159}{35840}f_{n+4} + \frac{162}{245}f_{n+\frac{9}{2}} - \frac{4131}{35840}f_{n+5}\right]$$

$$y_{n+5} = y_n + h\left[\frac{5435}{18144}f_n + \frac{21125}{14112}f_{n+1} + \frac{325}{1008}f_{n+2} + \frac{5375}{3024}f_{n+3} - \frac{125}{2016}f_{n+4} + \frac{4400}{3969}f_{n+\frac{9}{2}} + \frac{115}{2016}f_{n+5}\right]$$

……………………………….2.2.0

2.3 Six steps Block Hybrid Simpson's Method with one off-step point.

The parameters required for equation (2.9) are k=6, t=1, m= k+2; $\left(x_n , x_{n+1}\right)$,

$$\left(\bar{x}_0 = x_n \,,\, \bar{x}_1 = x_{n+1} \,,\, \bar{x}_2 = x_{n+2} \,,\, \bar{x}_3 = x_{n+3} \,,\, \bar{x}_4 = x_{n+4} \,,\, \bar{x}_5 = x_{n+5} \,,\, \bar{x}_{\frac{11}{2}} = x_{n+\frac{11}{2}} \,,\, \bar{x}_6 = x_{n+6}\right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7 \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7 \\ 0 & 1 & 2x_{n+\frac{11}{2}} & 3x_{n+\frac{11}{2}}^2 & 4x_{n+\frac{11}{2}}^3 & 5x_{n+\frac{11}{2}}^4 & 6x_{n+\frac{11}{2}}^5 & 7x_{n+\frac{11}{2}}^6 & 8x_{n+\frac{11}{2}}^7 \\ 0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7 \end{bmatrix} \tag{2.10}$$

Using the maple software environment to evaluate (2.10) at the grid points

$$x = x_{n+1} \,, x = x_{n+2} \,, x = x_{n+3} \,, x = x_{n+4} \,,\; x = x_{n+5} \,,\; x = x_{n+\frac{11}{2}} \,,\; x = x_{n+6}$$

We obtain the seven discrete schemes, namely,

$$y_{n+1} = y_n + h\left[\frac{4553}{15120}f_n + \frac{107293}{90720}f_{n+1} - \frac{3727}{3360}f_{n+2} + \frac{19001}{15120}f_{n+3} - \frac{4271}{3780}f_{n+4} + \frac{3559}{3360}f_{n+5} - \frac{400}{567}f_{n+\frac{11}{2}} + \frac{4381}{30240}f_{n+6}\right]$$

$$y_{n+2} = y_n + h\left[\frac{4027}{13860}f_n + \frac{4454}{2835}f_{n+1} - \frac{103}{420}f_{n+2} + \frac{52}{63}f_{n+3} - \frac{3047}{3780}f_{n+4} + \frac{82}{105}f_{n+5} - \frac{16384}{31185}f_{n+\frac{11}{2}} + \frac{137}{1260}f_{n+6}\right]$$

$$y_{n+3} = y_n + h\left[\frac{361}{1232}f_n + \frac{345}{224}f_{n+1} + \frac{243}{1120}f_{n+2} + \frac{859}{560}f_{n+3} - \frac{36}{35}f_{n+4} + \frac{1053}{1120}f_{n+5} - \frac{48}{77}f_{n+\frac{11}{2}} + \frac{143}{1120}f_{n+6}\right]$$

$$y_{n+4} = y_n + h\left[\frac{3034}{10395} f_n + \frac{880}{567} f_{n+1} + \frac{16}{105} f_{n+2} + \frac{1952}{945} f_{n+3} - \frac{386}{945} f_{n+4} + \frac{16}{21} f_{n+5} - \frac{16384}{31185} f_{n+\frac{11}{2}} + \frac{104}{945} f_{n+6}\right]$$

$$y_{n+5} = y_n + h\left[\frac{295}{1008} f_n + \frac{28025}{18144} f_{n+1} + \frac{125}{672} f_{n+2} + \frac{1975}{1008} f_{n+3} + \frac{125}{756} f_{n+4} + \frac{955}{672} f_{n+5} - \frac{400}{567} f_{n+\frac{11}{2}} + \frac{275}{2016} f_{n+6}\right]$$

$$y_{n+\frac{11}{2}} = y_n + h\left[\frac{905773}{3096576} f_n + \frac{7180745}{4644864} f_{n+1} + \frac{310123}{1720320} f_{n+2} + \frac{7643933}{3870720} f_{n+3} + \frac{2019127}{15482880} f_{n+4} + \frac{1476079}{860160} f_{n+5} \right.$$
$$\left. - \frac{4213}{9072} f_{n+\frac{11}{2}} + \frac{1925957}{15482880} f_{n+6}\right]$$

$$y_{n+6} = y_n + h\left[\frac{41}{140} f_n + \frac{54}{35} f_{n+1} + \frac{27}{140} f_{n+2} + \frac{68}{35} f_{n+3} + \frac{27}{140} f_{n+4} + \frac{54}{35} f_{n+5} + \frac{41}{140} f_{n+6}\right] \qquad 2.11$$

2.4 Seven steps Block Hybrid Simpson's Method with one off-step point.

The parameters required for equation (2/9) are k=7, t=1, m= k+2; (x_n , x_{n+1}) ;

$$\left(\bar{x}_0 = x_n ,\ \bar{x}_1 = x_{n+1} ,\ \bar{x}_2 = x_{n+2} ,\ \bar{x}_3 = x_{n+3} ,\ \bar{x}_4 = x_{n+4} ,\ \bar{x}_5 = x_{n+5} ,\ \bar{x}_6 = x_{n+6} ,\ \bar{x}_{\frac{13}{2}} = x_{n+\frac{13}{2}} ,\ \bar{x}_7 = x_{n+7}\right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D = \begin{bmatrix}
1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\
0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 \\
0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 \\
0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7 & 9x_{n+2}^8 \\
0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7 & 9x_{n+3}^8 \\
0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7 & 9x_{n+4}^8 \\
0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7 & 9x_{n+5}^8 \\
0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7 & 9x_{n+6}^8 \\
0 & 1 & 2x_{n+\frac{13}{2}} & 3x_{n+\frac{13}{2}}^2 & 4x_{n+\frac{13}{2}}^3 & 5x_{n+\frac{13}{2}}^4 & 6x_{n+\frac{13}{2}}^5 & 7x_{n+\frac{13}{2}}^6 & 8x_{n+\frac{13}{2}}^7 & 9x_{n+\frac{13}{2}}^8 \\
0 & 1 & 2x_{n+7} & 3x_{n+7}^2 & 4x_{n+7}^3 & 5x_{n+7}^4 & 6x_{n+7}^5 & 7x_{n+7}^6 & 8x_{n+7}^7 & 9x_{n+7}^8
\end{bmatrix} \qquad 2.12$$

Using the maple software environment to evaluate (2.12) at the grid points

$$x = x_{n+1} ,\ x = x_{n+2} ,\ x = x_{n+3} ,\ x = x_{n+4} ,\ x = x_{n+5} ,\ x = x_{n+6} ,\ x = x_{n+\frac{13}{2}} ,\ x = x_{n+7}$$

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We obtain the eight discrete schemes, namely,

$$y_{n+1} = y_n + h\left[\frac{6904181}{23587200}f_n + \frac{24976453}{19958400}f_{n+1} - \frac{7382233}{5443200}f_{n+2} + \frac{641023}{362880}f_{n+3} - \frac{3229573}{1814400}f_{n+4} + \frac{2523853}{1814400}f_{n+5} - \frac{2071633}{1814400}f_{n+6} + \frac{4345984}{6081075}f_{n+\frac{13}{2}} - \frac{35857}{259200}f_{n+7}\right]$$

$$y_{n+2} = y_n + h\left[\frac{209201}{737100}f_n + \frac{253552}{155925}f_{n+1} - \frac{75403}{170100}f_{n+2} + \frac{3502}{2835}f_{n+3} - \frac{75553}{56700}f_{n+4} + \frac{15172}{14175}f_{n+5} - \frac{50563}{56700}f_{n+6} + \frac{487424}{868725}f_{n+\frac{13}{2}} - \frac{1546}{14175}f_{n+7}\right]$$

$$y_{n+3} = y_n + h\left[\frac{83173}{291200}f_n + \frac{394389}{246400}f_{n+1} - \frac{3}{22400}f_{n+2} + \frac{8879}{4480}f_{n+3} - \frac{35829}{22400}f_{n+4} + \frac{27549}{22400}f_{n+5} - \frac{22529}{22400}f_{n+6} + \frac{15744}{25025}f_{n+\frac{13}{2}} - \frac{2727}{22400}f_{n+7}\right]$$

$$y_{n+4} = y_n + h\left[\frac{52498}{184275}f_n + \frac{250904}{155925}f_{n+1} - \frac{2264}{42525}f_{n+2} + \frac{7064}{2835}f_{n+3} - \frac{13574}{14175}f_{n+4} + \frac{15224}{14175}f_{n+5} - \frac{12944}{14175}f_{n+6} + \frac{3506176}{6081075}f_{n+\frac{13}{2}} - \frac{1592}{14175}f_{n+7}\right]$$

$$y_{n+5} = y_n + h\left[\frac{269245}{943488}f_n + \frac{1280525}{798336}f_{n+1} - \frac{5825}{217728}f_{n+2} + \frac{173875}{72576}f_{n+3} - \frac{27725}{72576}f_{n+4} + \frac{119765}{72576}f_{n+5} - \frac{76025}{72576}f_{n+6} + \frac{22400}{34749}f_{n+\frac{13}{2}} - \frac{8975}{72576}f_{n+7}\right]$$

$$y_{n+6} = y_n + h\left[\frac{2593}{9100}f_n + \frac{3096}{1925}f_{n+1} - \frac{33}{700}f_{n+2} + \frac{86}{35}f_{n+3} - \frac{369}{700}f_{n+4} + \frac{396}{175}f_{n+} \quad 2.13 \quad 299 + \frac{12288}{25025}f_{n+\frac{13}{2}} - \frac{18}{175}f_{n+7}\right]$$

$$y_{n+\frac{13}{2}} = y_n + h\left[\frac{132397603}{464486400}f_n + \frac{8212607897}{5109350400}f_{n+1} - \frac{59738627}{1393459200}f_{n+2} + \frac{227150027}{92897280}f_{n+3} - \frac{233609207}{464486400}f_{n+4} + \frac{1030790657}{464486400}f_{n+5} - \frac{57365867}{464486400}f_{n+6} + \frac{339677}{467775}f_{n+\frac{13}{2}} - \frac{52778531}{464486400}f_{n+7}\right]$$

$$y_{n+7} = y_n + h\left[\frac{959651}{3369600} f_n + \frac{4589683}{2851200} f_{n+1} - \frac{41503}{777600} f_{n+2} + \frac{128233}{51840} f_{n+3} - \frac{144403}{259200} f_{n+4} + \frac{597163}{259200} f_{n+5} \right.$$
$$\left. - \frac{82663}{259200} f_{n+6} + \frac{1047424}{868725} f_{n+\frac{13}{2}} + \frac{13391}{259200} f_{n+7}\right]$$

2.5 The Order and Error constants of the A-stable Block Hybrid Methods.
The hybrid block methods which are obtained in a block form with the help of maple software have the following order and error constants for each case.

Case k=5

Evaluating points	*Order*	*Error Constants*
$y(x = x_{n+1})$	7	$\frac{1759}{211680}$
$y(x = x_{n+2})$	7	$\frac{337}{52920}$
$y(x = x_{n+3})$	7	$\frac{57}{7840}$
$y(x = x_{n+4})$	7	$\frac{44}{6615}$
$y\left(x = x_{n+\frac{9}{2}}\right)$	7	$\frac{54351}{8028160}$
$y(x = x_{n+5})$	7	$\frac{275}{42336}$

Table 1: k=5 BHSM with one off-step point

The method k=5 is of order 7 as a block and has error constants

$$C_8 = \left(\frac{1759}{211680}, \frac{337}{52920}, \frac{57}{7840}, \frac{44}{6615}, \frac{54351}{8028160}, \frac{275}{42336}\right)^T$$

Case k=6		
Evaluating points	*Order*	*Error Constants*
$y(x = x_{n+1})$	8	$-\frac{209749}{29030400}$
$y(x = x_{n+2})$	8	$-\frac{653}{11340}$
$y(x = x_{n+3})$	8	$-\frac{2277}{358400}$
$y(x = x_{n+4})$	8	$-\frac{169}{28350}$
$y(x = x_{n+5})$	8	$-\frac{7325}{1161216}$
$y\left(x = x_{n+\frac{11}{2}}\right)$	8	$-\frac{46301497}{731782400}$
$y(x = x_{n+6})$	8	$-\frac{9}{1400}$

Table 2: k=6 BHSM with one off-step point

The method k=6 is of order 8 as a block and has error constants

$$C_9 = \left(-\frac{209749}{29030400}, -\frac{653}{11340}, -\frac{2277}{35840}, -\frac{169}{28350}, -\frac{7325}{1161216}, -\frac{46301497}{731782400}, -\frac{9}{1400}\right)^T$$

Case k=7		
Evaluating points	*Order*	*Error Constants*
$y(x = x_{n+1})$	9	$\frac{13789}{2177280}$
$y(x = x_{n+2})$	9	$\frac{3541}{680400}$
$y(x = x_{n+3})$	9	$\frac{251}{44800}$
$y(x = x_{n+4})$	9	$\frac{457}{85050}$
$y(x = x_{n+5})$	9	$\frac{2425}{435456}$
$y(x = x_{n+6})$	9	$\frac{3}{560}$
$y\left(x = x_{n+\frac{13}{2}}\right)$	9	$\frac{60323029}{11147673600}$
$y(x = x_{n+7})$	9	$\frac{8183}{1555200}$

Table 3: k=7 BHSM with one off-step point

The method k=7 is of order 9 as a block and has error constants

$$C_{10} = \left(\frac{13789}{2177280}, \frac{3541}{680400}, \frac{251}{44800}, \frac{457}{85050}, \frac{2425}{435456}, \frac{3}{560}, \frac{60323029}{11147673600}, \frac{8183}{1555200} \right)^T$$

2.6 Stability Regions of The Block Hybrid Simpson's Methods.

To compute and plot the absolute stability regions of the block hybrid Simpson's methods, the methods are reformulated as general linear methods expressed as;[4,5,6]

$$\begin{bmatrix} Y \\ -- \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} A & / & U \\ - & - & - \\ B & / & V \end{bmatrix} \begin{bmatrix} hf(Y) \\ \\ y_{i-1} \end{bmatrix}$$

where,

$$A = \begin{bmatrix} a_{11} & . & . & . & a_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ a_{s1} & . & . & . & a_{ss} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & . & . & . & b_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ b_{k1} & . & . & . & b_{ks} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_n \\ y_{n+1} \\ . \\ . \\ . \\ y_{n+k} \end{bmatrix} \qquad y_{i+1} = \begin{bmatrix} y_{n+k} \\ . \\ . \\ y_{n+k-1} \end{bmatrix} \qquad y_{i-1} = \begin{bmatrix} y_{n+k-1} \\ . \\ . \\ y_{n+k-2} \end{bmatrix}$$

and the elements of the matrices A,B,U and V are obtained from the interpolation and collocation and collocation points.

The elements of the matrices A, B, U and V are substituted into the stability matrix

$$M(z) = B_2 + zA_2(I - zA_1)^{-1} B_1 \qquad where \quad A_1 = A, A_2 = B, B_1 = U, B_2 = V$$

and the stability function

$$\rho(\eta, z) = \det(\eta I - M(z))$$

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Computing the stability function with Maple yields the stability polynomial of the method which is plotted in Matlab to produce the required absolute stability region of the method.

2.2 Absolute stability region of the block hybrid method K=5 [4]

The block hybrid methods (2.1) with one off-grid point are arranged as shown below;
The coefficients of these methods expressed in tabular form bellow gives the coefficients of the new method.

$$
\begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+\frac{9}{2}} \\ y_{n+5} \\ --- \\ y_{n+5} \\ y_{n+4} \\ y_{n+3} \\ y_{n+2} \\ y_{n+1} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & . & 0 & 0 & 0 & 0 & 1 \\
\frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{2151}{7168} & \frac{374463}{250880} & \frac{1215}{3584} & \frac{31077}{17920} & \frac{3159}{35840} & \frac{162}{245} & -\frac{4131}{35840} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{5435}{18144} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} & . & 0 & 0 & 0 & 0 & 1 \\
- & - & - & - & - & - & - & - & - & - & - & - & - \\
\frac{5435}{1814} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} & . & 0 & 0 & 0 & 0 & 1 \\
\frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} & . & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+\frac{9}{2}} \\ f_{n+5} \\ - \\ y_{n+4} \\ y_{n+3} \\ y_{n+2} \\ y_{n+1} \\ y_n \end{bmatrix}
$$

Substituting the values of A, B, U and V into the stability matrix and the stability function and using Maple software yields the stability polynomial of the block method.

Using a Matlab program, we obtained the stability region of the block hybrid Simpson's method for K= 5 as shown in figure 1. The stability region of the block method shows that it is A-stable.

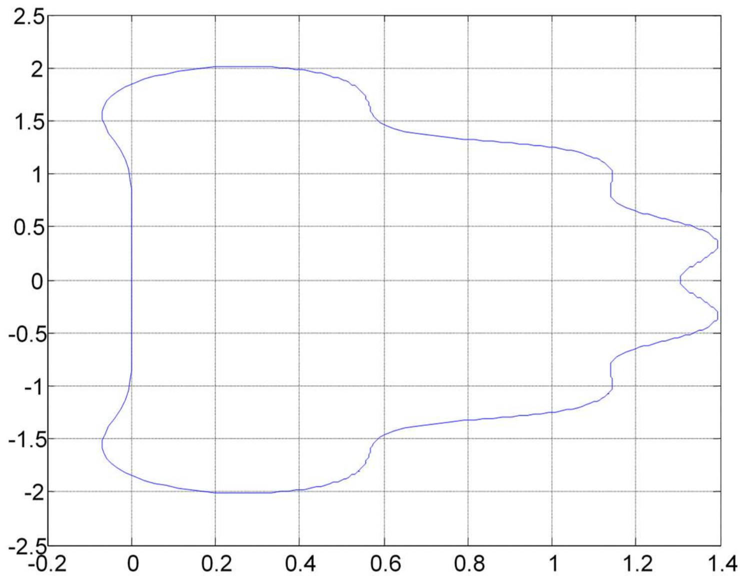

Figure 1: Stability region of the block hybrid Simpson's method K=5

Following the same procedure for k=6 and 7, the elements of the matrices A,B,U and V are substituted and computing the stability function with Maple software yield, the stability polynomial of the method which is then plotted in MATLAB environment to produce the required absolute stability region of the methods, as shown by the figures 2 and 3 below:

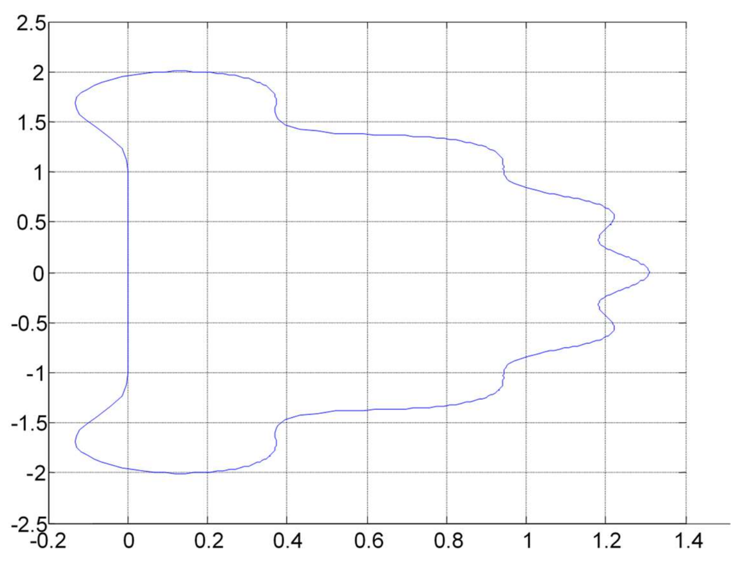

Figure 2 Stability region of the block hybrid Simpson's method K=6

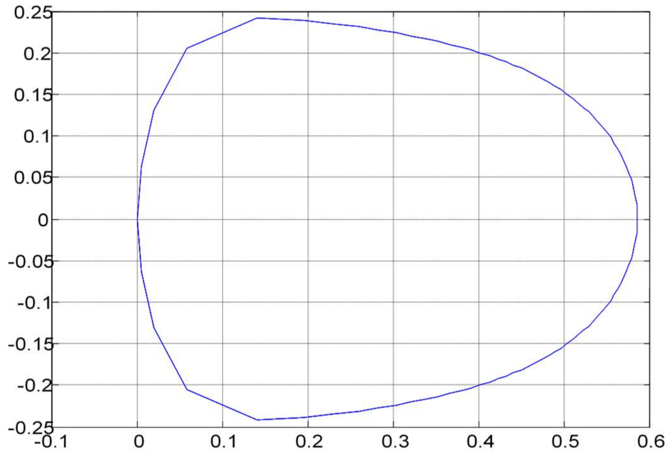

Figure 3: Stability region of the block hybrid Simpson's method K=7

3. NUMERICAL IMPLEMENTATION.

To study the efficiency of the block hybrid method for $2 \le k \le 7$, we present some numerical examples widely used by several authors such as [8,9,10,11].

$$Experiment 1 \quad y^1 = -1000000y, \quad where\ h = 0.1, \quad x\varepsilon[0,1.8]$$

$$Exact\,solution \quad y(x) = e^{-1000000x}$$

Table of absolute errors for experiment 1

Y	A-stable Hybrid Block Simpson's K=5	A-stable Hybrid Block Simpson's K=6	A-stable Hybrid Block Simpson's K=7
0.1	1.56×10^{-1}	1.36×10^{-1}	1.21×10^{-1}
0.2	5.56×10^{-2}	4.24×10^{-2}	3.30×10^{-2}
0.3	3.33×10^{-2}	2.27×10^{-2}	1.54×10^{-2}
0.4	2.22×10^{-2}	1.82×10^{-2}	1.20×10^{-2}
0.5	1.11×10^{-1}	1.52×10^{-2}	1.20×10^{-2}
0.6	1.73×10^{-2}	9.09×10^{-2}	1.20×10^{-2}
0.7	6.17×10^{-3}	1.24×10^{-2}	7.69×10^{-2}
0.8	3.70×10^{-3}	3.86×10^{-3}	9.30×10^{-3}
0.9	2.47×10^{-3}	2.07×10^{-3}	2.54×10^{-3}
1.0	1.23×10^{-2}	1.65×10^{-3}	1.18×10^{-3}
1.1	1.92×10^{-3}	1.38×10^{-3}	8.45×10^{-4}
1.2	6.86×10^{-4}	8.26×10^{-4}	8.45×10^{-4}
1.3	4.11×10^{-4}	1.13×10^{-3}	8.45×10^{-4}
1.4	2.74×10^{-4}	3.51×10^{-4}	5.92×10^{-3}
1.5	1.37×10^{-3}	1.88×10^{-4}	7.15×10^{-4}
1.6	2.13×10^{-4}	1.50×10^{-4}	1.95×10^{-4}
1.7	7.62×10^{-5}	1.25×10^{-4}	9.10×10^{-5}
1.8	4.57×10^{-5}	7.51×10^{-4}	6.50×10^{-5}

Absolute Errors Of A-Stable New BHSM At x $\in$[0, 1.8] $\lambda = -1000000$

4. CONCLUSION

The A - Stability, Convergence , Accuracy from the stability graph, the numerical table results is evident that the proposed schemes are indeed numerically viable and can handle stiff equations. The result on the Numerical table shows the strength at the 7th block and the convergence.
It therefore implies that to more the block the better the rate of convergence with a firm consideration of the stability of the scheme.

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