



Solution-adaptive Cross-diffusion Effects on Heat and Mass Transfer of Unsteady MHD Fluid Flow in Porous Media

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ABSTRACT

The influence of solution-adaptive Cross-diffusion Effects on Heat and Mass Transfer of Unsteady MHD Fluid Flow in Porous Media over inclined plate is the thrust of this paper. Its applications in energy renewal in industries and engineering continuously generating research works among academics. This study therefore, investigated solution-adaptive cross-diffusion effects on heat and mass transfer of unsteady MHD fluid flow over inclined sheet in the presence of heat generation and chemical reaction. The governing nonlinear partial differential equations of the model were transformed to a system of ordinary differential equations. The coupled system of equations were then numerically solved by a fourth order Runge-Kutta method along with shooting technique. A parametric study on the effect of variations in the fluid parameters on velocity, temperature and concentration were conducted and presented graphically. Also, the effect of radiation and chemical reaction on heat and mass transfer as well as effects on non-dimensional skin friction, Nusselt and Sherwood number and parameters were verified and discussed in detail. The effects of solution-adaptive cross-diffusion were critically explained including their significance for alternative economic growth for national development.

Keywords: Cross-diffusion, unsteady MHD flow, porous Media Inclined Plate.

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1. INTRODUCTION

The effect of natural convection on the accelerated boundary layer flow of two dimensional incompressible fluid over an inclined plate is implemented. The influence of solution-adaptive cross diffusion on heat and mass transfer of unsteady MHD fluid in porous media with suction and chemical reaction has important role in technological applications. Its importance and significance in astrophysical geophysical industries and engineering are thrust of this paper. The problems emanating from such studies require enormous research activities especially where the heating of rocks and buildings by the use of radiators is a familiar area where heat transfer by natural convection are needed. The study like this plays an important role in manufacturing industries for the design of steel rolling, power plants, fins, gas turbines and various devices for aircraft, material processing, energy generation, utilization, propulsion devices for space craft temperature and chemical reaction measurement. MHD fluid flow in porous media has wide and numerous applications in industry and environments (Kala, Singh and Kumar 2014; Qatanani and Mai Musmar, 2012).



Some of other areas of applications but not limited are : flow of ground water through soil and rocks (porous media) that are very important for agriculture and pollution control; extraction of oil and natural gas from rocks which are prominent in oil and gas industries; functioning of tissues in body (bone, cartilage and muscle and so on) belong to porous media, flow of blood and treatments through them; understanding various medical conditions (such as tumor growth, a formation of porous media) and their treatment (such as injection, a flow through porous media in medical sciences). In the similar manner, the subject of MHD has attracted the attention of a large number of scholars due to its diverse applications in several problems of technological importance. The ionized gas or plasma can be made to interact with the magnetic field and can frequently alter heat transfer and friction characteristics on the bounding surface. Heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering. In astrophysics and geophysics, it is mainly applied to study the cellular and solar structures, interstellar matter, radio propagation through the ionosphere.

Sharma (2004) analysed unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and heat transfer past a semi-infinite vertical porous plate. Makinde (2010) studied the MHD boundary layer flow with the heat and mass transfer over a moving vertical plate with magnetic field and convection heat exchange at the surface. In the same vein, Shekhar (2014) researched into boundary layer phenomena of MHD flow and heat transfer over exponentially stretching sheet embedded in a thermally stratified medium. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbine machinery, and aerospace technology. The rise of such flows is due to unsteady motion of either the boundary or the boundary temperature. Besides, the unsteadiness may also be due to the oscillatory free stream velocity or temperature. The phenomenon of heat and mass transfer is also very common in chemical process industries such as food processing and polymer production. Also the study of magneto hydrodynamics (MHD) incompressible viscous flows has many important engineering applications in devices such as MHD power generators, cooling of nuclear reactors, geothermal systems, aerodynamic processes, and heat exchange designs (Amoo and Idowu, 2017)

The unsteady MHD fluid flow over surface which is stretched with velocity that depends on time and position had been considered in this paper. Free convection flow involving heat transfer occurs frequently in an environment where difference between land and air temperature can give rise to complicated flow patterns. The study of effects of magnetic field on free convection flow is often found importance in liquid metals, electrolytes and ionized gasses. At extremely high temperatures in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. In engineering, the problem assumes greater significance in MHD pumps, MHD joint bearings and so on. Recently, it is of great interest to study the effects of magnetic field and other participating parameters on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. Chen (2004) investigated the effects of heat and mass transfer in MHD free convection from a vertical surface. In addition, the applications of the effect of Hall current on the fluid flow with variable concentration have been seen in MHD power generators, astrophysical and meteorological studies as well as in plasma physics. The Hall Effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall Effect is disregarded.

But if the strength of magnetic field is high and the number density of electrons is small, the Hall Effect cannot be ignored as it has a significant effect on the flow pattern of an ionized gas. Hall Effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. Model studies on the effect of Hall current on MHD convection flows have been carried out (Ahmed, *et al* 2010). Also the free convection flow in the presence of magnetic field is very important owing to its significant effect on the boundary layer control and on the performance of many engineering devices using electrically conducting fluids. This type of fluid flow finds application in MHD power generation, plasma studies, nuclear reactor using liquid metal coolant and geothermal energy extraction. The energy equation had led to a highly nonlinear partial differential equation with the inclusion of radiation effects. In this study Darcy Parameter, heat source, angle of inclination and chemical parameter were added to see the effect of cross-diffusion of unsteady MHD flow in porous media over inclined plate or surface. The objectives of the present study is to analyse numerically, the solution-adaptive cross-diffusion effects on heat and mass transfer of MHD flow in porous media over inclined stretching surface. The effects of various governing parameters on the fluid velocity, temperature concentration which were skin friction coefficient, Nusselt and Sherwood numbers presented in Tables 1 and 2 and were discussed.



2. MATHEMATICAL ANALYSIS

An unsteady two dimensional laminar boundary layer flow of an incompressible fluid over stretching inclined plate is considered. The fluid is assumed to be Newtonian, electrically conducting and its properties due to temperature and concentration are limited to fluid density. The density variation and the effect of the buoyancy are taken into consideration in the momentum equations. The x-axis is taken along the plate in the upward direction and y is taken normal to it. A uniform magnetic field is applied in the direction plate. The transverse applied magnetic field and magnetic Reynolds number assumed to be very small so that the induced magnetic field and all effect are negligible. Based on the above assumptions, the governing equations that explained the flow were:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} = \frac{\nu\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu k}{\rho} + g\beta_T (T - T_\infty) \cos(\alpha) + g\beta_C (C - C_\infty) \cos(\alpha) \quad (2)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + \frac{u\partial T}{\partial x} + \frac{v\partial T}{\partial y} \right) = \frac{k\partial^2 T}{\partial y^2} + Q(T - T_\infty) - \frac{\partial q_r}{\partial y} + \frac{D_m k_c \partial^2 C}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t} + \frac{u\partial C}{\partial x} + \frac{v\partial C}{\partial y} = \frac{D_m \partial^2 C}{\partial y^2} + \gamma(C - C_\infty) + \frac{D_m k_T \partial^2 T}{\partial y^2} \quad (4)$$

The boundary conditions for velocity, temperature and concentration fields are;

$$\begin{aligned} u = U, \quad v = V_w, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \quad t > 0 \end{aligned} \quad (5)$$

where

u , v , C , and T are velocity component in the x direction, velocity component in the y direction, concentration of the fluid species and fluid temperature respectively. L is the reference length, $B(x)$ is the magnetic field strength, U_0 is the reference velocity and V_0 is the permeability of the porous surface. The physical quantities K , ρ , ν , σ , D , k , C_p , Q_0 and γ are the permeability of the porous medium, density, fluid kinematics viscosity, electric conductivity of the fluid, coefficient of mass diffusivity, thermal conductivity of the fluid, specific heat, rate of specific internal heat generation or absorption and reaction rate coefficient respectively. g is the gravitational acceleration, β_T and β_C are the thermal and mass expansion coefficients respectively. q_r is the radiative heat flux in the y direction. The last terms in equations (3) and (4) are Dufour or diffusion thermo effect and the Soret or thermo-diffusion effect respectively. By using the Rosseland approximation according to Ibrahim and Suneetha (2015). The radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_0}{3\delta} \frac{\partial T^4}{\partial y} \quad (6)$$

where

σ_0 and δ are the Stefan-Boltzmann and the mean absorption coefficient respectively.



Assume the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, using Taylor series to expand T^4 about the free stream T_∞ and neglecting higher order terms, this gives the approximation

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

The magnetic field $B(x)$ is assumed to be in the form

$$B(x) = B_0 e^{\frac{x}{2L}} \quad (8)$$

where B_0 is the constant magnetic field. We introduced the stream function $\psi(x, y)$ such that $u = \frac{\partial \psi}{\partial y}$,

$$v = -\frac{\partial \psi}{\partial x} \quad (9)$$

We substituted equation (9) in equation (1) and the continuity equation satisfied Cauchy-Riemann equation. We transformed the equations (2), (3) and (4) into an ordinary differential equations following similarity transformation variables according to Sajid and Hayat (2008), Subhakar and Gangadhar (2012), Ibrahim and Suneetha (2015), Amoo and Idowu (2017). In view of equations (6) and (7), equation (3) becomes:

$$\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left(\frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{Dk \partial^2 T}{C_s \partial y^2} \quad (10)$$

In order to write the governing equations and the boundary conditions in dimensionless form of equation (11), the following non-dimensional quantities were introduced and by identity continuity equation (1) satisfied Cauchy-Riemann equation. Then equations (2), (4) and (10) became a system of ordinary differential equations with the following transformations when substituting the dimensionless parameters equation (11)

$$\begin{aligned} \eta &= y \sqrt{\frac{C}{v(1-\lambda t)}}, \quad \psi(x, y) = x \sqrt{\frac{Cv}{(1-\lambda t)}} f(\eta), \\ T &= T_\infty + T_w \left[\frac{Cx}{2v(1-\lambda t)^2} \right] \theta(\eta), \quad C = C_\infty + C_w \left[\frac{Cx}{2v(1-\lambda t)^2} \right] \theta(\eta), \\ M &= \frac{\sigma B_0^2 (1-\lambda t)}{\rho C}, \quad G_r = \frac{g B_T T_w}{2C\nu}, \\ Q &= \frac{CQ_0}{(1-\lambda t)}, \quad G_c = \frac{g\beta_C C_w}{2C\nu}, \quad D_f = \frac{D_m K_T C_w}{\nu C_s C_p T_w}, \quad S_r = \frac{D_m K_T T_w}{\nu T_m C_w}, \\ P_r &= \frac{\nu C_p}{k}, \quad S_c = \frac{\nu}{D_m}, \quad U = \frac{\lambda}{C}, \quad \lambda^* = \frac{(1-\lambda t)\gamma}{\rho C} \\ f_w &= V_0 \sqrt{\frac{(1-\lambda t)}{2C\nu}}, \quad D_a = \frac{(1-\lambda t)\nu}{Ck} \end{aligned} \quad (11)$$



where η is similarity variable, f is the dimensionless stream function, θ is dimensionless temperature ϕ is the dimensionless concentration, M is the magnetic field parameter, α is the angle of inclination, D_a is the Darcy porosity, G_c is the solutal Grashof number, Gr is the thermal Grashof number, Pr is the Prandtl number, Q is the heat generation parameter, Sc is the Schmidt number, D_f is the Dufour number, Sr is the Soret number, λ^* is the chemical reaction parameter, F_w is the permeability at the plate and U is the unsteadiness parameter. s and m are susceptibility and mean of diffusivity. In view of the systems of ordinary differential equations, the geometry of the model and coordinate system of ODEs describing the cross-diffusion of unsteady MHD fluid flow is shown in

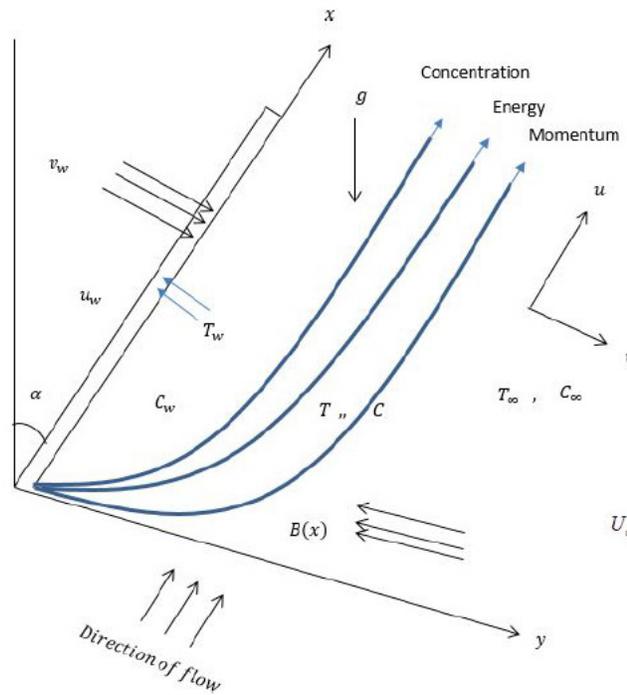


Figure 1: The Geometry of the Model and Coordinate System

The equations (12), (13) and (14) with boundary condition (15) to were:

$$f''' + ff'' - 2f'^2 - \frac{U}{2}\eta f' - (M + D_a)f' + Gr\theta \cos(\alpha) + G_c \cos(\alpha) = 0 \quad (12)$$

$$\theta'' + Pr f \theta' - Pr \frac{U}{2} \eta \theta' - Pr f' \theta + Pr Q \theta + Pr D_f \phi'' = 0 \quad (13)$$

$$\phi'' + Sc f \phi' - Sc \frac{U}{2} \eta \phi' - Sc f' \phi - Sc \lambda^* \phi + Sc Sr \theta'' = 0 \quad (14)$$



The corresponding boundary conditions take the form:

$$\begin{aligned} f &= f_w, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0 \\ f' &= 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \tag{15}$$

where the prime symbolizes the derivative with respect to η and from the f_w is the suction parameter from the numerical computation, the local skin-function coefficient, the local Nusselt number and Sherwood number which are proportional to $f''(0) - \theta'(0)$ and $-\phi(0)$ are involved out and the corresponding numerical values are presented in Table 2. The governing equation (12)-(14) subject to the boundary condition (15) were integrated using Shooting technique along with Runge-Kutta method.

Shooting technique along with Runge-Kutta method has been adopted as the numerical scheme for this research work. Runge-Kutta method for solving differential equation is widely used and affords a high degree of accuracy (Stroud, 1996; Stroud and Booth, 2003). It is a step by step process where a table of function values for a range of x is accumulated. Several intermediate calculations are required at each stage, but these are straight forward and present little difficulty.

These methods are also based on finite-difference numerical techniques. Consider the two-point boundary value problem such as:



$$y'' = f(x, y, y'), \quad y(a) = \alpha, \quad y(b) = \beta, \quad (16)$$

where $a < b$ and $x \in [a, b]$.

Making an initial guess for $y(a)$ and denote by $y(x_i)$ the solution of the initial value problem is

$$y'' = f(x, y, y'), \quad y(a) = \alpha, \quad y'(a) = \beta, \quad (17)$$

Introducing the notation $Y(x_i) = y(x_i)$ and $v(x; 0) = \frac{\partial}{\partial x}y(x; 0)$, equation (17) is rewritten as

$$\begin{aligned} \frac{\partial}{\partial x}Y(x; 0) &= v(x; 0) \\ \frac{\partial}{\partial x}v(x; 0) &= f(x, Y(x; 0), v(x; 0)) \end{aligned} \quad (18)$$

The solution $Y(x_i)$ of the initial value problem (18) coincided with the solution $y(x)$ of the initial value problem (17) provided we can find a value of $Y(x)$ such that

$$\phi(x) \equiv Y(b_j) - \beta = 0 \quad (19)$$

The basic ideal of the shooting method for the numerical solution of the boundary value problem (17) is to find a root to the equation (19). Here, fourth order Runge-Kutta techniques is used to find the root and the scheme for the fourth order Runge-Kutta according to (LeVeque, 2007) is

$$y_{n+1} = y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4) \quad (20)$$

where

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ k_2 &= f\left(x_n + \frac{h}{2}, y_n + \frac{hk_1}{2}\right) \\ k_3 &= f\left(x_n + \frac{h}{2}, y_n + \frac{hk_2}{2}\right) \\ k_4 &= f(x_n + h, y_n + hk_3) \end{aligned}$$



In this study, a numerical code that incorporated the methods described above was developed by the researchers using Maple 18 to solve the models. Numerical results are reported in the Tables 1 and 2. The Prandtl number is taken to be $Pr = 0.71$ which corresponds to air, the value of Schmidt number (Sc) were chosen to be $Sc = 0.24, 0.62, 0.78, 2.62$, representing diffusing chemical species of most common interest in air like H_2, H_2O, NH_3 and Propyl Benzene respectively.

3. RESULTS AND DISCUSSION

The system of nonlinear ordinary differential equations (12) to (14) together with boundary condition (15) were solved. The numerical scheme used were the Runge-Kutta Fourth order method along with shooting technique. Table 1 shows the validation with earlier research endeavours. In order to benchmark our numerical results, the present results for the skin function, Nusselt number and Sherwood number in the absence of M, U, fw, Q, D_f, S_r

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Table 1: Comparison of numerical values of $\theta'(0)$

Pr	Present Results	Subhkar and Gangadhar(2012)	Srinivasachary and Ram Ready (2011)	Magyari and Keller (1999)
0.5	-0.63895	-0.667192	-0.59438	-0.59434
1.0	-0.96253	-1.00426	-0.95478	-0.95478
3.0	-1.86755	-1.92168	-1.86908	-1.86908
0.5	-2.49895	-2.55594	-2.50015	-2.50014

From Table 1, it could be deduced that the variations of Pr . for numerical values of $\theta'(0)$ when compared with the existing literature were in close agreement. A comparison of the previous studies over years had been improvement of $\theta'(0)$ for varying Pr . and the present study followed the same trends. The possible reasons for the trend's variation might be due to effectiveness of the method of solution and system of equations or models considered



Table 2: Effect of M , u , Sc , Pr , Da and λ on $f''(0)$, $\theta'(0)$ and $\phi'(0)$ (P-Parameters)

P	Values	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	P	values	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
M	1	1.2327	0.3208	1.41395	Da	0.2	0.63535	0.908775	1.19407
	3	0.4713	0.1848	1.3946		1.0	0.83358	0.25163	1.40293
	5	-0.1650	0.0577	1.3842		2.0	1.11033	-1.02625	1.79392
	7	-0.7113	0.0613	1.3803		4.0	1.9521	-16.2638	6.4653
Gc.	2	0.0538	0.1548	1.3860	Gr	0.5	-0.8929	-0.4397	1.4390
	5	0.83363	0.2516	1.4029		5	0.8336	0.2516	1.4029
	7	1.3439	0.3083	1.4149		7	1.4755	0.3508	1.4197
	10	2.00952	0.3833	1.4334		10	2.3798	0.4570	1.4459
f_w	1	0.83359	0.25163	1.40293	α	5	-1.4814	-0.4145	1.4110
	2	0.363091	0.44991	1.73378		30°	-1.9798	-0.8486	1.4871
	3	-0.39147	0.75866	2.08255		45°	-0.6410	0.0576	1.3829
	4	-1.37759	1.2124	2.46598		75°	0.6016	0.2152	1.3977
u	0.01	0.71098	0.46728	1.43757	Q	0.5	0.8335	0.2516	1.4029
	2.4	1.039086	-0.11824	1.37588		1.5	1.3420	-1.07122	1.8079
	5.0	1.07584	-0.18645	1.11774		3.5	1.3992	-1.8791	2.0012
	10.0	0.87587	0.12736	0.80538		5.0	0.5918	-0.6005	1.5660
Pr.	0.01	0.94183	0.2495	1.4151	Df.	0.2	0.63535	0.908775	1.19407
	2.0	0.6881	0.2383	1.3985		1.0	0.83358	0.25163	1.40293
	2.5	0.6498	0.1400	1.14280		2	1.11033	-1.02625	1.79392
	4.0	0.6014	1.0524	1.1482		4	1.95211	-16.2638	6.4653
Sc.	0.35	0.9086	0.6416	0.9001	λ	0.5	0.8336	0.2516	1.4029
	0.62	0.8336	0.2516	1.4029		1.5	0.7887	-0.01549	1.7514
	1.50	0.6469	-1.5469	3.8802		2.5	0.7566	-0.2229	2.02752
	2.00	0.5629	-4.0702	7.3893		4.0	0.7207	-0.4769	2.3702
Sr.	0.035	0.832078	0.246188	1.39159					
	0.5	0.83358	0.25163	1.40293					
	1	0.83997	0.25263	1.42171					
	2	0.87679	0.015332	1.78531					

The effects of various parameters on velocity profiles in the boundary layer are depicted in Table 2. The effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease. Increase in magnetic parameter led to decrease in Skin friction, Nusselt numbers and Sherwood numbers. Similar trend of slight decrease in the fluid velocity near the inclined plate is observed with an increase in suction parameter Q. Whereas there was decrease in nusselt number but increased Sherwood number. Numerical analysis showed the effect of thermal Grashof number Gr on the velocity field. The thermal Grashof number signifies the relative of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number, i.e. free convection effects.



It is seen that as the thermal Grashof number Gr influence the velocity field almost in the boundary layer when compared to far away from the plate. It is seen that as Grashof number Gr increased, the velocity field increased. The effect showed that increased Gr increased skin friction, Nusselt and Sherwood numbers respectively. The effect of mass (solutal) Grashof number Gc on the velocity, thermal and concentration thereby increased the skin friction, Nusselt number and Sherwood numbers. The mass (solutal) Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the solutal Grashof number.

The effect of the Schmidt number Sc on the velocity. The Sc embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass transfer (concentration) boundary layer. It is noticed that as Sc increased the velocity field decreased. Similar trend of slight decrease in the fluid velocity near the inclined plate is observed with an increase in Pr . The variation of the velocity boundary layer with the heat generation/absorption parameter Q . It was noticed that the velocity boundary layer thickness increased with an increase in heat generation/absorption parameter. There was increase/decrease in skin friction showing the effect of suction/injection. The variation of the velocity boundary layer with the Df , it was observed that the velocity boundary layer (skin friction) thickness increased with an increase in the Dufour number Df parameter. The variation of the velocity boundary layer with the Sr . It was found that the velocity boundary layer thinness increased with a decrease in the Sr . The variation of the velocity boundary layer with the U . It was found that the velocity boundary layer thickness decreased with an increase in the unsteadiness parameter. The effects of porosity on boundary layer flow or exponential velocity, an increase in Darcy porosity led to increase in exponential velocity.

The behaviour of magnetic parameters on exponential velocity showed an increase in magnetic parameters led to decrease in exponential velocity. This slow down the rate of fluid flow thereby thinning the velocity boundary layer. The influence of inclined parameter on the exponential velocity, it was discovered that increase in angle of inclination led to corresponding increase in boundary layer flow. Chemical reaction effects showed decrease in boundary layer flow. In effect, it decreased the boundary layer flow. The effect of the M on the temperature was illustrated. It was observed that as the M increased, the temperature decreased. The effect of the suction parameters fw on the temperature. It was noticed that as suction parameter increased, the temperature increased that was injection. The thermal boundary thickness with an increase in the thermal or Solutal Grashof number Gr or Gc . The effect of Sc on the temperature was noticed that as the Sc increased, an increasing trend in the temperature field is noticed. Much of significant contribution of Sc was noticed as we move far away from the plate.

The effect of Pr on the temperature. It was noticed that as the Pr increased the temperature increased. On the effects of heat generation/absorption parameter Q on the temperature. It was noticed that as the heat generation/absorption parameter increased the temperature increased. The variation of the thermal boundary layer with the Df . It was noticed that the thermal boundary layer thickness increased with an increase in the Df . The variation of the thermal boundary layer with the Sr . It was observed that the thermal boundary layer thickness decreased with an increase in the Sr . The effect of unsteadiness parameter U on the temperature, as the unsteadiness parameter increased the thermal boundary layer is found to be increasing. The effect of the Darcy porosity on the temperature. An increase in Darcy porosity led to decrease in the temperature profile. The influence of variation of angle of inclination on exponential temperature. An increase in angle of inclination led to decrease in the temperature profile. On the variation of chemical parameter on temperature profile, the variation indicated that increase in λ led to decrease in the temperature profile.

On chemical species concentrating against span wise coordinate η for varying values of physical parameters in the boundary layer. The species or concentration is highest at the plate surface and decrease to zero far away from the plates satisfying the boundary condition. The effect of M on the concentration field as the magnetic parameter increases the concentration is found to be increasing. The effect of suction parameter fw on the concentration field was illustrated, as the suction parameter increased the concentration was found to be increasing. However, as we move away from the boundary layer, the effect is not significant. The effect of buoyancy parameters, (Gr , Gc) on the concentration field illustrated showed that the concentration boundary layer thickness increased with an increase in the thermal or solutal Grashof numbers Gr and Gc . The effect of Schmidt number increased, as there was an increase in the concentration field.



The influence of the heat generation/absorption parameter Q on the concentration field was analysed. It was noticed that the concentration increased/decreased monotonically with the increase of the heat generation absorption parameter. On the variation of the concentration boundary-layer with Df parameter. It was observed that the concentration boundary layer thickness increased with an increase in the Dufour number. The variation of the concentration boundary-layer with the Sr , it was found that the concentration boundary layer thickness with an increase in the Soret number. On the unsteady parameter on concentration profile, an increase in U -parameter decreased the concentration profile. On the variation of Darcy porosity parameter on the concentration profile. An increase in Darcy porosity parameter led to increase in the concentration profile. On the influence of Prandtl number on the concentration profile, an increase in Prandtl parameter led to increase in the concentration profile. On the effects of varied angle of inclination on the concentration profile. An increase in this parameter led to increase in the concentration profile. The effects of varied λ or chemical reaction of inclination on the concentration profile. There was a correspondent increase in chemical parameter and concentration profile.

4. CONCLUSION

The present paper analysed the solution-adaptive cross-diffusion effect of the convective heat and mass transfer on the unsteady boundary layer flow over a stretching plate by taking heat generation/absorption, chemical Soret and Dufour effects into account. The governing equations are approximated to a system of non-linear ordinary differential equations using similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It was observed that the local heat and mass transfer rates at the plate increased with an increase in the buoyancy forces. It was noticed that the local skin-friction coefficient, local heat and mass transfer rates at the plane decreased with an increase in the magnetic parameter. As the Prandtl number increased, skin-friction decreased but Nusselt and Sherwood numbers increased. It was found that the local heat and mass transfer rate at the plate increases, but Skin-friction coefficient decreased with an increase in the unsteadiness parameter. As the Dufour number or heat generation/absorption parameter increased, whereas the Nusselt number decreased. The effect of the Soret number showed increase the Skin-friction, the Nusselt number and Sherwood number.

Solution-adaptive cross-diffusion effects could be explored for energy efficiency and environmental advantages. In the tropics, the practical use of solar energy could be made to work for the nation and save energy and money. Switching and investing in solar energy will help beautify our environment e.g. street, market and most of highways. Solar energy could be conserved to light our homes to safeguard us from heat, preserve our food. During harmattan we can use heat our homes to balance the room conditions. Solar electric energy power generation could be invested in by converting sun radiation directly to electricity. Other sectors could find this topic useful in any porous media



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