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Free Vibration Analysis of Uniform Shear Beam On Winkler Foundation

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ABSTRACT

In this study, differential transform method (DTM) is employed to investigate free vibration of uniform shear beams with constant shear distortion and constant stiffness resting on Winkler foundation DTM is an efficient technique for the solution of problems defined by linear or non-linear differential equations. This research shows that DTM is an effective method for free vibration analysis of uniform shear beam with constant shear distortion and constant stiffness resting on Winkler foundation. The model equation is obtained and solved numerically using DTM, tabular and graphical results are presented for the natural frequencies and mode shapes.

Keywords: Winkler Foundation, Shear Beam, Differential Transform Method.

1. INTRODUCTION

Beams on elastic foundation model is widely used in the formulation of practical applications in Geo-technical engineering. For practical applications, the lateral stability and dynamic behaviour of Shear beam-columns, Shear beams, and Shear building are of great importance in structural and earthquake engineering. The vibration and seismic responses of Shear beam and framed structures modelled as a shear building have been studied by many researchers and treated extensively in the technical literature by Thomson (1972), Belvins (1986), Berg (1989), Paz (1990), Clough(1996) et al., among many others using different methods, mainly matrix analysis and lumped masses. Free vibration of shear beams with finite rotatory inertia was studied by X-F(2011) et al., non-classical modes of vibration of shear beams was carried out by J.Dairo Aristizabal-Ochoa (2004) and Exact solutions of free vibration shear type structures was studied by Q.S Li (2001).

An exact formulation of the beam problem was first investigated in terms of general elasticity equations by Pochammer (1876) and Chree(1889). They derived the equations that describe a vibrating solid cylinder. However, it is not practical to solve the full problem because it yields more information than usually needed in applications. Therefore, approximate solutions for transverse displacement as a solution. The Euler-Bernoulli beam theory, sometimes called the classical beam theory, Euler beam theory, Bernoulli beam theory or Bernoulli-Euler beam theory, is the most commonly used because it is simple and provides reasonable engineering approximations for many problems. However, the EulerBernoulli model tends to slightly overestimate the natural frequencies. This problem is exacerbated for the natural frequencies of the higher modes. Also, the prediction is better for slender beams than non-slender beams.

The shear beam models adds shear distortion to the Euler-Bernoulli model. It should be noted that this is different from the pure shear model which includes the shear distortion and rotary inertia only or the simple shear beam which includes the shear distortion and lateral displacement only. Neither the pure shear nor the simple shear model fits our purpose of obtaining an improved model to the Euler-Bernoulli model because it exclude the most important factor, the bending effect. By adding shear distortion to the Euler-Bernoulli beam, the estimate of the natural frequencies improves considerably. There are several foundation models such as Winkler, Pasternak, Vlasor etc. that have been used in the analysis of beam vibration. The most used foundation problems is the Winkler foundation model in which the soil is modelled as uniformly distributed linear elastic vertical springs which produce distributed reaction in the direction of the deflected beam. There are also different beam models theory, the mostly used model is EulerBernoulli beam which is suitable for slender beams.

If the beam is short and thick then, Timoshenko beam has to be performed in these analysis. Vibration and dynamic buckling of shear beam on elastic foundation under moving load was studied by Seong-Minkim (2005) et al. using Fourier transformation method. Static and dynamic stability of uniform Shear building under generalized boundary condition was analyzed by J.Dario Aristizabal-Ochon (2004). Avramids(2006) et al. analysed beam bending problem on three-parameter elastic foundation. De Rosa(1995) analysed free vibration of Timoshenko beams on two-parameter elastic foundation. El-Moushy (1999) investigated fundamental frequencies of Timoshenko beams on Pasternak foundation.Also N.R Naidu(1995) et al. analysed the vibration of initially stressed uniform beams on a two-parameter elastic foundation. Seon M. Hun (1999) et al. analysed the dynamics of transversely vibrating beams using four engineering theories. Mutman U(2013) analysed free vibration of an Euler beam of variable width foundation using homotopy perturbation method.

The Winkler-model (one-parameter model) which has been originally developed for the analysis of rail road tracks, has credit for its mathematical simplicity. However, one of the most important deficiencies of the Winkler foundation model is that a displacement continuity appears between the loaded and the unloaded part of the foundation surface. The mechanical modelling of the foundation using the Pasternaks foundation converges to the wrinkle foundation if the second parameter in Pasternaks foundation is neglected. In this study, free vibration of Shear beam resting on a one-parameter elastic foundation (Winkler) is considered. Differential transform method is applied to determine the natural frequencies and the mode shapes have also been investigated.

Although, Seon M.Han(1999) et al. gave full development/analysis for the transverse vibrating uniform beam using Shear beam which was not placed on an elastic foundation as an example, the present study deals with a one-parameter model foundation whose shear distortion term and the stiffness of the foundation element were varied.

2. SHEAR BEAM MODEL ON A ONE-PARAMETER ELASTIC FOUNDATION

If a shear beam resting on a one parameter elastic (Winkler) foundation, the governing differential equation for the system without damping effect and if the effect of rotatory inertial is neglected and only the effect of shear distortion on the dynamic deflection of beam is considered and its subjected to a static axial force can be written in a Cartesian coordinate system {*x,y*} as

$$m\frac{\partial y^2(x,t)}{\partial t^2} - S\Big(\frac{\partial y^2(x,t)}{\partial x^2} - \frac{\partial \phi(x,t)}{\partial x}\Big) - P\frac{\partial^2 y(x,t)}{\partial x^2} + Ky(x,t) = 0$$

$$EI\frac{\partial^2 \phi(x,t0}{\partial x^2} + S\Big(\frac{\partial y(x,t)}{\partial x} - \phi(x,t)\Big) = 0$$

$$0 < x < L \tag{1}$$

where $m = \rho A$ is the mass of the beam per unit length, k is the stiffness of the foundation per unit length, P is the axial force (positive and negative signs represented tension and compression), E is Youngs Modulus of elasticity, I is the second moment of inertial, $y(x,t)$ is the vertical displacement of the beam, $\varphi(x,t)$ is the rotation of the beam, and $S = k^{\prime}GA$ is the shear distortion of the beam where $K^{\prime}$ is the effective shear area and G is the shear modulus of the beam. Due to the end conditioned of the beam, different boundary conditions have to be imposed to obtain the desired solution.

Some of these conditions are as follows:

(a) For Hinged-Hinged(Simply-Supported) beam the end conditions are;
(b)

$$\frac{\partial \phi}{\partial x} = 0, y = 0 \quad at \quad x = 0, L \tag{2}$$

(b) For Clamped-Clamped beam the end conditions are;

$$\varphi = 0, y = 0 \qquad at\ x = 0, L \tag{3}$$

(c) For Clamped-Free(Cantilever) beam the end conditions are;

$$\phi = 0, y = 0 \qquad at \quad x = 0 \tag{4}$$

$$\frac{\partial \phi}{\partial x} = 0, S\Big(\frac{\partial y}{\partial x} - \phi\Big) = 0 \qquad at \quad x = L \tag{5}$$

3. FREE VIBRATION ANALYSIS

Now free vibration analysis of the uniform shear beam on a one parameter elastic (Winkler) foundation is discussed as follows: The solution is separated due to its variables as given in the following form to formulate the analysis of the presented problem

$$Y(x,t) = H(x)e^{i\omega t}, \qquad \phi(x,t) = G(x)e^{i\omega t} \tag{6}$$

Where ω is the circular frequency for the vibration Substituting equation (16) into the governing equation, the equation of motion becomes as follows: For the first equation in equation in the governing equation we have,

$$mi^2\omega^2 H(x)e^{i\omega t} - S\left[\frac{d^2}{dx^2}H(x)e^{i\omega t} - \frac{d}{dx}G(x)e^{i?t}\right] - p\frac{d^2}{dx^2}H(x)e^{i\omega t} + kH(x)e^{i\omega t} = 0 \tag{7}$$

Which can be further simplified as;

$$(k - m\omega^2)H(x) - S\left[\frac{d^2}{dx^2}H(x) - \frac{d}{dx}G(x)\right] - p\frac{d^2}{dx^2}H(x) = 0 \tag{8}$$

Also for the second equation in equation in the governing equation we have,

$$EI\frac{d^2}{dx^2}G(x)e^{i\omega t} + S\left[\frac{d}{dx}H(x)e^{i\omega t} - G(x)e^{i\omega t}\right] = 0 \tag{9}$$

Equation (9) can be further simplified as;

$$EI\frac{d^2}{dx^2}G(x) + S\left[\frac{d}{dx}H(x) - G(x)\right] = 0 \tag{10}$$

Without loss of generality the following dimensionless quantities can be introduced;

$$X = \frac{x}{L}, \quad G(X) = \frac{G(x)}{L}, \quad H(X) = \frac{H(x)}{L} \tag{11}$$

Inserting equation (11) into equation (8) we have,

$$S\frac{d^2}{d(LX)^2} + P\frac{d^2LH(X)}{d(LX)^2} - S\frac{dLG(X)}{d(LX)} - (k - m\omega^2)LH(X) = 0 \tag{12}$$

We further simplify equation (12) to become,

$$\frac{d^2}{dX^2}H(X) - \lambda\frac{d}{dX}G(X) - (\mu - \gamma\omega^2)H(X) = 0 \tag{13}$$

Where

$$\lambda = \frac{SL}{(S+P)}, \quad \mu = \frac{L^2k}{(S+P)}, \gamma = \frac{L^2m}{(S+P)} \tag{14}$$

Inserting equation (11) into equation (10) we have,

$$EI\frac{d^2}{d(LX)^2}LG(X) + S\left[\frac{d}{d(LX)}LH(X) - LG(X)\right] = 0 \tag{15}$$

We further simplify equation (15) to become,

$$\frac{d^2}{d(X)^2}G(X) + \alpha\frac{d}{dX}H(X) - \beta G(X) = 0 \tag{16}$$

Where

$$\alpha = \frac{SL}{EI}\beta = \frac{SL^2}{EI}$$

The boundary conditions in view of equation (6) now becomes; (i) For Hinged-Hinged (Simply-Supported) end;

$$\frac{d}{dX}G(X) = 0, \quad H(X) = 0 \quad at \quad X = 0, L \tag{17}$$

(ii) For Clamped-Clamped end;

$$G(X) = 0, \quad H(X) = 0 \quad \text{at} \quad X = 0, L \tag{18}$$

(iii) For Clamped-Free(Cantilever) end;

$$G(X) = 0, \quad H(X) = 0 \quad \text{at} \quad X = 0 \tag{19}$$

$$S\left[\frac{d}{dX}H(X) - G(X)\right] = 0, \quad \frac{d}{dX}G(X) = 0 \quad at \quad X = L \tag{20}$$

Table 1 Properties of a one- parameter shear beam-column on an Elastic (Winkler) foundation

Young modulus of area moment of inertia EI	363.35kN m^1
Cross-Sectional area A	0.0097389 m^2
Stiffness of the foundation k	Varying values-0,1.0MPa,10MPa and 50MPa
Shear distortion S	Varying values-10MN,15MN,20MN and 40MN
Length L	1m
Mass m	297.5kg/m
Axial force P	0
Density ρ	7830kg/m^3

4. NUMERICAL TECHNIQUES

In this research we used the differential transform method (DTM) which is a numerical method based on Taylors expansion. This method constructs an analytical solution in form of a polynomial. Unlike the traditional high order Taylor's series method which requires a lot of symbolic computations, the differential transform method is an analytical solution in the form of a polynomial. But it is different from Taylor series method that requires computation of the high order derivatives. The differential transform method is an iterative procedure that is described by the transformed equations of original functions for solution of differential equations. Also, the boundary conditions of the system are transformed into a set of algebraic equations in term of the differential transform of the original functions and the solution of this algebraic equations give the desired solution of the problem. Consider the functions $w(x)$ which is analytic in a domain D and $x = x_0$ represent any point in D. the function is represented by a power series whose centre is located at x_0. The differential transform of the function w(x) is given as;

$$W(k) = \frac{1}{k!}\left[\frac{d^k w(x)}{dx^k}\right](x = x_0) \qquad (21)$$

Where $w(x)$ is the original function and $W(k)$ is the transformed function. The inverse transformation is defined as:

$$W(x) = \sum_{k=0}^{\infty}(x - x_0)^k W(k) \qquad (22)$$

Combining equations (21) and (22) give

$$W(x) = \sum_{k=0}^{\infty}\frac{(x - x_0)}{k!}\left[\frac{d^k w(x)}{dx^k}\right]_{x=x_0} \qquad (23)$$

The fundamental mathematical operation performed by differential transform method and boundary conditions are given in the tables 2 and 3 below which was used by Alev Kacar[2011] et al. in the free vibration analysis of beams on Winkler foundation by using differential transform method.

Table 2: The fundamental operations of DTM.

Original function	Transformed function
$w(x) = g(x)$ 6= $h(x)$	$W(k) = G(k) \pm H(k)$
$W(x) = \lambda g(x)$	$W(k) = \lambda G(k)$
$w(x) = \frac{\partial g(x)}{\partial x}$	$W(k) = (k+1)G(k+1)$
$w(x) = \frac{\partial^m g(x)}{\partial x^m} g(x)$	$W(k) = (k+1)(k+2)\cdots(k+m)G(k+m)$
$w(x) = 1$	$W(k) = \partial(k)$
$w(x) = g(x)h(x)$	$W(k) = \sum_{m=0}^{k} H(m)G(k-m)$

Table 3: Theorems for differential transform method for boundary conditions.

$x = 0$		$x = 1$	
Original B.C	Transformed B.C	Original B.C	Transformed B.C
$W(0) = 0$	$W(0) = 0$	$W(1) = 0$	$\sum_{k=0}^{\infty} W(k) = 0$
$\frac{dW(0)}{dx} = 0$	$W(1) = 0$	$\frac{dW(1)}{dx} = 0$	$\sum_{k=0}^{\infty} KW(k) = 0$
$\frac{d^2(0)}{dx} = 0$	$W(2) = 0$	$\frac{d^2W(1)}{dx^2}$	$\sum_{k=0}^{\infty} k(k-1)W(k) = 0$
$\frac{d^3(0)}{dx} = 0$	$W(3) = 0$	$\frac{d^3W(1)}{dx^3} = 0$	$\sum_{k=0}^{\infty}(k-1)(k-2)W(k) = 0$

5. DTM FORMATION

Taking the differential transforms of equation (13), we have

$$(k+1)(k+2)\bar{H}(k+2) - \lambda[(k+1)\bar{G}(k+1)] - \mu\bar{H}(k) = 0 \qquad (24)$$

We got the below recurrence relation

$$\bar{H}(k+2) = \frac{\lambda[(k+1)\bar{G}(k+1)] + (\mu - \gamma\omega^2)H\bar{H}(k)}{(k+1)(k+2)} \qquad (25)$$

Also taking the differential transform of equation (15), we have

$$(k+1)(k+2)\bar{G}(k+2) + \alpha[(k+1)\bar{H}(k+1)] - \bar{G}(k) = 0 \qquad (26)$$

We got the below recurrence relation

$$\bar{G}(k+2) = \frac{\beta\bar{G}(k) - \alpha[(K+1)\bar{H}(k+1)]}{(k+1)(k+2)} \qquad (27)$$

Several iterations are carried out during the analysis procedure and three boundary conditions for each case are rewritten by using the solution for displacement of the beam. Each boundary condition produces an equation containing two unknowns due to the initial approximation. These boundary conditions in non-dimensionless form are;

(i) Hinged-Hinged (Simply-Supported) end

$$\bar{G}(1) = 0, \quad \bar{H}(0) = 0 \quad \text{at} \quad \bar{X} = 0 \qquad (28)$$

$$\sum_{k=0}^{\infty} k\bar{G}(k) = 0, \quad \sum_{k=0}^{\infty} \bar{H}(k) = 0 \quad \text{at} \quad \bar{X} = 1 \qquad (29)$$

(ii) Clamped-Clamped end

$$\bar{G}(0) = 0, \quad \bar{H}(0) = 0 \quad \text{at} \quad \bar{X} = 0 \qquad (30)$$

$$\sum_{k=0}^{\infty} \bar{G}(k) = 0, \quad \sum_{k=0}^{\infty} \bar{H}(k) = 0 \quad \text{at} \quad \bar{X} = 1 \qquad (31)$$

(iii) Clamped-Free(Cantilever) end

$$\bar{G}(0) = 0, \qquad \bar{H}(0) = 0 \qquad \text{at} \quad \bar{X} = 0 \qquad (32)$$

$$\sum_{k=0}^{\infty} k\bar{G}(k) = 0, \quad \sum_{k=0}^{\infty} k\bar{H}(k) - \sum_{k=0}^{\infty} \bar{G}(k) = 0 \quad \text{at} \quad \bar{X} = 1 \qquad (33)$$

The corresponding differential transforms for the boundary conditions (28) to (33) are given in table (3). Hence, two equations in two unknowns may be written with respect to the boundary conditions of the problem. These equations can be represented in matrix form as follows;

$$[M\bar{\omega}]A = \{0\} \qquad (34)$$

Where $A^T = A, B$

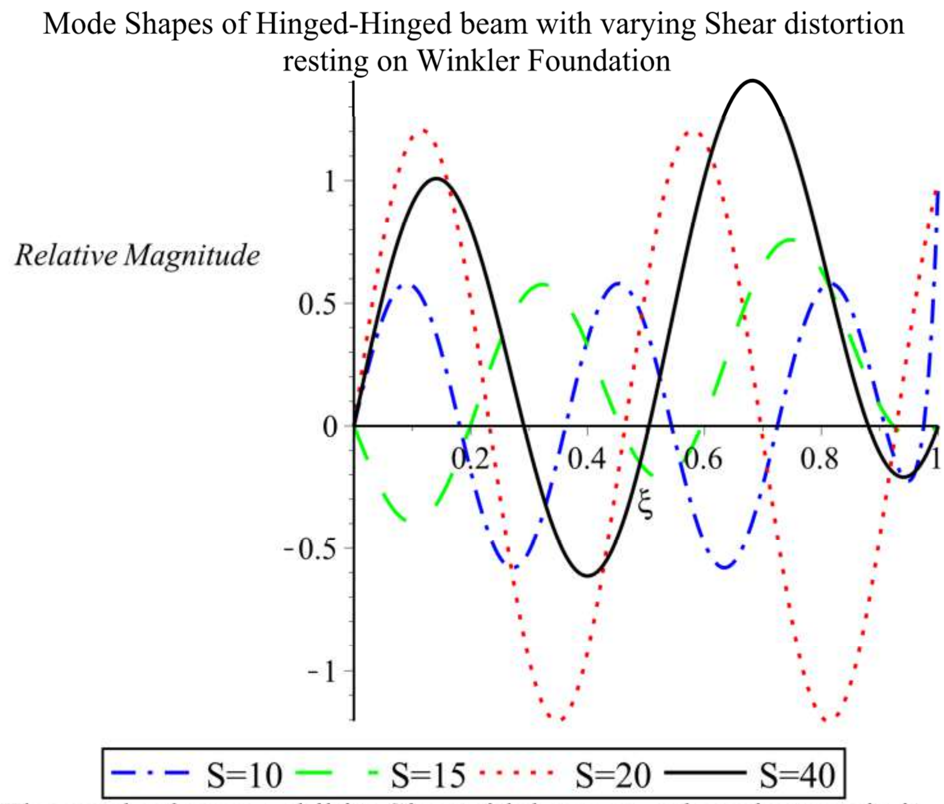
For a non-trivial solution, determinant of coefficient matrix must be zero. Determinant of the coefficient matrix yields a characteristic equation in terms of ω. Positive real roots of this equation are the normalised free vibration frequencies for the case considered.

6. NUMERICAL RESULTS FOR CASE A

A number of case studies are carried out with respect to parameter S that lead to a variation of Shear distortion of the beam with the aid of mathematical computational software (MAPLE 18). The results are in tables 4-6 below, and mode shapes of the cases are also represented graphically.

TABLE 4: Free Vibration Frequencies for Hinged-Hinged (Simply Supported) End resting on Winkler Foundation with varying Shear distortion at constant stiffness of the foundation (K=77.17MPa)

S	10	15	20	40
ω_1	17.55915917	14.97316049	13.67336632	11.68998207
ω_2	47.16791677	44.58184603	43.28195044	41.29788666
ω_3	96.51592493	93.92980413	92.62995641	90.64743090



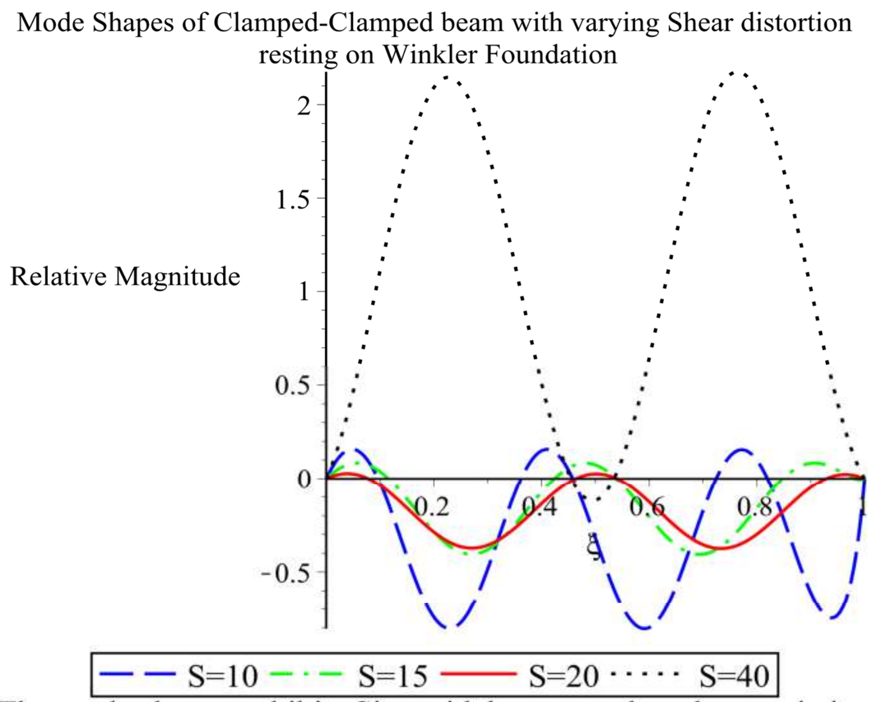
The mode shapes exhibits Sinusoidal curve and are harmonic,it also completes four period at S=15 and S=20 but completes three and five period at S=40 and S=10 respectively. It is also a symmetric graph at S=15.

Figure 1

The first three natural frequencies for hinged-hinged (simply-supported) end beam on Winkler foundation are presented in table 4, it is observed that the value for each frequency decreases as S increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the Hinged-Hinged (simply-supported) end beam oscillates at those frequencies.

TABLE 5: Free Vibration Frequencies for Clamped-Clamped End resting on Winkler Foundation with varying Shear distortion at constant stiffness of the foundation(K=77.17MPa)

S	10	15	20	40
ω_1	17.58139355	15.00645693	13.71768845	11.77804327
ω_2	47.16795507	44.58193016	43.28210275	41.29849203
ω_3	96.51837568	93.93327886	92.63491449	90.65574068



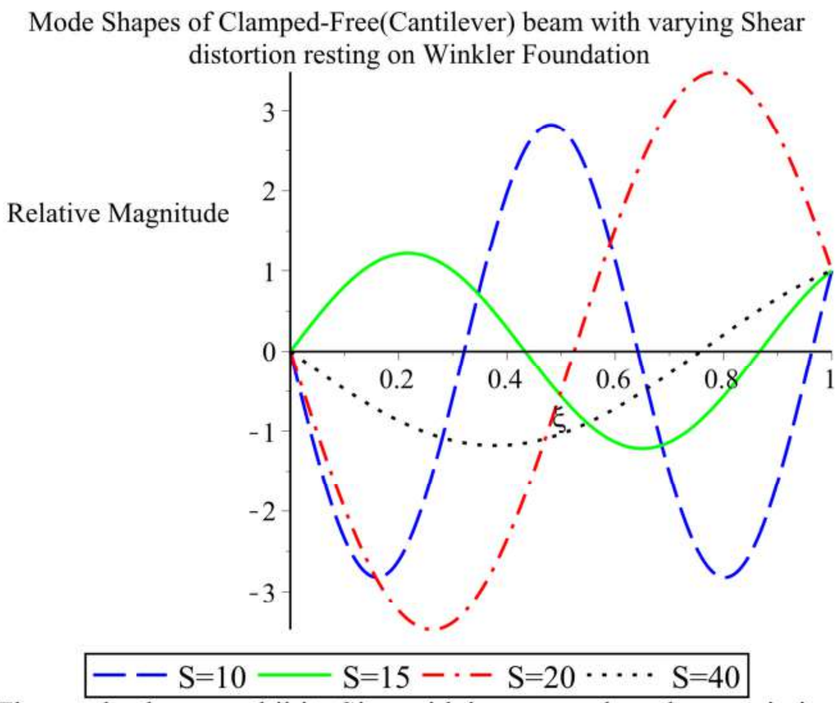
The mode shapes exhibits Sinusoidal curve and are harmonic,it also completes three period each at S=15,S=40 and S=20 but completes Five period at S=10. It is also a symmetric graph except at S=10.

Figure 2

The first three natural frequencies for clamped-clamped end beam on Winkler foundation are presented in table 5, it is observed that the value for each frequency decreases as S increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the clamped-clamped end beam oscillates.

TABLE 6: Free Vibration Frequencies for Clamped-Free (Cantilever) End resting on Winkler Foundation with varying Shear distortion at constant stiffness of the Foundation (K=77.17MPa)

S	10	15	20	40
ω_1	10.17197641	7.593473258	6.301164853	4.347625521
ω_2	29.81859187	27.19442080	25.85685651	23.72630622
ω_3	69.33383389	66.72770030	65.40784593	63.35220948



The mode shapes exhibits Sinusoidal curve and are harmonic,it also completes two period each at S=15 and S=20 but completes one and three period at S=40 and S=10 respectively. It is also a symmetric graph at S=10.

Figure 3

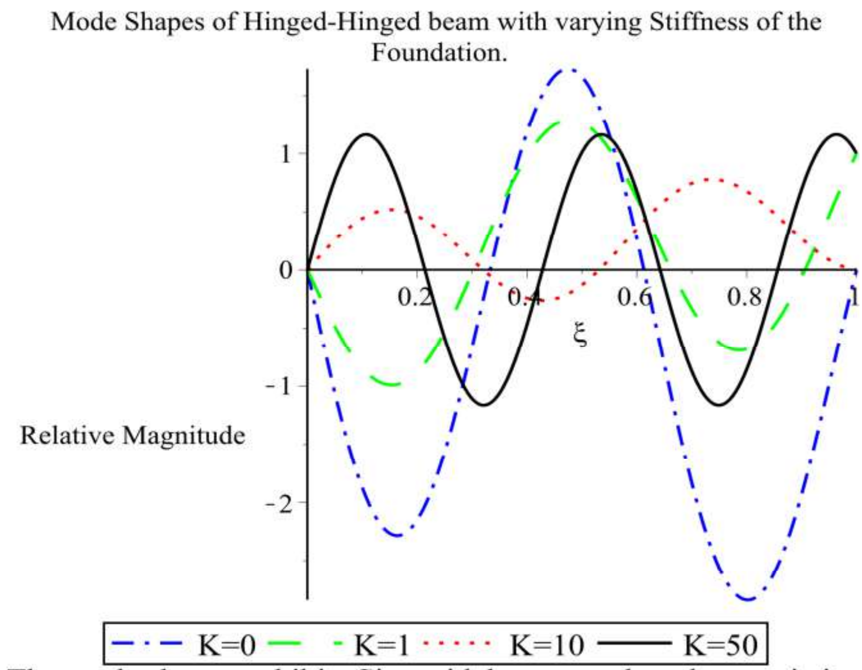
The first three natural frequencies for Clamped-Free (Cantilever) end beam on Winkler foundation are presented in table 6, it is observed that the value for each frequency decreases as S increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the Clamped-Free (Cantilever) end beam oscillates.

7. Numerical Results for Case B

A number of case studies are carried out with respect to parameter K that lead to a variation of the stiffness of the foundation of the beam with the aid of mathematical computational software (MAPLE 18). The results are in tables 7-9 below, and mode shapes of the cases are also represented graphically.

TABLE 7: Free Vibration Frequencies for Hinged Hinged(Simply-Supported) End resting on Winkler Foundation with varying stiffness of the foundation at constant shear distortion(S=10MN)

K	0	1.0	10	50
ω_1	9.842159257	9.942159244	10.84215926	14.84215924
ω_2	39.45091560	39.55091546	40.40091532	44.45091620
ω_3	88.79890591	88.89891660	89.79890105	93.79890439



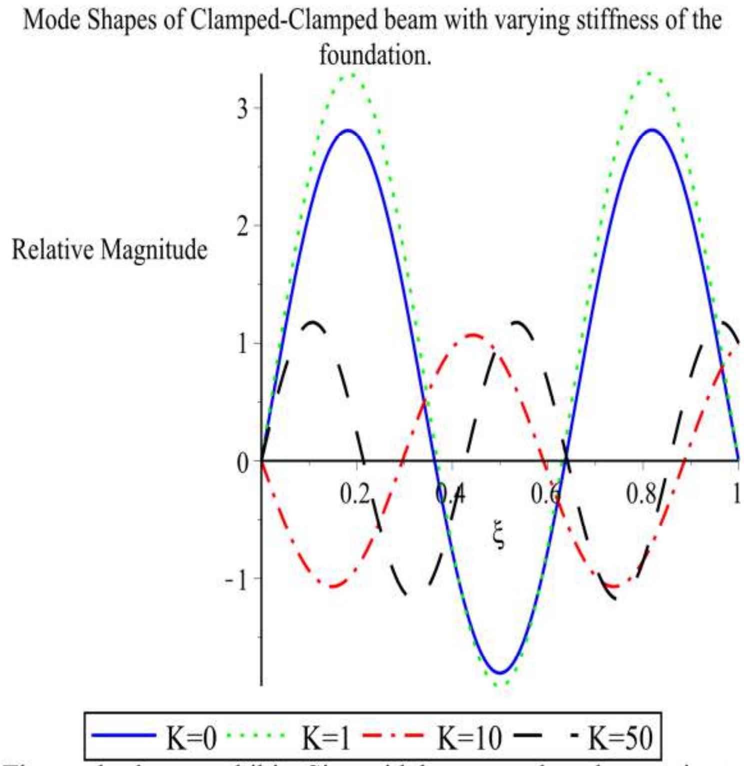
The mode shapes exhibits Sinusoidal curve and are harmonic,it also completes four period at K=50 but completes three period at K =0 , K=1 and K=10. It is also a symmetric graph at K=1 and K=0..

Figure 4

The first three natural frequencies for hinged-hinged (simply-supported) end beam on Winkler foundation are presented in table 7, it is observed that the value for each frequency increases as K increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the Hinged-Hinged (simply-supported) end beam oscillates at those frequencies.

TABLE 8: Free Vibration Frequencies for Clamped-Clamped End resting on Winkler Foundation with varying stiffness of the foundation at constant shear distortion (S=10MN)

K	0	1.0	10	50
ω_1	9.864393634	9.964393613	10.86439364	14.86439356
ω_2	39.45095313	39.55095358	40.45095372	44.45095582
ω_3	88.80141095	88.90140449	89.80138435	93.80137713



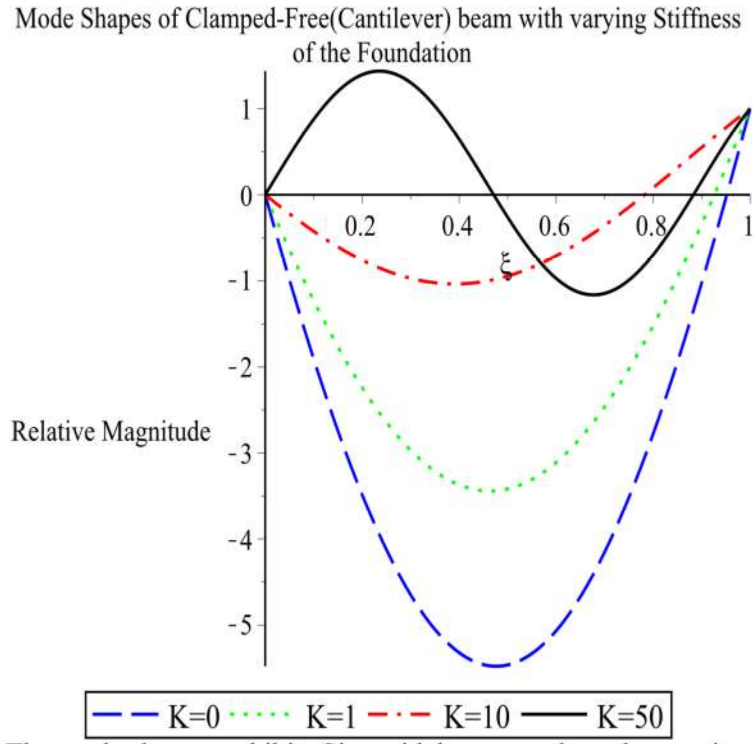
The mode shapes exhibits Sinusoidal curve and are harmonic at all values of K.,it also completes three period each at K=0,K= 1 and K=10, it completes four periods at K=50. It is also a symmetric graph at K=0 and K=1.

Figure 5

The first three natural frequencies for clamped-clamped end beam on Winkler foundation are presented in table 8, it is observed that the value for each frequency increases as K increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the clamped-clamped end beam oscillates.

TABLE 9: Free Vibration Frequencies for Clamped-Free (Cantilever) End resting on Winkler Foundation with varying stiffness of the foundation at constant shear Distortion (S=10MN)

K	0	1.0	10	50
ω_1	2.454976439	2.554976441	3.454976440	7.454976452
ω_2	22.10159128	22.20159120	23.10159127	27.10159100
ω_3	61.61683730	61.71684223	62.61684148	66.62353418



The mode shapes exhibits Sinusoidal curve and are harmonic. The graph is also symetric at K=0 and K=1. The graph completes one period at K=0,K=1 and K=10 and completes two periods at K=50.

Figure 6

The first three natural frequencies for Clamped-Free (Cantilever) end beam on Winkler foundation are presented in table 9, it is observed that the value for each frequency increases as **K** increases and as end conditions of the beam differs. The natural modes at the first frequency ω_1, $\omega 2$, $\omega 3$, describes the situation in which the Clamped-Free (Cantilever) end oscillates.

8. SUMMARY AND CONCLUSION

In the study, DTM was used for the Free Vibration analysis of Shear beam with constant Shear rotation and constant stiffness on Winkler Foundation. To represent a Variation in the Shear distortion a horizontal Shear beam with varying shear distortion and varying stiffness of the foundation is considered. The analysis were expanded for various cases. DTM also produced reasonable results for the vibration of constant shear distortion and constant stiffness of the foundation shear beams on Winkler Foundation showing the efficacy of the method. In the case of vibration in constant shear distortion and constant stiffness of the foundation, the governing equation becomes an equation with constant coefficients and it is not easy to obtain analytical solutions for these type of problems. However, DTM produced very good approximations after performing some iterations with the method. Also normalized mode shape of Hinged-Hinged (Simply-Supported) End, Clamped-Clamped End and Clamped-Free (Cantilever) End with constant shear distortion and constant stiffness of the foundation are investigated.

We also discovered that as we increase the values of shear distortion parameter, the value of the natural frequency decreases at each boundary conditions we considered. Also as we increase the stiffness of the foundation the natural frequency increases at each boundary conditions we considered.

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