

## Optimization of an Oil Immersed Power Transformer Loading Using Thermal Model with Environmental Variables

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### ABSTRACT

In this work, the loading capacity of an oil immersed power transformer was optimized using environmental variables-based thermal model. An existing semi-physical model of a transformer top-oil temperature model was modified to take into account the effect of solar radiation. Using data gotten from 132/33 kV power transformer situated at Uyo, Nigeria, multivariate linear regression algorithm was used to obtain the top-oil model coefficients. The Newton Raphson algorithm was applied for the optimization of the transformer loading capacity. The algorithms were implemented in MATLAB software. From the results obtained from simulation, the top-oil temperature was tested with different set of data from the one used for the training. The computed  $R^2$  values for top-oil temperature multivariate linear regression model with training and testing datasets are 0.9398 and 0.9219 respectively. Also, based on results for the optimal transformer loading, it was seen that the transformer loading was at its maximum values in the early hours of the day when there was solar radiation, while the top-oil temperature recorded its lowest values. Whereas at the mid-day, when the solar radiation recorded its maximum value, the optimal transformer loading capacity was at its minimum value and the top-oil temperature recorded its highest value. This shows that solar radiation contributes enormous heat to the transformer oil thus leading to the rise in oil temperature, which in turn can lead to reduction in the transformer life span. To checkmate the reduction in the lifespan of the transformer, the transformer maximum loading capacity will have to be reduced.

**Keywords:** Power Transformers, Hot-Spot Temperature, Top Oil Temperature, Solar Radiation, Loss Of Life, Thermal Model, Optimal Loading, Multivariate Linear Regression.

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### 1. INTRODUCTION

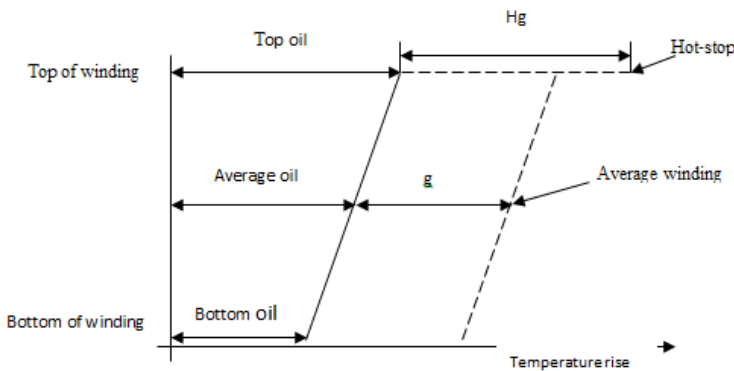
The diversity in location of electrical loads and the inception of commercial electric power has created a vast network of transmission systems so that power generated at one station may be fed to grid system and may be distributed over large areas and states. The main issue encountered during the transmission of electrical power over very long distances is the efficiency of the power system. This efficiency is greatly compromised by the losses experienced along the line (Østergaard *et al.*, 2001) which are reduced using transformers to boost voltage levels and correspondingly reduce current levels at the generating stations before transmission and distribution at suitable levels for use by consumers since the major losses on a transmission line are current related. These transformers make it possible to operate different parts of the system at different voltages and thereby retaining the respective advantages of higher and lower voltages because in the design of power systems, greater energy efficiency of high voltage and low current must be weighed against safety and capital cost (Kaintzyk *et al.*, 2003).

The reliability of the power system can be adversely affected by the failure and mal-operation of transformers as a result of the failure of insulation caused by high stress under abnormal operating and environmental conditions or in cases when heat generated in a power transformer is not dissipated efficiently by the surrounding medium. Non-uniformity in heat dissipation from sources such as core losses, winding losses, stray losses in the tank and metal support structures cause the transformer oil temperature and winding temperature to rise developing hot spots in the power transformers leading to thermal stress (Muhamad *et al.*, 2015). This heat is transferred to the surrounding oil as  $I^2R$  losses under steady state conditions which continue until the heat generated by the windings equals the heat taken away by the oil under continuous load. This rise of transformer temperature is not just a function of the internal heat dissipation but environmental factors such as ambient temperature and solar radiation which contribute to making the temperature rise and the lifespan of the transformer location and climate dependent. The part of the transformer operating at the highest temperature (the hot spot temperature) experiences the greatest deterioration and plays a very important role in estimating transformer insulation life.

As reported in the IEEE and IEC standards (IEEE Standard C57.19, 1995, IEC Standard, 1991 and Swift *et al.*, 2001), the calculation of the internal transformer temperature is complicated and at best an average value obtained using simplifying assumptions. Also, the existing transformer thermal models used in predicting the top-oil temperature of transformer do not take into consideration the effects of solar radiation on the transformer temperature. This work will develop a robust thermal model for an oil immersed transformer taking into solar radiation. A model for optimizing the loading capacity of the transformer will also be developed and simulations will be done with MATLAB.

## 2. TRANSFORMER THERMAL MODELLING

Power losses are converted into heat in a transformer. These losses are composed of no-load losses and load losses. The no load losses are comprised of eddy-current and hysteresis losses of the core. The load losses are comprised of resistive losses (windings losses, joint point losses and connector losses), winding eddy losses and the stray losses. In thermal modelling of a transformer, two types of temperatures are usually modelled – the top-oil temperature (TOT) and the hot-spot temperature (HST). The temperature distribution inside a transformer is extremely complex and difficult to model accurately, consequently the thermal characteristics of transformers are normally analysed in the simple thermal diagram. A typical thermal diagram using the IEC 354 and IEEE C57.91-1995 standards is shown in figure 1 below



**Figure 1: Transformer thermal diagrams (Swift *et al.*, 2001)**

### 2.1 Top-oil Temperature Models

There are several fundamental models for predicting a transformer’s TOT. In this section the various models which are relevant to our research are discussed.

#### *Model #1 Top-oil-rise Model ANSC C57.92, C57.115*

The classical model for predicting a transformer’s TOT is described in IEEE/ANSI standard C57.92 (updated as C57.115) and is called IEEE/ANSI C57.115 model. The IEEE/ANSI C57.115 model for TOT rise over ambient temperature is governed by the first order differential equation (IEEE Standard C57.19, 1995 and IEC Standard, 1991):

$$T_o \frac{d\theta_o}{dt} = -\theta_o + \theta_{w1} \dots \dots \dots (1)$$

The solution of above differential equation is

$$\theta_o = (\theta_{w1} - \theta_i) \left( 1 - e^{-t/T_o} \right) + \theta_i \dots \dots \dots (2)$$

Where,

$$\theta_{w1} = \theta_{f1} \left( \frac{k^2 R + 1}{R + 1} \right)^n \dots \dots \dots (3)$$

$$T_o = \frac{C \theta_{f1}}{P_{f1}} \dots \dots \dots (4)$$

- $\theta_o$ : Top-oil rise over ambient temperature (°C);
- $\theta_u$ : Ultimate top-oil rise for load L (°C);
- $\theta_i$ : Initial top oil rise for  $t = 0$  (°C)
- $\theta_{fi}$ : Top-oil rise temperature over ambient temperature at rated load (°C);
- $T_o$ : Time constant (h);
- $C$ : Thermal capacity (MW h/°C);
- $P_{fi}$ : Total loss at rated load (MW);
- $n$ : Oil exponent;
- $K$ : Ratio of load to rated load or per unit load current;
- $R$ : Ratio of load loss to no-load loss at rated load.

Applying the forward Euler discretization rule on equation (2),

$$\frac{d\theta_o[k]}{dt} = \frac{\theta_o[k] - \theta_o[k-1]}{\Delta t} \dots \dots \dots (5)$$

Where  $\Delta t$  is the sampling period, and solving we get,

$$\theta_o[k] = \frac{T_o}{T_o + \Delta t} \theta_o[k-1] + \frac{\Delta t \theta_{fi}}{T_o + \Delta t} \left( \frac{(I[k])^2 R + 1}{R + 1} \right)^n \dots \dots \dots (6)$$

where  $I[k]$  is the per-unit transformer current (based on the rated value of the transformer) at time-step index  $k$ .

When the load current is near its rating, or  $R \gg 1$  and  $K^2 R \gg 1$ , TOT rise over ambient temperature is given by,

$$\begin{aligned} \theta_o[k] &= \frac{T_o}{T_o + \Delta t} \theta_o[k-1] + \frac{\Delta t \theta_{fi} R}{(T_o + \Delta t)(R + 1)} \left( \frac{I[k]}{I_{rated}} \right)^{2n} + \frac{\Delta t \theta_{fi}}{(T_o + \Delta t)(R + 1)} \\ &= K_1 \theta_o[k-1] + K_2 I[k]^{2n} + K_3 \dots \dots \dots (7) \end{aligned}$$

But this simplified model does not accurately account for the effects of ambient temperature dynamics on TOT (Tylavsky *et al.*, 2000).

**Model #2 Top-oil model**

The top-oil model (as opposed to top-oil-rise model) proposed by Lesieutre *et al.* (1997), accounts for dynamic variations in ambient temperature ( $\theta_{amb}$ ). It is based on the differential equation that defines the TOT  $\theta_{top}$  by,

$$T_o \frac{d\theta_{top}}{dt} = -\theta_{top} + \theta_u + \theta_{amb} \dots \dots \dots (8)$$

The solution of above differential equation is

$$\theta_{top} = (\theta_u + \theta_{amb} - \theta_{topi}) \left( 1 - e^{-t/T_o} \right) + \theta_{topi} \dots \dots \dots (9)$$

Where  $\theta_{topi}$  is the initial TOT for  $t=0$ .

Applying the forward Euler discretization and setting  $n = 1$ ,

$$\theta_{top}[k] = \frac{T_o}{T_o + \Delta t} \theta_{top}[k-1] + \frac{T_o}{T_o + \Delta t} \theta_{amb}[k] + \frac{\Delta t \theta_{fi} R}{(T_o + \Delta t)(R + 1)} \left( \frac{I[k]}{I_{rated}} \right)^{2n} + \frac{\Delta t \theta_{fi}}{(T_o + \Delta t)(R + 1)} \dots (10)$$

Where  $I[k]$  is the per-unit transformer current (based on the rated value of the transformer) at time-step index  $k$ . Equation (10) can also be written in a form amenable to multivariate linear regression with coefficients  $K_1 - K_3$  as,

$$\theta_{top}[k] = K_1\theta_{top}[k-1] + (1 - K_1)\theta_{amb}[k] + K_2I[k]^2 + K_3 \dots \dots \dots (11)$$

Lesieutre et al. (1997) showed that the top-oil model outperforms the top-oil-rise model.

**Model #3 Semi-physical Model**

It has been shown (Tylavsky *et al.*, 2000) that if  $(1 - K_1)$  in equation (11) is replaced by another coefficient and reassign the coefficients,

$$\theta_{top}[k] = K_1\theta_{top}[k-1] + K_2\theta_{amb}[k] + K_3I[k]^2 + K_4 \dots \dots \dots (12)$$

The resulting model is known as a semi-physical model because it is not based entirely on physical principles. Comparing these three different models for the prediction of TOT and Model #3 gave the best prediction.

**2.2 Hot Spot Temperature Equations**

Hot Spot Temperature is the sum of ambient temperature, top oil rise over ambient temperature and winding hot spot rise over top oil temperature.

$$\theta_H = \theta_A + \Delta\theta_{top} \dots \dots \dots (13)$$

- $\theta_A$ : Ambient temperature
- $\theta_{top}$ : Top-oil rise over ambient temperature in °C
- $\Delta\theta_H$ : Winding hot spot rise over top-oil temperature

$$\theta_{HST} = \theta_{top} + \theta_{HSM} \left( \frac{I(t)}{I_{rated}} \right)^{2m} \dots \dots \dots (14)$$

Where  $\theta_{HSM}$  is the maximum HST over TOT in the rating load provided by the manufacturer.

**2.3 Transformer Insulation Life Characteristics**

Insulation aging is a function of temperature and other environmental factors. Today with modern cooling systems, the effect of solar heat flux can be reduced, but the temperature is a limiting factor that should not be exceeded a predetermined value. Since, in most apparatus, the temperature distribution is not uniform, that part which is operating at the highest temperature will normally undergo the greatest deterioration. Therefore, in aging studies, it is usual to consider the aging effects produced by hottest spot temperature. The aging effects on power Transformer are estimated from the aging acceleration factor.

**Aging Acceleration Factor ( $F_{AA}$ )**

Aging Acceleration Factor ( $F_{AA}$ ) for a given hottest-spot temperature is the rate at which transformer insulation aging is accelerated compared with the aging rate at a reference HST. The Power Transformer winding was rated for a hot spot rise temperature of 65°C. The mathematical formulation for  $F_{AA}$  is given in the following equation (Heathcote, 1998):

$$F_{AA} = e^{\left[ \frac{B}{\theta_{HSTref} + 273} - \frac{B}{\theta_{HST} + 273} \right]} \dots \dots \dots (15)$$

Where  $B$  is the aging rate constant of which a value of 15,000 is considered to be appropriate and  $\theta_{HSTref}$  is the winding-hot-spot reference temperature.

**Equivalent Aging Acceleration Factor**

The equivalent aging acceleration factor at the reference temperature in a given time period for the given temperature cycle is defined as (IEEE Standard C57.91, 1995):

$$F_{EAA} = \frac{\sum_{n=1}^N F_{AA,n} \Delta t_n}{\sum_{n=1}^N \Delta t_n} \dots \dots \dots (16)$$

Where,  $F_{EAA}$  is the equivalent aging acceleration factor for the total time period  $N$ ,  $N$  is the total number of time intervals,  $\Delta t_n$  is the  $n$ th time interval and  $F_{AA,n}$  is aging acceleration factor for the temperature which exists during the time interval  $\Delta t_n$ . Transformer insulation's aging is directly connected with the hot spot winding temperature.

$$\% \text{ of Insulation life} = A \cdot e^{\left[ \frac{15000}{\theta_{HST} + 273} \right]} \dots \dots \dots (17)$$

$$\text{Per unit life} = 9.8 \times 10^{-18} e^{\left[ \frac{15000}{\theta_{HST} + 273} \right]} \dots \dots \dots (18)$$

**Percentage Loss of Life**

The equivalent loss of life in the total time period is determined by multiplying the equivalent aging by the time period (t) in hours. In this case total time period used is 24 hours. Therefore, the equation of percentage loss of life equation is as follows (Kumar and Mahajan, 2011)),

$$\% \text{ Loss of Life} = \frac{F_{EAA} \times t \times 100}{\text{Normal Insulation Life}} \dots \dots \dots (19)$$

**3. METHODOLOGY**

A major challenge with the semi-physical model in equation (12) is the absence of the effect of solar radiation which is a significant source of heat flux in temperate regions. From the basic heat transfer principles, heat energy absorbed by a power transformer's oil due to solar radiation is given as

$$H_{top} = mc \Delta \theta \dots \dots \dots (20)$$

$$H_{top} = mc (\theta_{top}(k) - \theta_{top}(k - 1)) \dots \dots \dots (21)$$

where  $m$  and  $c$  are the mass and specific heat capacity  $c$  of the transformer oil respectively.

Also, the heat content in solar radiation  $S_{rad}$  (W/m<sup>2</sup>) on a surface area  $A$  (m<sup>2</sup>) of the transformer is given as

$$H_{rad} = S_{rad}(k) \times A \dots \dots \dots (22)$$

According to the law of conservation of energy,

$$H_{top} = H_{rad}$$

$$mc (\theta_{top}(k) - \theta_{top}(k - 1)) = S_{rad}(k) \times A$$

$$\therefore \theta_{top}(k) = \frac{S_{rad}(k) \times A}{mc} - \theta_{top}(k - 1) \dots \dots \dots (23)$$

Since the surface area of the transformer  $A$ , the mass  $m$  and the specific heat capacity  $c$  of transformer oil are constant, equations (23) is simplified thus

$$\theta_{top}(k) = K S_{rad}(k) - \theta_{top}(k - 1) \dots \dots \dots (24)$$

Thus, including solar radiation effect in equation (3.1) and reassigning the coefficients gives:

$$\theta_{top}(k) = K_1 I(k)^2 + K_2 \theta_{amb}(k) + K_3 \theta_{top}(k - 1) + K_4 S_{rad}(k) + K_5 \dots \dots \dots (25)$$

### 3.1 Determination of Top-Oil Temperature Model Coefficients

In this work, multivariate linear regression will be used to determine the optimal coefficients for the top-oil temperature model in equation (25) from measured data. The criterion for linear regression is a least-squares criterion to minimize the error squared over all data points.

For a data set with one input, X, and one output, Y, and assuming there exists a linear relationship between X and Y, linear regression can be used to find the optimal coefficients, A and k, which gives a best fit to equation (25).

$$Y = A + kX \dots \dots \dots (26)$$

Where:

X: independent variable

K: coefficient to be determined

Y: dependent variable

The value of k in equation (26) is determined using multivariate linear regression by selecting the square of the error between the actual output Y and the predicted output  $\hat{Y}$  while the predicted output  $\hat{Y}$  is minimized. As can be observed in equation (25), there are four (4) independent variables (I(k),  $\theta_{amb}(k)$ ,  $\theta_{top}(k - 1)$  and  $S_{rad}(k)$ ). Thus, multivariate linear regression, which is an extension of that used for single regression, is needed in this case.

For a model with four (4) independent variables, the generalized form of equation (26) is given as:

$$Y = k_1X_1 + k_2X_2 + k_3X_3 + k_4X_4 + k_5 \dots \dots \dots (27)$$

Normalizing equation (27) gives:

$$Y = k_1(X_1 - \bar{X}_1) + k_2(X_2 - \bar{X}_2) + k_3(X_3 - \bar{X}_3) + k_4(X_4 - \bar{X}_4) + k_5 + \varepsilon$$

$$= \hat{Y} + \varepsilon \dots \dots \dots (28)$$

Where:

Y : actual value of top-oil temperature (dependent variable)

$\hat{Y}$  : Predicted value of top-oil temperature

$\varepsilon$  : Error between Y and  $\hat{Y}$

$X_1$  : Load value

$\bar{X}_1$  : Mean value of  $X_1$

$X_2$  : Ambient temperature

$\bar{X}_2$  : Mean value of  $X_2$

$X_3$  : Top-oil temperature (independent variable)

$\bar{X}_3$  : Mean value of  $X_3$

$X_4$  : Solar radiation value

$\bar{X}_4$  : Mean value of  $X_4$

$k_1 - k_5$  : Coefficients to be determined

Expressing equation (28) in matrix form:

$$y_j = [x_j][k] + \varepsilon_j \dots \dots \dots (29)$$

Where:

$$x_j = X_j - \bar{X}_j \dots \dots \dots (30)$$

Where:

$y_j$ : Normalized dependent variable

$[x_j]$ : Vector of independent variables with element  $x_j$

$[k]$ : Vector of coefficients with element  $k_i$

j: subscript index for data points

Equation (27) can also be written using scalar notations:

$$y_j = \sum_i x_{ji}k_i + \varepsilon_j$$

$$\Rightarrow \hat{y}_j = \sum_i x_{ji}k_i \dots \dots \dots (31)$$

Where,  $i$  is the subscript index for the  $i$ th regression coefficient.

To determine the optimal coefficients, the values of the coefficient that give minimum error squared between the actual and the predicted top-oil temperature is selected. This criterion is expressed as:

$$\min \sum_j (y_j - \hat{y}_j)^2 = \min \sum_j \varepsilon_j^2 \dots \dots \dots (32)$$

Substituting equation (31) into equation (32) gives:

$$\min(\sum_j \varepsilon_j^2) = \min[\sum_j (y_j^2 - 2y_j(\sum_i x_{ji}k_i) + (\sum_i x_{ji}k_i)^2)]$$

$$= \min \left[ \sum_j (y_j^2 - 2y_j[x_j][k] + ([x_j][k])^2) \right] \dots \dots \dots (33)$$

In order to find the coefficients that minimize the error squared, equation (33) is differentiated with respect to the  $i$ th coefficient,  $k_i$ , with the result set equal to zero.

$$\sum_j 2y_j x_{ji} = \sum_j \frac{d}{dk_i} (x_{j1}k_1 + x_{j2}k_2 + \dots)^2$$

$$= 2 \sum_j x_{ji} (x_{j1}k_1 + x_{j2}k_2 + \dots)$$

$$\Rightarrow \sum_j y_j x_{ji} = \sum_j x_{ji} (x_{j1}k_1 + x_{j2}k_2 + \dots) \dots \dots \dots (34)$$

Where  $i = 1 - 5$

Using Matrix notation, equation (34) becomes:

$$[X^T Y] = [X^T X][k] \dots \dots \dots (35)$$

The vector of coefficients is solved by inverting  $[X^T X]$  matrix.

$$[k] = [X^T X]^{-1}[X^T Y] \dots \dots \dots (36)$$

Using equation (36) along with measured data, the optimal coefficients of the modified semi-physical model that minimizes the error between the actual and predicted top-oil temperature are determined.

**3.2 Determination of the Maximum Transformer Loading**

Once the optimal coefficients for the predicted top oil temperature have been obtained, the ultimate task will be to use the top oil and hot spot temperature models to determine the maximum load that a transformer can tolerate without exceeding the maximum top-oil and hot-spot temperature at a particular time taking into consideration environmental variables. On substituting the optimal coefficients for the predicted top-oil temperature and the forecasted ambient temperature ( $\theta_{amb}$ ) and solar radiation ( $S_{rad}$ ) value as well as the present value of top-oil [ $\theta_{top}(t-1)$ ] into equation (2), then simplifying we will have

$$\theta_{top} = A + BI^2 \dots \dots \dots (37)$$

where  $A$  is the constant and  $B$  is coefficient of  $I^2$

Substituting equation (37) into (14)

$$\theta_{hs} = A + BI^2 + CI(t)^{2m} \dots\dots\dots (38)$$

Where,

$$C = \frac{\theta_{hm}}{I_{rated}^{2m}}$$

Rewriting equation (38) as an objective function gives

$$f_{HS}(I) = A + BI(t)^2 + CI(t)^{2m} - \theta_{hs(max)} = 0 \dots\dots\dots (39)$$

Note that equation (39) is a function of the load current with  $\theta_{hs(max)}$  as the limit of hot-spot temperature of the transformer.

To find the maximum load violating maximum top-oil and hot-spot temperature criteria, Newton-Raphson iterative algorithm is applied on equation (39).

Newton-Raphson algorithm is based on the fact that if the initial guess of the roots  $f(I) = 0$  is at  $I_i$ , then if a tangent to the curve at  $f(I_i)$  is drawn, the point  $I_{i+1}$  where the tangent crosses the I-axis is an improved estimate of the root.

Using the definition of the slope of a function, at  $I = I_i$

$$f'(I_i) = \frac{f(I_i) - 0}{I_i - I_{i+1}} \dots\dots\dots (40)$$

Which gives

$$I_{i+1} = I_i - \frac{f(I_i)}{f'(I_i)} \dots\dots\dots (41)$$

Starting with an initial guess,  $I_i$ , the next guess,  $I_{i+1}$ , is found using equation (41). The process is repeated until the root within a desirable tolerance is found. This root is the desired load current I from iteration. After finding the load current that satisfies the maximum hot-spot temperature from Newton-Raphson algorithm, the top-oil temperature value is obtained using equation (37). If the computed value of top-oil temperature exceeds the maximum top-oil temperature, the maximum allowable hot-spot temperature is reduced by a factor  $\alpha$ , and the desired load current recomputed using Newton-Raphson algorithm.

$$\theta_{HS(max)}(t + 1) = r \times \theta_{HS(max)}(t) \dots\dots\dots (42)$$

### 3.3 Data Collection

For the determination of the coefficients  $K_1 - K_5$  in top-oil temperature model using multiple linear regression, data were obtained from a 45 MVA, 132/33 kV live power transformer situated at Uyo, Nigeria. The specification of the power transformer is given in Table 1. The data collected were top-oil temperature, secondary load current, ambient temperature and solar radiation on an hourly interval for two days. The data obtained are divided into two for the purpose of training and testing of the top-oil temperature model.



**Table 1: Specification of the Power Transformer**

Parameter	Value
Rating	3-phase, 132/33kV, 45/60 MVA, 50Hz
Top-oil temperature rise @ 50°C ambient temperature	60 °C
Hot-spot temperature rise @ 50°C ambient temperature	65 °C
Cooling method	ONAN/ONAF
Mass of oil	20000 kg
Total mass	85000 kg
Volume of oil	22420 lts
Vector group	YNd11

#### 4. SIMULATION AND RESULTS

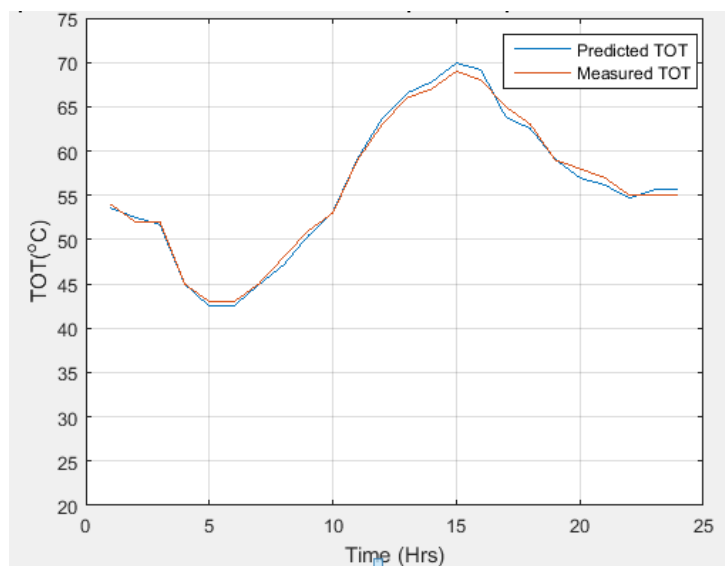
In this research work, MATLAB software was used to implement both the multivariate linear regression and Newton Raphson algorithm for the determination of top-oil temperature model coefficients as well as the optimization of 132/33 kV power transformer loading capacity. The top-oil temperature model coefficients obtained are:

$$K_1 = 17.00517, K_2 = 0.525688, K_3 = 0.517386, K_4 = 0.005878, K_5 = 2.034151$$

Substituting  $K_1 - K_5$  into top-oil model in equation (38) gives

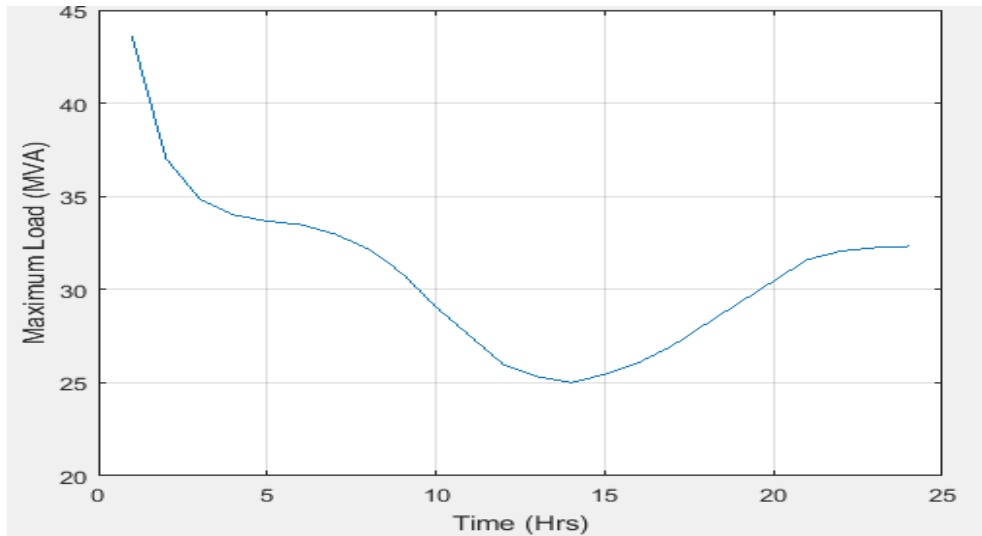
$$\theta_{top}(k) = 17.005171(k)^2 + 0.525688\theta_{amb}(k) + 0.517386\theta_{top}(k-1) + 0.005878S_{rad}(k) + 2.034151 \dots \dots \dots (4.1)$$

The computed  $R^2$  values for top-oil temperature multivariate linear regression model with training and testing datasets are 0.9398 and 0.9219 respectively. These high values of  $R^2$  indicate that the estimated regression line fits the data well. Figure 2 shows a plot of the testing data and predicted top-oil temperatures over a 24 hour period.

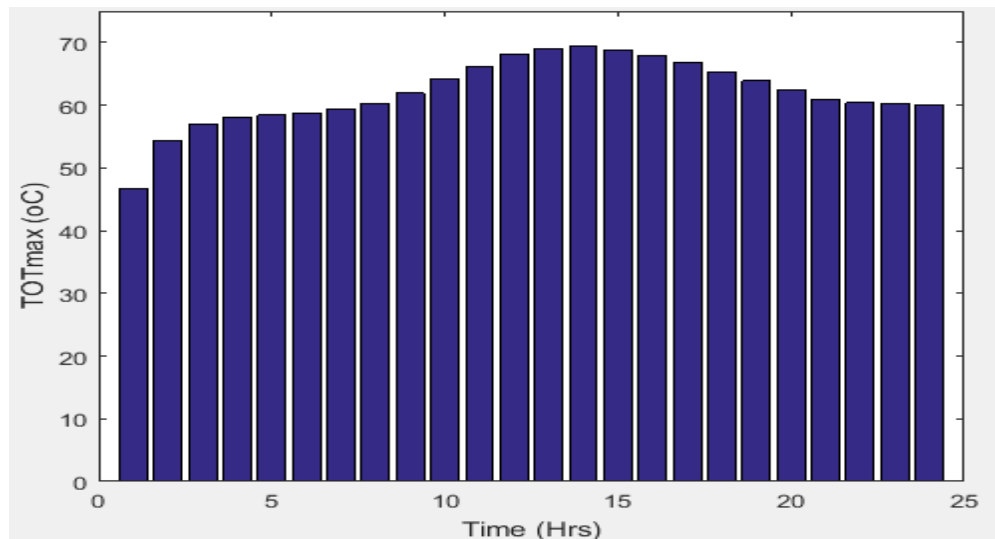


**Figure 2: Comparison of testing data and predicted top-oil temperature over a 24 hour period**

Using MATLAB software, the case-studied power transformer loading capacity was optimized for varying environmental factors (solar radiation and ambient temperature) over a 24 hour period under the condition that top-oil and hot-spot temperature should not exceed 105 °C and 110 °C respectively. The results of the simulation are presented in figures 3 and 4. It is observed that at the beginning of the simulation at 1:00 am when the solar radiation is 0, the optimal transformer loading capacity was at its maximum value while the top-oil temperature recorded its lowest value. Whereas about the mid-day, when the solar radiation was at its maximum, the optimal transformer loading capacity was at its minimum value while the top-oil temperature recorded its highest value. The simple explanation to this is the fact that solar radiation contributes enormous heat to the transformer oil thus leading to the rise in oil temperature, which in turn can lead to reduction in the transformer life span. Hence to check-mate the reduction in the lifespan of the transformer, the transformer maximum loading capacity will have to be reduced. This is the reason for the low loading capacity at high top-oil temperature.



**Figure 3: Optimal transformer loading over a 24 hour period**



**Figure 4: Optimal top-oil temperature over a 24 hour period**

## 5. CONCLUSION

Transformer insulation is the heart of a power transformer design and maximum performance during loading depends on the insulations credibility. Therefore, a proper computation of all the factors contributing to the temperature rise of the transformer's insulation and the thermal limitations of the transformer is necessary to apply the thermal settings. As determined in this paper, the fundamental principles of thermal loading of a transformer is a critical factor and if not properly managed will lead to the loss of life of the transformer. Therefore the optimization of an oil immersed power transformer loading using thermal model with environmental variables was presented. The results of the optimization using multiple linear regression and Newton Raphson algorithm implemented with MATLAB software on the data collected showed that solar radiation contributes enormous heat to the transformer oil thus leading to the rise in oil temperature which in turn can lead to the reduction in transformer life span. This information can help to checkmate the transformer maximum loading capacity and reduce this appropriately under maximum solar radiation conditions.

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