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On the Modified Uniform Distribution

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ABSTRACT

This study considered the modified Uniform Distribution, the Quasi-Transmuted Uniform distribution based on the new quasi-transmutation technique. The study also derived some essential properties of the modified distribution based on the baseline Uniform (0, 1) density function to justify the flexibility of the proposed modified distribution. The result indicates improved performance of the new distribution as expected of a generalized distribution.

Keywords: Quasi-transmuted uniform distribution, Moment generating function, Cumulant generating function, Hazard function, Order statistics

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1. INTRODUCTION

Exploring the inherent flexibility in basic density functions has been a subject of significant interest among researchers. This is the basis for the derivations of many generalized distributions from selected baseline distributions.

Subramanian and Rather (2018) noted that generalized distributions are needed in cases where existing basic distributions may not fit well the data under consideration. Several generalized distributions exist in literature like the Inverted Exponential distribution by Keller et al. (1982); Generalized Inverted exponential distribution by Abouammoh and Alshingiti (2009); Transmuted Inverse exponential distribution by Oguntunde and Adejumo (2014); Extended Generalized distribution by Olapade (2014); Harris Extended distribution by Pinho et al. (2015); Extended Poisson Exponential distribution by Fatima and Roohi (2015); Fractional Beta Exponential distribution by Anake et al. (2015); Exponentiated Generalized Extended Exponential distribution by Thiago et al. (2016); Transmuted Generalized Uniform distribution by Subramanian and Rather (2018); Gompertz Alpha Power Inverted Exponential distribution by Eghwerido et al.(2019) and Odd Generalized Exponential Power Function distribution by Hassan et al. (2019).

The baseline distribution selected for this study is the Uniform distribution defined below

$$U(a,b) = f(x) = \frac{1}{b-a}, a < x < b \qquad \text{......................................(1.0)}$$

This implies that U (0, 1) is defined as

$$U(0,1) = f(x) = 1, 0 < x < 1 \qquad \text{.................................(2.0)}$$

For (2.0), the cumulative distribution function (c. d. f) is given as

$$F(x) = x, 0 < x < 1 \qquad \text{..(3.0)}$$

The quasi-transmutation technique is derived from the general quadratic equation

$$Ax^2 + Bx + C = 0 \qquad \text{..(4.0)}$$

Thus, a random variable X is said to have a Quasi-Transmuted distribution if its cdf is given as

$$G^*(x) = a[F(x)]^2 + (1-a)F(x), a > 0 \qquad \text{..................................(5.0)}$$

where F(x) is the cdf of the selected baseline distribution

The probability density function (p. d. f.) of the Quasi-Transmuted distribution is obtained from (5.0) using (6.0).

That is,

$$g^*(x) = [G^*(x)]' = 2af(x)F(x) + (1-a)f(x), a > 0 \qquad \text{............................. (6.0)}$$

We note that $G^*(x)$ is a valid cdf since $G^*(-\infty) = 0$ and $G^*(\infty) = 1$. The implication of this is that $g^*(x)$ is also a valid pdf.

2. THE QUASI-TRANSMUTED UNIFORM DISTRIBUTION

The Quasi-transmuted distribution is a continuous density function with cdf and pdf defined as

$$G^*(x) = ax^2 + (1\text{-}a)x, \ 0 < x < 1; \ a \ \varepsilon \ (0, \frac{1}{2}, 1) \qquad \text{..................(7.0)}$$

$$g^*(x) = 2ax + (1\text{-}a), \ 0 < x < 1; \ a \ \varepsilon \ (0, \frac{1}{2}, 1) \qquad \text{............... ...(8.0)}$$

We shall consider some essential properties of the Quasi-transmuted distribution next.

2.1 Some Characterizations Of The Quasi-Transmuted Uniform Distribution

2.1.1 Moments

The k^{th} moment about zero for a random variable X from the Quasi-transmuted uniform distribution is

$$\mu_k^1 = E(X^K) = \int_0^1 x^k \ g^* \ (x) \, dx$$

$$= \int_0^1 [2ax^{k+1} + (1-a)x^k] dx$$

$$= \frac{2a}{k+2} + \frac{1-a}{k+1}, \qquad a = 0, \frac{1}{2} \& 1$$

$$\therefore\ \mu_1^1 = \mathrm{E}(X) = \frac{2a}{3} + \frac{1\text{-}a}{2} = \frac{1}{2}, \frac{7}{12}, \frac{2}{3} \text{ for a} = 0, \frac{1}{2} \text{and } 1 \text{ respectively}$$

The kth moment about the mean for a random variable X from the Quasi-transmuted uniform distribution is defined as

$$\mu_k = \mathrm{E}(X-\mu)^k$$
$$= \int_0^1 (x-\mu)^k g^*(x)dx = \int_0^1 (x-\mu)^k \left[2ax + (1-a)x\right] dx$$

Specifically, when k = 2, μ_k becomes the variance. Hence,

$$Variance\ (X) = \sigma_X^2 = \mathrm{E}(X^2) - \left[\mathrm{E}(X)\right]^2$$
$$= 0.083, 0.076 \text{ and } 0.056 \text{ for a } = 0, \frac{1}{2} \text{ and } 1 \text{ respectively}$$

2.1.2 Moment Generating Function

The moment generating function (m.g.f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$M_X(t) = \mathrm{E}(e^{tX}) = \int_0^1 e^{tX} g^*(x)dx$$
$$= \int_0^1 \left[1 + tX + \frac{(tX)^2}{2!} + \ldots\right] g^*(x)dx$$
$$= \int_0^1 \sum_{j=0}^{\infty} \frac{t^j}{j!} X^j g^*(x)dx$$
$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \mu_j^1$$
$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \left[\frac{2a}{j+2} + \frac{1-a}{j+1}\right], \text{ a } \varepsilon\ (0, \frac{1}{2}, 1)$$

2.1.3 Characteristic Function

The characteristic function (c. f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$
\begin{aligned}
\phi_x(t) &= M_X(it) \\
&= \mathrm{E}(e^{itX}) \\
&= \int_0^1 e^{itX} g^*(x)dx \\
&= \sum_{J=0}^{\infty} \frac{(it)^j}{J!}\left(\frac{2a}{j+2} + \frac{1-a}{j+1}\right), \text{ a } \varepsilon \ (0, \frac{1}{2}, 1)
\end{aligned}
$$

2.1.4 Cumulant Generating Function

The cumulant generating function (c. g. f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$
\begin{aligned}
K_X(t) &= In\left[M_X(t)\right] \\
&= In\left[\sum_{J=o}^{\infty} \frac{t^j}{j!} \mu_j^1\right] \\
&= In\left[\sum_{J=0}^{\infty} \frac{t^j}{j!}\left(\frac{2a}{j+2} + \frac{1-a}{j+}\right)\right], \text{ a } \varepsilon \ (0, \frac{1}{2}, 1)
\end{aligned}
$$

2.1.5 Hazard Function

The hazard function (h. f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$
\begin{aligned}
h(x) &= \frac{g^*(x)}{1-G^*(x)} \\
&= \frac{2ax+(1-a)}{1-\left[ax^2+(1-a)x\right]} \\
&= \frac{2ax+(1-a)}{1-ax^2-x+ax}, \text{ a } \varepsilon \ (0, \frac{1}{2}, 1)
\end{aligned}
$$

2.1.6 Reverse Hazard Function

The reserve hazard function (r. h. f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$
\begin{aligned}
h_r(x) &= \frac{g^*(x)}{G^*(x)} \\
&= \frac{2ax+(1-a)}{ax^2+(1-a)x}, \quad a \in (0, \frac{1}{2}, 1)
\end{aligned}
$$

2.1.7 Survival Function

The survival function (s.f.) of a random variable X from the Quasi-transmuted uniform distribution is defined as

$$S_X(x) = 1 - G^*(x)$$

$$= 1 - ax^2 - x + ax, a \in (0, \frac{1}{2}, 1)$$

2.1.8 Order Statistics

Suppose $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ are the order statistics from the random sample $X_1, X_2, \ldots, X_n$ from the Quasi-transmuted uniform distribution with $g^*(x)$ and $G^*(x)$ as the pdf and cdf, the r^{th} order statistic where $1 \le r \le n$ is given as

$$h_{(r)}(x) = \frac{n!}{(r-1)!(n-r)!} g^*(x)\left[G^*(x)\right]^{r-1}\left[1-G^*(x)\right]^{n-r}$$

$$= \frac{n!}{(r-1)!(n-r)!} \left[2ax + (1-a)\right]\left[ax^2 + (1-a)x\right]^{r-1}\left[1 - ax^2 - x + ax\right]^{n-r}$$

By setting r = n and r =1 in the function ($h_{(r)}(x)$) above, we have the distributions for the largest and lowest order statistics. For the largest we have that

$$h_{(r=n)}(x) = \frac{n!}{(n-1)!}\left[G^*(x)\right]^{n-1} g^*(x)$$

$$= n\left[ax^2 + (1-a)x\right]^{n-1}(2x + 1 - a), \ a \in (0, \frac{1}{2}, 1)$$

For the lowest order statistic, we have that

$$h_{(r=1)}(x) = n\left[1 - G^*(x)\right]^{n-1} g^*(x)$$

$$= n\left[1 - ax^2 - x + ax\right]^{n-1}(2ax + 1 - a), \ a \in (0, \frac{1}{2}, 1)$$

2.1.9 Random Number Generation

By the quantile function (i.e the inverse of the cdf), random numbers can be generated for the Quasi-transmuted uniform distribution as follows.

$$Let\ ax^2 + (1-a)x \ = d \qquad so\ that$$
$$ax^2 + (1-a) - d \ = 0$$
$$\Rightarrow x = \frac{(1-a) \pm \sqrt{(1-a)^2 + 4ad}}{2a},\ \text{a}\ \varepsilon\ (0, \frac{1}{2}, 1)$$

where d is a random variable from the Uniform (0, 1) distribution. Thus, x can be estimated from set values of a and d.

3.0 CONCLUSION

The Quasi-transmuted Uniform distribution has been successfully derived including its essential properties. The variance was found to reduce as the value of was increasing. This is what is expected of a generalized distribution when compared to the baseline distribution. The quasi-transmutation method is a therefore a plausible method of generating other distributions.

REFERENCES

1. Abouammoh, A. M. and A. M. Alshingiti (2009). Reliability estimation of generalized inverted exponential distribution. *Journal of Statistical Computation and Simulation 79 (11), 1301-1315.*
2. Anake, T. A., P. E. Oguntunde and A. O. Odetunmibi, (2015). On a fractional beta- exponential distribution. *International Journal of Mathematics and Computations 26 (1), 26-34.*
3. Eghwerido, J. T., Zelibe, S. C and E. Eyefia, (2019). Gompertz-Alpha power inverted exponential distribution: properties and applications. J*ournal of Thailand Statisticians. Article in the press.*
4. Fatima, A. and Roohi, A. (2015). Extended Poisson exponential distribution. *Pakistan Journal of Statistics and Operation Research*. 11(3), 361-375.
5. Hassan, A., E. Elshrpieny and R. Mohamed, (2019). Odd Generalized Exponential Power Function Distribution: Properties and Applications. Gazi University Journal of Science 32(1), 351-370.
6. Keller, A. Z., A. R. R. Kamath and U. D.Perera, (1982). Reliability Aanalysis of CNC machine tools. Reliability Engineering 3 (6), 449-473
7. Oguntunde, P E. and A. O. Adejumo, (2014). The transmuted inverse exponential distribution. *International Journal of Advanced Statistics and Probability 3 (1), 1-7.*
8. Pinho, L. G. B., G. M. Cordeiro and J. S. Nobre, (2015). The Harris extended exponential distribution. *Communications in Statistics -Theory and Methods 44 (16), 3486-3502.*
9. Subramanian, C. and Rather, A.(2018).Transmuted Generalized Uniform Distribution, *International Journal of Scientific Research in Mathematical and Statistical Sciences, 5(3):25-32*
10. Thiago A. N. de Andrade, M. Bourguignon and G. M. Cordeiro. (2016). The exponentiated generalized extended exponential distribution. Journal of Data Science 14, 393-414