Advances In Multidisciplinary & Scientific Research
AIMS Research Journal
A Multidisciplinary & Interdisciplinary Journal

Volume 6, 2020 Edition

On Some Computational Methods for Solving an Ordinary Differential Equation with Initial Condition.

[1]Oshinubi I. K., [2]Adeyeye, F.J., [3]Amao, F.A & [4]Longe, O.B.
[1]Laboratoire Jean Kuntzmann, Mathématiques applique informatique
Universite Grenoble, Alpes, France
[2]Department of Math/Computer Science, Federal University of Petroleum Resources, Effurun, Nigeria
[3]Department of Mathematics, Faculty of Science, Adeleke University, Ede, Nigeria.
[4]School of IT & Computing, American University of Nigeria, Yola, Nigeria
E-mails: [1]kayode.oshinubi@univ-grenoble-alpes.fr, [2]adeyeye.fola@fupre.edu.ng; [3]fola_amao@yahoo.com; [4]longeolumide@fulbrightmail.org

ABSTRACT

Abstract: This research presents computational methods for solving ordinary differential equation with initial condition. Ogunrinde et al.(2012) presented Euler's method and Runge Kutta method for solving initial value problem in ordinary differential equation. We shall use Adomian decomposition method to solve the problem solved by Ogunrinde et al.(2012) and compare the results and error. The error incurred is undertaken to determine the accuracy and consistency of the three methods

Keywords: Adomian Decomposition Method (ADM), Differential Equation, Euler's Method, Error, Stability Runge Kutta Method,.

Aims Research Journal Reference Format:
Oshinubi I. K., Adeyeye, F.J. Amao, F.A. & Longe, O.B. (2020): On Some Computational Methods for Solving an Ordinary Differential Equation with Initial Condition. Advances in Multidisciplinary Research Journal. Vol. 6. No. 1, Pp 1–8.
Article DOI: dx.doi.org/10.22624/AIMS/V6N1P1

1. INTRODUCTION

Ordinary Differential equations can describe nearly all system undergone change. Many mathematicians have studied the nature of these equations and many complicated systems can be described quite precisely with compact mathematical expressions. However, many systems involving differential equations are so complex. It is in these complex systems where computer simulations and numerical approximations are useful. The techniques for solving differential equations based on numerical approximations were developed before programmable computers existed. The problem of solving ordinary differential equations is classified into initial value and boundary value problems, depending on the conditions specified at the end points of the domain.

There are numerous methods that produce numerical approximations to solution of initial value problems in ordinary differential equations such as Euler's method which was the oldest and simplest method originated by Leonhard Euler in 1768, Improved Euler's method and Runge Kutta methods described by Carl Runge and Martin Kutta in 1895 and 1905 respectively.

There are many excellent and exhaustive texts on this subject that may be consulted, such as [8], [4], [6], [5],[17],[18] and [1] just to mention few. In this work we present the practical use of Adomian decomposition method for solving ordinary differential equations with initial conditions.

2. NUMERICAL METHOD

Numerical method forms an important part of solving initial value problems in ordinary differential equations, most especially in cases where there is no closed form solution. Next we present three numerical methods namely Euler's Method, Runge Kutta method, and Adomian Decomposition method.

2.1 Runge Kutta Method

Runge Kutta method is a technique for approximating the solution of ordinary differential equation. This technique was developed around 1900 by the mathematicians Carl Runge and Wilhelm Kutta. Runge Kutta method is popular because it is efficient and used in most computer programs for differential equation.
The following are the orders of Runge Kutta Method as listed below:
-Runge Kutta method of order one is called Euler's method.
-Runge Kutta method of order two is the same as modified Euler’s or Heun's Method.
-Runge Kutta method of order four is called classical Runge Kutta method

2.2 Euler’s Method

Euler’s method is also called tangent line method and is the simplest numerical method for solving initial value problem in ordinary differential equation, particularly suitable for quick programming which was originated by Leonhard Euler in 1768. This method subdivided into three namely,
2.2.1 Forward Euler’s method.
2.2.2 Improved Euler’s method.
2.2.3 Backward Euler’s method.

2.3 Adomian Decomposition Method

The Adomian decomposition method, proposed by Adomian initially with the aims to solve frontier physical problem, has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, biology and chemical reactions etc. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly. It is well known that the key of the method is to decompose the nonlinear term in the equations into a peculiar series of polynomials are the so-called Adomian polynomials. Adomian formally introduced formulas that can generate Adomian polynomials for all forms of nonlinearity. Recently, a great deal of interests has been focused to develop a practical method for the calculation of Adomian polynomials An.

However, the methods developed by [9]-[15] also require a huge size of calculations. [16] Established a promising algorithm that can be easily programmed in Maple, and be used to calculate Adomian polynomials for nonlinear terms in the differential equations. Let us first recall the basic principles of the Adomian decomposition methods for solving differential equations.

Consider the general equation Fu ¼ g, where F represents a general nonlinear differential operator involving both linear and nonlinear terms, the linear term is decomposed into L þ R, where L is easily invertible and R is the remainder of the linear operator. For convenience, L may be taken as the highest order derivate. Thus the equation may be written as:

Lu+ Ru+ Nu= g, (1)

Where Nu represents the nonlinear terms. Solving Lu from (1), we have Lu = g – Ru- Nu (2)
Because L is invertible, the equivalent expression is Lu = L^{-1}g – L^{-1}Ru- L^{-1}Nu (3)
If L is a second order operator, for example, L^{-1} is a twofold integration operator and L^{-1}Lu= u- u(0) -t (0), then (3) for u yields

u = a+ bt+ $L^{-1}g - L^{-1}Ru - L^{-1}Nu$ (4)

Therefore, u can be presented as a series
u= $\sum_{n=0}^{\infty} u_n$ (5)

with u_0 identified as a + bt + $L^{-1}g$ and $u_n\ (n > 0)$ is to be determined. The nonlinear term Nu will be decomposed by the infinite series of Adomian polynomials.

$$Nu = \sum_{n=0}^{\infty} A_n, \qquad (6)$$

where A_n's are obtained by writing

$$v(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n, \qquad (7)$$

$$N(v(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n. \qquad (8)$$

Here λ is a parameter introduced for convenience. From(7)and(8), we deduce

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N(v(\lambda))\right]_{\lambda=0}, \quad n = 0, 1, \ldots \qquad (9)$$

Now, substituting(5) and (6)into(4), we obtain

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n.$$

Consequently, we can write

$$u_0 = a + bt + L^{-1}g,$$
$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0,$$
$$\vdots$$
$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n.$$

All of u_n are calculable, and $u = \sum_{n=0}^{\infty} u_n$. Since the series converges and does so very rapidly, the n-term partial sum $\varphi_n = \sum_{i=0}^{n-1} u_i$ can serve as a practical solution.

3. NUMERICAL EXPERIMENTS

We shall now proceed by using ADM Method In order to confirm the applicability and suitability of the methods for solution of initial value problems in ordinary differential equations, it was computerized in Maple software package. The performance of the methods was checked by comparing their accuracy and efficiency with methods used by Ogunrinde et al.(2012). The efficiency was determined from the number iterations counts and number of functions evaluations per step while the accuracy is determined by the size of the discretization error estimated from the difference between the exact solution and the numerical approximations.

Advances In Multidisciplinary & Scientific Research
AIMS Research Journal
A Multidisciplinary & Interdisciplinary Journal
Volume 6, 2020 Edition

Example:
We use Adomian Decomposition method to approximate the solution of the initial value problem y', $y(0) = 2$, y with step size $h = 0.1$on the interval $0 \leq x \leq 1$whose exact solution is given by $y(x) = e^x + x + 1$. The results obtained shown in Table 1 and Table 2, the comparison of the method to Euler, Runge Kutta and exact solution and the error incurred respectively.

3.1 Table of Results

Table 1: The Comparative Result Analysis of Adomian Decomposition Method, Runge Kutta Method and Euler's Method

n	x_n	$y(x_n)$	*Runge Kutta*	*Euler*	*Adomian*
0	0.0	2.0000	2.0000	2.0000	2.0000
1	0.1	2.2052	2.2051	2.2000	1.8048
2	0.2	2.4214	2.4213	2.4100	1.6186
3	0.3	2.6498	2.6497	2.6310	1.4402
4	0.4	2.8918	2.8918	2.8641	1.2684
5	0.5	3.1487	3.1486	3.1105	1.1018
6	0.6	3.4221	3.4221	3.3716	0.9392
7	0.7	3.7137	3.7137	3.6487	0.7792
8	0.8	4.0255	4.0255	3.9436	0.6203
9	0.9	4.3596	4.3596	4.2579	0.4611
10	1.0	4.7182	4.7182	4.5937	0.3000

Table 2: Error incurred in Adomian Decomposition Method, Runge Kutta Method and Euler's Method

n	x_n	*Runge kutta Error*	*Euler Error*	*Adomian Error*
0	0.0	0.0000	0.0000	0.0000
1	0.1	0.0001	0.0052	0.4004
2	0.2	0.0001	0.0114	0.8028
3	0.3	0.0001	0.0188	1.2096
4	0.4	0.0000	0.0277	1.6234
5	0.5	0.0001	0.0382	2.0469
6	0.6	0.0000	0.0505	2.4829
7	0.7	0.0000	0.0650	2.9345
8	0.8	0.0000	0.0819	3.4052
9	0.9	0.0000	0.1017	3.8985
10	1.0	0.0000	0.1245	4.4182

3.2 Graph of Result

Given below is the graph of results

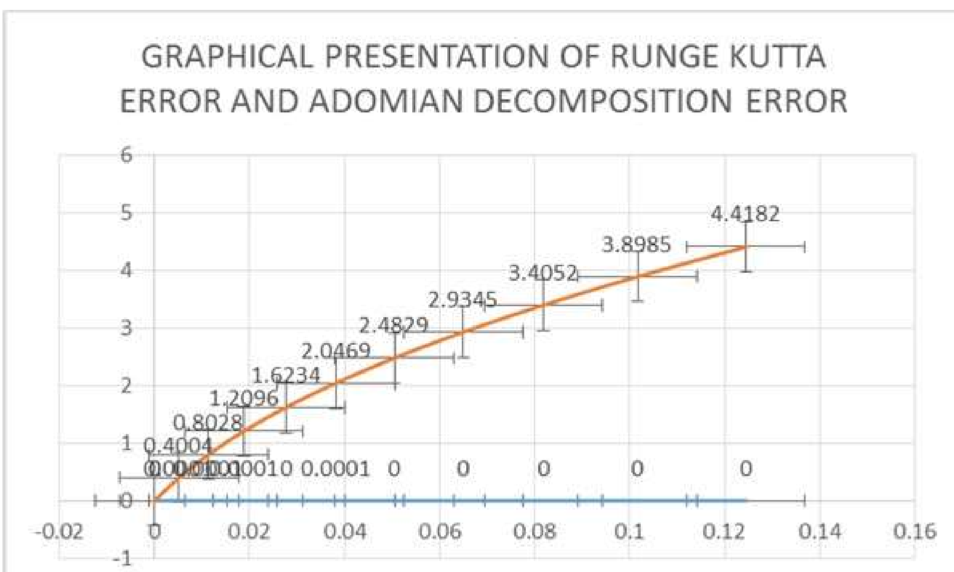

Fig. 1: Graphical Presentation of Runge Kutta4. DISCUSSION OF RESULTS

We noticed that in Table 2, the error incurred in Adomian Decomposition Method is greater than that of Runge Kutta method and Euler Method and at the same time get larger as *n* increases. Hence Runge Kutta method is more accurate than its counterpart Euler's method and Adomian Decomposition Method as we can see from Table 2.

5. CONCLUSION

We have in our disposal three numerical methods for solving initial value problems in ordinary differential equations. In general, numerical method has its own advantages and disadvantages of use. From the problem solved using MAPLE, it is observed that a lot of useful insights into numerical solution of initial value problem have been gained. We conclude that Runge Kutta method is consistent, convergent, quite stable and more accurate than Euler's method and Adomian Decomposition Method and it is widely used in solving initial value problems in ordinary differential equations.

REFERENCES

[1] W. E. Boyce and R. C. DiPrima, *Elementary differential equation and boundary value problems*, (John Wiley and Son, 2001).

[2] L. Collatz, *Numerical treatment of differential equations*, (Springer Verlag Berlin, 1960).

[3] K. Erwin *Advanced Engineering Mathematics*, Eighth Edition, (Wiley Publisher, 2003).

[4] Gilat, *Matlab: An Introduction with application*, (John Wiley and Sons, 2004).

[5] N. Kockler, *Numerical methods and scientific computing*, (Clarendon Press, Oxford London, 1994).

[6] J. D. Lambert, *Numerical method for ordinary systems of Initial value problems*, (John Wiley and Sons, New York, 1991).

[7] D. C. Samuel, *Elementary numerical analysis (an algorithm approach),* Third Edition, (Mc Graw International Book Company, 1981).

[8] M. P. Stephen, *To compute numerically, concepts and strategy*, (Little Brown and Company, Canada, 1983).

[9] R. Rach, A convenient computational form for the Adomian polynomials, J. Math. Anal. Appl. 102 (1984) 415–419.

[10] V. Seng, K. Abbaoui, Y. Cherruault, Adomian polynomials for nonlinear operators, Math Comput. Modell. 24 (1996) 59–65.

[11] K. Abbaoui, Y. Cherruault, Convergence of Adomians method applied to differential equations, Comput. Math. Appl. 28 (5) (1994) 103–109.

[12] S. Guellal, Y. Cherruault, Practical formulae for calculation of Adomians polynomials and application to the convergence of the decomposition method, Int. J. Biomed. Comput. 36 (1994) 223–228.

[13] A.M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Appl. Math. Comput. 111 (2000) 53–69

[14] Y. Cherruault, G. Saccomandi, B. Some, New results for convergence of Adomians method applied to integral equations, Math. Comput. Modell. 16 (2) (1992) 85–93.

[15] K. Abbaoui, Y. Cherruault, New ideas for proving convergence of decomposition methods, Comput. Math. Appl. 29 (7) (1995) 103–108.

[16] C. Wenhai, L. Zhengyi, An algorithm for Adomian decomposition, Applied Mathematics and Computation 159 (2004) 221–235.

[17] Ogunrinde R. Bosede , Fadugba S. Emmanuel , Okunlola J. Temitayo , On Some Numerical Methods for Solving Initial Value Problems in Ordinary Differential Equations. *IOSR Journal of Mathematics, Volume 1, Issue 3 (July-Aug 2012), PP 25-31*

[18] K.I Oshinubi and R.B Ogunrinde. An Adomian Decomposition Method for Autonomous and Non- Autonomous Ordinary Differential Equations, Asian Journal of Mathematics and Computer Research. Vol 19, Issue 2 (2017) 65-74