Advances In Multidisciplinary & Scientific Research
A Multidisciplinary & Interdisciplinary Journal

Volume 6, 2020 Edition

A Bayesian Approach to the Sensitivity of Parameters Misspecification in The Seemingly Unrelated Regression Models

Alaba, Oluwayemisi Oyeronke
Department of Statistics
University of Ibadan
Ibadan, Nigeria
E-mail: oluwayemisioyeronke@yahoo.com

ABSTRACT

The omission of important variables is often encountered in econometrics, which often results in the misspecification of the model. This study considered a misspecified joint seemingly unrelated regression (SUR) model within the Bayesian context. The explanatory variables were generated from a uniform distribution for sample sizes 5, 10, 30, 50, 100 and 1000. The coefficients of β were generated from the multivariate normal distribution. Different levels of correlation (0.2, 0.4, 0.5, 0.7 and 0.9) among the independent variables were considered. The choice of hyperprior distribution was explored. Due to the incorrectly defined model, the intended attribute of unbiasedness in the estimated coefficients, which depends on the fitted model's correspondence with the actual underlying data-generating process, was not attained. Irregular patterns were seen in the posterior means and standard deviations except for a few large sample sizes.

Keywords: Misspecification, Multicollinearity, Posterior mean, Posterior standard deviation, Seemingly unrelated regression.

Aims Research Journal Reference Format:
Alaba, Oluwayemisi Oyeronke (2020): A Bayesian Approach to the Sensitivity of Parameters Misspecification in The Seemingly Unrelated Regression Models. Advances in Multidisciplinary and Scientific Research. Vol. 6. No. 1, Pp 133-152. www.isteams.net/aimsjournal. dx.doi.org/10.22624/AIMS/V6N1P13

1. INTRODUCTION

In econometrics, omitting key variable(s) from a model specification is common (Lawal and Alaba, 2019). This may be done to simplify the model in an effort to conserve the degree of freedom or because some important variables are not available. Omitting pertinent variables has several consequences, one of which is that the inaccuracy is likely to result in a biased estimation of the coefficients (Clarke, 2005). Omitted variable bias is likely to happen when a model is incorrectly described because of the excluded variable. When a model includes an appropriate explanatory variable but lacks a link between the explanatory and response variables, it also exhibits a functional type of misspecification. Several studies have reported that one of the causes for the exclusion of variables is multicollinearity (Kim, 2019; Zainodin and Yap, 2013; Das *et al,* 2011; Graham, 2003).

Instrumental variables or proxies are usually employed in dealing with omitted variable concerns (Wooldridge, 2016; Hausman, 1975), but the effective application of these methodologies by researchers remains difficult on how to appropriately model the omitted variable that influences the dependent variable (Branson and Lovell, 2000). Clarke (2005) in his application reported that including an additional proxy may increase or decrease the bias of the estimated coefficients. A fundamental challenge in model specification is over- and underfitting a model (Rohlfing, 2008; Bozdogan, 1987).

Even when the omitted variables are orthogonal to the variables included in the model, it is well documented that removing relevant variables results in incorrect inference. This is because of inherent relationships between the included and omitted variables, which results in unbiased estimates of the coefficients but insufficient for producing an unbiased estimator of the variances (Midi *et al*, 2010). Different approaches have been used to tackle model misspecification including, Christopher and Western (2016) who used the classical approach and McGuirk *et al*, 1993, they defined an updating rule that specifies a mapping from prior beliefs and the signal of an agent to subjective posterior, and modelling an agent as a Bayesian learner with a misspecified model.

In the Seemingly Unrelated Regression (SUR) model (Alaba and Akinrelere, 2019; Alaba and Lawal, 2019; Alaba *et al*, 2019; Alaba *et al*, 2010), misspecifying by omitting relevant regressors from one or more equations leads to biases of individual OLS estimates of the parameters (Aigner, 1974). In the case of FGLS estimator mis-specification in the system results in the inconsistency of the model. The omission of relevant regressors in SUR model has an extreme consequence for the estimators. This is because there is no correlation between the included and omitted variables, which results in unbiased estimates of the coefficients but insufficient for producing an unbiased estimator of the variances (Frost, 1979). There is a dearth of studies on the misspecification of SUR model parameters within the Bayesian context. This study, therefore, is aimed at investigating the effects of misspecification in the SUR model within the Bayesian context.

2. THE MODEL

Consider a true equation of the SUR model:

$$y_1 = X_1\beta_1 + \gamma_1 x + u_1 \quad (1)$$

$$y_2 = X_2\beta_2 + \gamma_2 x + u_2 \quad (2)$$

where y_i is a $(T \times 1)$ vector of observations on the dependent variable, X_i is a $(T \times K_i)$ matrix of values of the explanatory variables, β_i is a $(K_i \times 1)$ vector of parameters and ε_i is a $(T \times 1)$ vector of disturbances.

In matrix form;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (3)$$

or

$$y = X\beta + (I_2 \otimes x)\gamma + u \quad (4)$$

where,

$$E(u_1) = E(u_2) = 0;\ \mathrm{E}(u_i u_j^{'}) = \sigma_{ij} I_T\ \ (\mathrm{i, j} = 1, 2)$$

The error term across equations is contemporaneously correlated.

Suppose that γ_1 and γ_2 are omitted from the true model in (1) and (2), the disturbance terms become;

$$\begin{aligned} \varepsilon_1 &= \gamma_1 x + u_1 \\ \varepsilon_2 &= \gamma_2 x + u_2 \end{aligned} \quad (5)$$

In contrast, the misspecified model in (1) and (2) is;

$$y = X\beta + \varepsilon \quad (6)$$

Estimating (6) by OLS, we obtain;

$$b_0 = (X'X)^{-1} X'y \quad (7)$$

From (2) and (4), we obtain

$$E(b_0 - \beta) = (X'X)^{-1} X'(I_2 \otimes x)\gamma \quad (8)$$

Similarly;

$$\begin{aligned} M(b_0) &= E(b_0 - \beta)(b_0 - \beta)' \\ &= (X'X)^{-1} X'\left[(\Sigma \otimes I_T) + (I_2 \otimes x)\gamma\gamma'(I_2 \otimes x')\right] X(X'X)^{-1} \end{aligned} \quad (9)$$

Considering (6), the likelihood function for β and Σ in the model (10) is;

$$\begin{aligned} \ell(\beta, \Sigma^{-1} / y) \alpha\, |\Sigma^{-1}|^{T/2} \exp\left\{-1/2(y - X\beta)'(\Sigma^{1} \otimes I_T)(y - X\beta)\right\} \\ \alpha\, |\Sigma^{-1}|^{T/2} \exp\left\{-1/2 trF\Sigma^{-1}\right\} \end{aligned} \quad (10)$$

Where F is an $(M \times M)$ symmetric matrix with $(ij)'\ th$ element equal to $(y_i - X_i\beta_i)'(y_j - X_j\beta_j)$.

In the Bayesian context, uncertain prior information about the unknown parameters is reflected in the formulation of a prior density.

$$P(\beta,\Sigma^{-1}) = P(\beta)P(\Sigma^{-1}) \quad (11)$$

$$P(\beta)\,\alpha\ \text{constant}$$

$$P(\Sigma^{-1})\,\alpha\,|\Sigma^{-1}|^{-(M+1)/2} \quad (12)$$

With β_{ij} as the j^{th} coefficient in the i^{th} equation of the SUR model. The joint posterior density for all of the unknown parameters is

$$\begin{aligned} P(\beta,\Sigma^{-1}/y)\alpha\,|\Sigma^{-1}|^{(T-M-1)/2} & \exp\left\{-1/2(y-X\beta)'(\Sigma^{1}\otimes I_T)(y-X\beta)\right\} \\ \alpha\,|\Sigma^{-1}|^{(T-M-1)/2} & \exp\left\{-1/2trF\Sigma^{-1}\right\} \end{aligned} \quad (13)$$

From (13) the conditional posterior density for β given Σ^{-1} is

$$P(\beta,\Sigma^{-1}/y)\alpha\exp\left\{-1/2(y-X\beta)'(\Sigma^{1}\otimes I_T)(y-X\beta)\right\} \quad (14)$$

Which is multivariate normal in form with mean vector.

$$\begin{aligned} E(\beta\,|\,\Sigma^{-1},y) &= \left[X'(\Sigma^{-1}\otimes I_T)X\right]^{-1}X'(\Sigma^{-1}\otimes I_T)y \\ &= b_G \end{aligned} \quad (15)$$

$$\begin{aligned} V(\beta\,|\,\Sigma^{-1},y) &= \left[X'(\Sigma^{-1}\otimes I_T)X\right]^{-1} \\ &= \Omega \end{aligned} \quad (16)$$

So, conditional on knowledge of Σ and with diffuse prior information, the posterior mean of β is just the GLS estimator. The marginal posterior density for β according to Tiao and Zellner (1964) is

$$P(\beta\,|\,y)\alpha\,|F|^{-T/2} \quad (17)$$

The marginal posterior density of Σ^{-1} is

$$P(\Sigma^{-1}\,|\,y) = \int P(\beta,\Sigma^{-1}\,|\,y)d\beta \quad (18)$$

This research utilized the Bayesian method for possibility of uncertainty in model specification. In the context of this research, we assume that the value of the distribution of $\hat{\beta}$ falls on the choice of the prior distribution.

3. SIMULATION EXPERIMENT

Data series were generated considering a system of SUR equations with three distinct linear equations contemporaneously and serially correlated. A positive 3 × 3 definite variance-covariance matrix was defined and derived-error terms were obtained. It consists of two parts (i) Simulation design (ii) Results and Discussion.

3.1 Simulation Design

The Monte Carlo experiment was performed by generating data according to the following algorithm.

1. Generate the explanatory variables from $MVN_3(0, \Sigma_x)$
2. Set the true values of β to $(1,1,1,1)'$.
3. Simulate the vector random error from $MVN_3(0, \Sigma_e)$.
4. For a given X structure, transform the original model to the canonical form.
5. The dependent variables were formed and a misspecified model including only the predictors x_1 and x_2
6. Repeat the above step 10000 times.

Table 1: Factors that vary for different models

Factors	Symbol	Design
Number of equations	M	3
Number of observations	T	5, 10, 30, 50, 100
Correlation among the explanatory variables	ρ_x	0.2, 0.4, 0.5, 0.7, 0.9
Contemporaneous correlation between corresponding errors among the equations	ρ_Σ	0.0, 0.2, 0.4, 0.6, 0.8, 0.9

Table 1: Posterior Mean for $\rho_x = 0.2$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-5.306	-5.329	-5.313	-0.595	-0.596	-0.594	4.307	4.330	4.312
10	-0.265	-0.283	-0.276	-1.740	-1.725	-1.724	-0.931	-0.940	-0.931
30	0.096	0.099	0.099	0.664	0.670	0.667	0.359	0.352	0.355
50	0.594	0.594	0.592	1.682	1.683	1.680	-0.202	-0.203	-0.199
100	1.907	1.900	1.903	1.571	1.571	1.568	-0.163	-0.160	-0.161
1000	2.364	2.363	2.363	2.535	2.536	2.536	-0.762	-0.761	-0.761
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-2.307	-2.314	-2.332	-0.023	-0.012	-0.003	1.525	1.519	1.530
10	1.181	1.189	1.180	-1.033	-1.045	-1.045	-3.815	-3.814	-3.813
30	1.759	1.765	1.763	1.354	1.348	1.348	-0.478	-0.474	-0.476
50	0.239	0.240	0.242	-1.523	-1.524	-1.529	-0.977	-0.980	-0.979
100	1.414	1.421	1.414	0.876	0.875	0.879	1.783	1.777	1.779
1000	0.075	0.077	0.077	1.023	1.023	1.021	1.220	1.219	1.220
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	2.266	2.302	2.291	-0.449	-0.449	-0.454	-1.687	-1.729	-1.725
10	-0.643	-0.656	-0.677	0.440	0.449	0.460	1.018	1.018	1.030
30	-1.673	-1.670	-1.672	3.678	3.682	3.676	1.880	1.877	1.886
50	-0.636	-0.636	-0.641	-0.730	-0.728	-0.729	-1.018	-1.017	-1.014
100	-1.788	-1.787	-1.786	-0.707	-0.710	-0.707	0.217	0.220	0.218
1000	-0.454	-0.455	-0.455	0.026	0.027	0.027	-0.705	-0.705	-0.706
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	2.328	2.300	2.310	1.902	1.895	1.898	0.405	0.429	0.424
10	1.471	1.458	1.463	0.004	0.021	0.018	0.536	0.531	0.537
30	1.425	1.429	1.430	-2.130	-2.130	-2.132	0.701	0.698	0.704
50	1.964	1.962	1.965	0.633	0.633	0.641	0.232	0.233	0.225
100	-2.011	-2.008	-2.013	-1.755	-1.754	-1.753	-1.309	-1.310	-1.310
1000	0.632	0.631	0.632	1.147	1.147	1.148	1.170	1.170	1.167
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-2.165	-2.180	-2.174	2.423	2.409	2.424	2.523	2.550	2.532
10	1.253	1.258	1.261	-3.532	-3.533	-3.528	0.674	0.677	0.674
30	-2.620	-2.629	-2.624	-1.107	-1.104	-1.105	-0.825	-0.816	-0.822
50	-1.649	-1.652	-1.652	0.113	0.120	0.118	2.403	2.402	2.401
100	-0.850	-0.848	-0.851	-0.680	-0.688	-0.684	-0.081	-0.077	-0.077
1000	0.816	0.817	0.817	1.225	1.224	1.225	-1.012	-1.012	-1.013

ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-6.449	-6.497	-6.498	6.627	6.707	6.648	-6.367	-6.426	-6.356
10	3.194	3.195	3.187	3.281	3.285	3.283	2.733	2.718	2.742
30	-0.336	-0.330	-0.326	1.197	-1.205	-1.207	-1.445	-1.448	-1.450
50	0.203	0.202	0.201	-1.639	-1.641	-1.644	0.993	0.999	1.003
100	-0.690	-0.692	-0.691	1.803	1.808	1.803	1.078	1.077	1.076
1000	-1.787	-1.786	-1.786	-1.446	-1.446	-1.446	-0.468	-0.469	-0.468

Table 2: Posterior Mean for $\rho_x = 0.4$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-0.935	-0.939	-0.939	-0.725	-0.731	-0.719	-0.493	-0.478	-0.476
10	2.320	2.324	2.324	-0.040	-0.048	-0.046	1.454	1.455	1.454
30	0.063	0.059	0.062	0.728	0.735	0.730	-0.295	-0.295	-0.293
50	1.055	1.052	1.056	1.561	1.559	1.561	-1.761	-1.757	-1.762
100	-1.731	-1.733	-1.734	0.476	0.471	0.471	1.265	1.272	1.271
1000	0.983	0.984	0.984	2.421	2.421	2.420	1.628	1.628	1.629
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	1.164	1.119	1.150	-1.026	-0.952	-1.017	-3.353	-3.394	-3.343
10	-1.466	-1.464	-1.468	0.146	0.157	0.157	0.126	0.118	0.123
30	-0.299	-0.305	-0.303	0.442	0.452	0.449	1.151	1.149	1.145
50	0.755	0.755	0.762	-0.637	-0.634	-0.637	0.217	0.217	0.216
100	-1.478	-1.482	-1.482	0.533	0.532	0.534	-1.002	-0.998	-0.999
1000	-1.938	-1.938	-1.939	-2.933	-2.933	2.932	-0.746	-0.746	-0.745
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	9.708	9.662	9.683	-5.494	-5.476	-5.493	-3.809	-3.817	-3.799
10	-4.096	-4.091	-4.104	0.016	0.019	0.039	0.571	0.576	0.557
30	0.886	0.884	0.878	1.324	1.324	1.326	-1.259	-1.267	-1.257
50	-0.973	-0.970	-0.973	-1.362	-1.364	-1.361	-2.375	-2.374	-2.376
100	-1.154	-1.153	-1.154	1.912	1.910	1.909	-1.304	-1.304	-1.302
1000	0.787	0.787	0.788	0.116	0.116	0.116	-1.619	-6.20	-6.20
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	1.147	1.129	1.140	-1.416	-1.406	-1.404	-0.630	-0.625	-0.629
10	1.219	1.216	1.224	1.222	1.215	1.220	-2.468	-2.463	-2.473
30	0.048	0.047	0.054	0.531	0.532	0.529	-1.614	-1.613	-1.619
50	0.604	0.601	0.603	3.617	3.621	3.617	0.121	0.122	0.122
100	2.001	1.993	1.997	-0.555	-0.552	-0.552	0.031	0.037	0.037
1000	1.660	1.659	1.659	1.100	1.099	1.101	0.651	0.652	0.651

ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-3.723	-3.755	-3.721	2.827	2.859	2.826	0.875	0.890	0.882
10	0.007	-0.002	-0.003	-2.401	-2.392	-2.384	-1.324	-1.314	-1.328
30	-1.096	-1.089	-1.092	1.627	1.620	1.627	2.220	2.219	2.220
50	1.697	1.696	1.691	0.073	0.073	0.076	0.647	0.646	0.649
100	-1.132	-1.135	-1.136	0.635	0.638	0.636	2.504	2.505	2.507
1000	-0.016	-0.017	-0.015	1.016	1.016	1.016	0.851	0.853	0.852
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.076	0.057	0.052	-3.598	-3.561	-3.581	-0.265	-0.282	-0.251
10	1.196	1.185	1.174	-0.766	-0.764	-0.753	1.032	1.042	1.043
30	-0.898	-0.903	-0.900	0.687	0.690	0.691	-0.122	-0.122	-0.127
50	0.570	0.571	0.570	0.733	0.729	0.731	-1.641	-1.633	-1.634
100	-0.933	-0.933	-0.928	0.224	0.231	0.224	-1.229	-1.237	-1.233
1000	1.130	1.129	1.130	2.675	2.676	2.675	1.939	1.938	1.939

Table 3: Posterior Mean for $\rho_x = 0.5$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.571	0.585	0.598	3.856	3.858	3.854	2.102	2.10	2.089
10	-2.028	-2.037	-2.034	-0.904	-0.913	-0.920	0.629	0.659	0.655
30	3.190	3.183	3.189	0.074	0.080	0.079	-1.603	-1.602	-1.604
50	-1.983	-1.984	-1.985	1.032	1.035	1.033	-0.107	-0.110	-0.110
100	-0.821	0.826	0.820	-1.177	-1.179	-1.174	0.738	0.736	0.736
1000	-0.241	-0.242	-0.243	-0.919	-0.917	-0.917	0.404	0.403	0.404
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	1.252	1.248	1.264	0.712	0.716	0.704	-2.225	-2.214	-2.229
10	0.833	0.837	0.841	-0.693	-0.705	-0.702	0.069	0.074	0.073
30	1.882	1.878	1.883	0.478	0.482	0.479	1.974	1.975	1.977
50	-1.854	-1.860	-1.866	0.450	0.452	0.454	0.431	0.434	0.441
100	-0.514	-0.513	-0.511	0.074	0.072	0.071	0.455	0.458	0.458
1000	0.886	0.886	0.886	1.671	1.672	1.672	1.871	1.870	1.869
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-0.581	-0.599	-0.584	0.225	0.211	0.221	5.876	5.901	5.861
10	1.188	1.190	1.200	-0.622	-0.623	-0.642	2.119	2.122	2.132
30	0.348	0.352	0.346	0.715	0.708	0.717	1.399	1.404	1.406
50	0.256	0.264	0.257	1.720	1.718	1.720	0.415	0.413	0.416
100	-2.149	-2.144	-2.143	3.346	3.345	3.346	1.697	1.697	1.695
1000	0.908	0.908	0.909	1.850	1.848	1.849	0.724	0.725	0.725

ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.792	0.775	0.789	1.790	1.783	1.785	-1.522	-1.492	-1.511
10	-0.175	-0.158	-0.161	1.684	1.670	1.665	2.770	2.769	2.781
30	0.219	0.219	0.217	-1.141	-1.143	-1.149	-0.922	-0.918	-0.914
50	2.904	2.905	2.911	0.200	0.196	0.196	1.397	1.398	1.393
100	-0.185	-0.184	-0.183	1.675	1.668	1.673	1.129	1.136	1.131
1000	-0.873	-0.873	-0.871	0.219	-0.219	-0.221	-1.633	-1.632	-1.632
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-1.014	-1.020	-1.016	-0.258	-0.248	-0.244	1.474	1.471	1.482
10	-1.128	-1.139	-1.139	-3.643	-3.639	-3.641	-2.874	-2.856	-2.850
30	-0.095	-0.095	-0.094	-0.994	-0.990	-0.994	0.537	0.531	0.534
50	0.597	0.595	0.593	1.787	1.789	1.794	-0.684	-0.681	-0.683
100	-1.549	-1.552	-1.551	-1.372	-1.367	-1.370	-2.029	-2.032	-2.032
1000	-0.163	-0.162	-0.162	-1.073	-1.073	-1.073	0.717	0.715	0.715
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-5.546	-5.557	-5.546	5.223	5.254	5.240	0.817	0.803	0.810
10	0.210	0.207	0.206	-2.257	-2.241	-2.259	0.460	0.444	0.455
30	2.797	2.792	2.788	0.054	0.06	0.058	1.307	1.304	1.309
50	1.705	1.703	1.701	0.760	0.755	0.757	-0.135	-0.133	-0.127
100	0.786	0.783	0.786	-2.181	-2.178	-2.182	0.001	-0.004	-0.002
1000	1.076	1.076	1.075	2.548	2.547	2.548	3.413	3.413	3.413

Table 4: Posterior Mean of $\rho_x = 0.7$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-1.413	-1.419	-1.452	0.042	0.052	0.030	3.771	3.754	3.801
10	0.960	0.946	0.960	-1.725	-1.697	-1.715	0.812	0.800	0.801
30	-0.044	-0.041	-0.045	-2.071	-2.076	-2.067	-1.008	-1.004	-1.010
50	0.989	0.986	0.991	-1.100	-1.097	-1.099	-1.286	-1.285	-1.289
100	-0.389	-0.386	-0.386	0.675	0.671	0.676	0.195	0.194	0.191
1000	-0.282	-0.283	-0.282	-1.661	-1.659	-1.660	-0.077	-0.078	-0.077
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.620	0.622	0.633	0.543	0.537	0.543	0.992	1.004	0.994
10	0.185	0.192	0.183	-1.449	-1.464	-1.463	0.774	0.780	0.787
30	-3.849	-3.848	-3.852	-0.777	-0.776	-0.775	-2.259	-2.267	-2.261
50	0.381	0.385	0.385	-0.070	-0.074	-0.073	0.101	0.106	0.101
100	0.675	0.669	0.673	2.509	2.508	2.507	2.323	2.329	2.324
1000	-0.233	-0.233	-0.233	0.368	0.368	0.367	0.008	0.008	0.008

ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.512	0.520	0.514	0.378	0.353	0.342	0.268	0.270	0.276
10	-1.959	-1.926	-1.944	-0.084	-0.108	-0.097	0.504	0.491	0.497
30	-0.023	-0.031	-0.025	-0.017	-0.014	-0.015	0.527	0.533	0.528
50	1.091	1.082	1.096	-0.277	-0.275	-0.280	-0.681	-0.678	-0.683
100	0.215	0.213	0.214	-3.354	-3.352	-3.357	-1.056	-1.056	-1.054
1000	0.640	0.640	0.640	-0.312	-0.313	-0.312	0.953	0.953	0.952
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-0.599	-0.582	-0.565	1.245	1.241	1.238	5.766	5.755	5.779
10	-0.887	-0.885	-0.884	-1.224	-1.227	-1.213	0.447	0.443	0.427
30	-0.438	-0.445	-0.440	-1.157	-1.158	-1.156	0.567	0.571	0.567
50	-0.099	-0.956	-0.090	-1.859	-1.856	-1.855	1.421	1.415	1.409
100	0.108	0.108	0.107	-2.014	-2.012	-2.014	-0.046	-	-0.046
1000	1.099	1.099	1.099	-0.147	-0.149	-0.149	0.547	0.547	0.549
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	1.691	1.687	1.687	3.451	3.450	3.461	3.513	3.546	3.536
10	-0.150	-0.155	-0.150	-0.551	-0.555	-0.546	0.059	0.064	0.051
30	-2.482	-2.479	-2.483	0.647	0.639	0.644	-0.211	-0.205	-0.205
50	-0.136	-0.141	-0.140	-0.315	-0.310	-0.313	1.300	1.294	1.300
100	-1.506	-1.508	-1.510	-0.849	-0.850	-0.847	-1.270	-1.270	-1.269
1000	-1.609	-1.609	-1.610	1.113	1.113	1.113	0.834	0.834	0.834
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-1.376	-1.382	-1.389	0.486	0.497	0.497	2.831	2.826	2.825
10	2.697	2.693	2.713	-1.248	-1.256	-1.262	-0.188	-0.179	-0.193
30	-1.228	-1.222	-1.226	-0.837	-0.848	-0.843	-1.634	-1.625	-1.625
50	-0.870	-0.868	-0.867	0.741	0.739	0.735	-0.882	-0.887	-0.882
100	-1.310	-1.312	-1.313	-0.581	-0.577	-0.578	0.304	0.300	0.302
1000	-1.848	-1.848	-1.848	-1.641	-1.641	-1.641	0.154	0.154	0.153

Table 5: Posterior Mean for $\rho_x = 0.9$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.921	0.910	0.906	-1.655	-1.667	-1.657	0.627	0.652	0.641
10	- 0.853	-0.855	-0.851	1.667	1.671	1.671	1.215	1.206	1.203
30	0.445	0.450	0.556	1.059	1.056	1.049	4.049	4.049	4.059
50	0.137	0.139	0.137	1.827	1.828	1.826	-1.028	-1.033	-1.027
100	0.580	0.580	0.577	-0.033	-0.033	-0.031	-0.605	-0.605	-0.606
1000	0.956	0.957	0.957	1.205	1.205	1.205	0.867	0.866	0.865

ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-0.527	-0.546	-0.551	-0.563	-0.547	-0.554	0.493	0.493	0.504
10	-3.318	-3.324	-3.324	1.569	1.588	1.587	-1.890	-1.896	-1.894
30	-5.103	-5.107	-5.102	-3.266	-3.260	-3.267	-2.632	-2.628	-2.625
50	1.652	1.646	1.640	0.582	0.588	0.590	0.386	0.385	0.387
100	1.462	1.460	1.463	-0.619	-0.619	-0.618	-1.078	-1.078	-1.083
1000	0.153	0.151	0.153	-1.676	-1.676	-1.677	-0.385	-0.385	-0.386
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-0.601	-0.610	-0.618	-1.414	-1.419	-1.416	1.665	1.661	1.684
10	-2.860	-2.864	-2.863	-1.612	-1.614	-1.612	2.147	2.145	2.155
30	-0.957	-0.950	-0.952	-1.053	-1.046	-1.060	-0.687	-0.699	0.690
50	1.473	1.468	1.466	2.078	2.075	2.072	0.385	0.393	0.396
100	1.730	1.732	1.729	2.530	2.527	2.530	2.769	2.767	2.769
1000	1.469	1.470	1.470	2.830	2.830	2.829	1.592	1.590	1.591
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	1.401	1.424	1.418	-3.031	-3.037	-3.056	3.138	3.121	3.136
10	-1.857	-1.858	-1.851	-2.740	-2.717	-2.731	2.222	2.198	2.201
30	-1.275	-1.273	-1.279	-1.114	-1.117	-1.103	-1.225	-1.229	-1.233
50	2.368	2.367	2.370	-0.656	-0.659	-0.662	0.843	0.845	0.848
100	-0.158	-0.158	-0.158	1.193	1.192	1.191	0.414	0.416	0.415
1000	0.165	0.165	0.165	0.441	0.440	0.441	-0.414	-0.412	-0.412
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-1.205	-1.214	-1.218	0.880	0.888	0.880	0.569	0.562	0.578
10	-0.856	-0.858	-0.847	-4.260	-4.258	-4.256	-0.829	-0.821	-0.841
30	-0.043	-0.041	-0.039	0.203	0.209	0.199	0.998	0.992	0.997
50	-1.140	-1.138	-1.140	1.068	1.068	1.062	-0.085	-0.085	-0.079
100	0.790	0.791	0.788	0.018	0.019	0.016	-1.343	-1.346	-1.342
1000	-1.777	-1.778	-1.776	-2.840	-2.840	-2.840	-1.117	-1.116	-1.116
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	-2.961	-2.976	-2.950	-5.343	-5.356	-5.340	0.840	0.862	0.849
10	-1.497	-1.492	-1.492	-4.246	-4.254	-4.260	0.135	0.146	0.147
30	0.982	0.978	0.975	1.358	1.354	1.364	-0.111	-0.106	-0.111
50	-0.127	-0.123	-0.129	-0.760	-0.757	-0.758	-0.393	-0.395	-0.391
100	1.620	1.621	1.618	3.208	3.209	3.209	0.003	-0.001	0.002
1000	0.894	0.893	0.893	1.898	1.899	1.899	2.044	2.044	2.043

Table 6: Posterior Standard Deviation for $\boldsymbol{\rho_x = 0.2}$ when $\boldsymbol{\rho_e}$ = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.414	0.393	0.407	0.286	0.275	0.295	0.446	0.433	0.444
10	0.189	0.185	0.187	0.296	0.289	0.292	0.292	0.283	0.285
30	0.135	0.129	0.129	0.104	0.097	0.094	0.138	0.134	0.133
50	0.090	0.091	0.089	0.109	0.112	0.109	0.098	0.101	0.096
100	0.067	0.068	0.068	0.064	0.067	0.065	0.066	0.066	0.068
1000	0.020	0.020	0.020	0.020	0.021	0.020	0.021	0.021	0.021
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.321	0.310	0.322	0.250	0.255	0.243	0.314	0.297	0.309
10	0.218	0.215	0.211	0.205	0.199	0.199	0.222	0.218	0.214
30	0.154	0.157	0.149	0.136	0.133	0.130	0.109	0.113	0.113
50	0.091	0.093	0.090	0.113	0.118	0.122	0.098	0.099	0.094
100	0.064	0.066	0.064	0.063	0.061	0.061	0.067	0.066	0.066
1000	0.020	0.020	0.022	0.021	0.020	0.020	0.021	0.021	0.021
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.489	0.499	0.506	0.608	0.608	0.621	0.684	0.650	0.644
10	0.250	0.236	0.235	0.408	0.400	0.395	0.246	0.254	0.242
30	0.122	0.119	0.124	0.143	0.144	0.139	0.130	0.131	0.128
50	0.094	0.095	0.094	0.087	0.085	0.085	0.086	0.083	0.086
100	0.071	0.070	0.069	0.066	0.065	0.062	0.061	0.057	0.057
1000	0.020	0.020	0.020	0.021	0.020	0.020	0.021	0.020	0.020
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.357	0.372	0.378	0.213	0.207	0.210	0.291	0.289	0.298
10	0.217	0.232	0.224	0.214	0.221	0.210	0.194	0.200	0.193
30	0.128	0.126	0.124	0.125	0.125	0.123	0.148	0.148	0.143
50	0.093	0.092	0.097	0.118	0.118	0.117	0.094	0.090	0.094
100	0.070	0.071	0.074	0.068	0.067	0.066	0.069	0.068	0.067
1000	0.020	0.020	0.021	0.020	0.020	0.020	0.020	0.020	0.020
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.302	0.282	0.284	0.398	0.403	0.404	0.456	0.449	0.443
10	0.325	0.340	0.323	0.288	0.283	0.287	0.230	0.232	0.225
30	0.128	0.139	0.133	0.123	0.127	0.126	0.108	0.109	0.110
50	0.102	0.106	0.105	0.096	0.099	0.098	0.094	0.096	0.092
100	0.064	0.063	0.063	0.062	0.064	0.062	0.073	0.074	0.073
1000	0.021	0.021	0.021	0.020	0.020	0.020	0.021	0.020	0.020

ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.891	0.868	0.837	1.257	1.213	1.172	0.803	0.756	0.759
10	0.200	0.206	0.204	0.170	0.171	0.169	0.326	0.328	0.313
30	0.119	0.117	0.118	0.113	0.112	0.108	0.121	0.122	0.123
50	0.099	0.099	0.099	0.095	0.095	0.093	0.104	0.108	0.107
100	0.059	0.058	0.058	0.061	0.061	0.064	0.065	0.064	0.068
1000	0.020	0.020	0.021	0.021	0.021	0.021	0.021	0.020	0.020

Table 7: Posterior Standard Deviation for $\rho_x = 0.4$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.197	0.202	0.205	0.387	0.370	0.394	0.298	0.298	0.313
10	0.235	0.228	0.227	0.178	0.178	0.181	0.156	0.154	0.153
30	0.142	0.147	0.141	0.140	0.144	0.139	0.111	0.112	0.109
50	0.103	0.100	0.101	0.093	0.098	0.097	0.098	0.099	0.094
100	0.071	0.071	0.073	0.073	0.076	0.073	0.060	0.063	0.059
1000	0.019	0.021	0.020	0.021	0.021	0.020	0.020	0.021	0.020
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.584	0.585	0.586	0.708	0.703	0.715	0.642	0.630	0.652
10	0.186	0.183	0.185	0.206	0.201	0.209	0.234	0.235	0.247
30	0.113	0.111	0.112	0.114	0.113	0.118	0.119	0.114	0.121
50	0.090	0.089	0.089	0.105	0.098	0.101	0.092	0.090	0.091
100	0.068	0.064	0.067	0.064	0.064	0.066	0.066	0.062	0.063
1000	0.021	0.021	0.021	0.020	0.020	0.021	0.021	0.021	0.0209
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.556	0.564	0.520	0.257	0.258	0.242	0.403	0.416	0.423
10	0.277	0.291	0.289	0.185	0.188	0.183	0.238	0.226	0.231
30	0.125	0.125	0.121	8.121	0.121	0.119	0.146	0.145	0.141
50	0.090	0.094	0.091	0.086	0.085	0.083	0.094	0.092	0.091
100	0.068	0.068	0.068	0.068	0.068	0.066	0.069	0.070	0.069
1000	0.019	0.020	0.021	0.020	0.021	0.020	0.020	0.020	0.022
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.327	0.337	0.334	0.350	0.342	0.347	0.433	0.451	0.434
10	0.251	0.257	0.252	0.307	0.327	0.316	0.245	0.270	0.257
30	0.109	0.115	0.108	0.156	0.153	0.152	0.130	0.129	0.135
50	0.095	0.091	0.094	0.107	0.105	0.106	0.094	0.091	0.087
100	0.066	0.062	0.064	0.067	0.063	0.064	0.068	0.066	0.068
1000	0.019	0.019	0.019	0.020	0.020	0.020	0.019	0.019	0.019

ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.540	0.541	0.534	0.675	0.670	0.664	0.241	0.239	0.249
10	0.211	0.205	0.216	0.189	0.203	0.197	0.241	0.225	0.219
30	0.131	0.132	0.136	0.134	0.131	0.125	0.119	0.123	0.119
50	0.091	0.095	0.094	0.092	0.096	0.097	0.091	0.092	0.095
100	0.066	0.067	0.067	0.069	0.071	0.072	0.069	0.069	0.072
1000	0.020	0.020	0.021	0.021	0.019	0.020	0.021	0.021	0.021
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.367	0.354	0.359	0.464	0.442	0.471	0.238	0.231	0.229
10	0.300	0.291	0.289	0.264	0.251	0.252	0.275	0.271	0.262
30	0.126	0.121	0.112	0.121	0.125	0.123	0.114	0.117	0.112
50	0.094	0.095	0.096	0.102	0.099	0.104	0.105	0.105	0.103
100	0.065	0.063	0.062	0.061	0.065	0.067	0.067	0.069	0.068
1000	0.021	0.021	0.021	0.020	0.020	0.020	0.020	0.021	0.021

Table 8: Posterior Standard Deviation for $\boldsymbol{\rho_x = 0.5}$ when $\boldsymbol{\rho_e}$ = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.468	0.431	0.452	0.284	0.271	0.278	0.280	0.268	0.275
10	0.320	0.322	0.328	0.271	0.277	0.278	0.278	0.285	0.286
30	0.103	0.099	0.101	0.128	0.126	0.125	0.123	0.121	0.124
50	0.091	0.089	0.087	0.116	0.114	0.114	0.090	0.093	0.093
100	0.059	0.064	0.063	0.061	0.061	0.063	0.066	0.066	0.065
1000	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.022	0.021
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.269	0.272	0.265	0.341	0.354	0.343	0.271	0.261	0.260
10	0.162	0.162	0.152	0.233	0.225	0.230	0.167	0.162	0.160
30	0.109	0.112	0.111	0.112	0.110	0.116	0.127	0.121	0.125
50	0.085	0.083	0.086	0.109	0.115	0.111	0.100	0.098	0.097
100	0.058	0.060	0.063	0.064	0.065	0.064	0.066	0.066	0.069
1000	0.020	0.020	0.020	0.020	0.020	0.020	0.019	0.020	0.020
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.615	0.600	0.611	0.241	0.239	0.231	0.875	0.865	0.883
10	0.284	0.290	0.305	0.262	0.261	0.261	0.190	0.192	0.194
30	0.137	0.129	0.126	0.140	0.143	0.143	0.125	0.121	0.125
50	0.085	0.087	0.085	0.094	0.092	0.095	0.091	0.090	0.096
100	0.066	0.065	0.063	0.062	0.065	0.061	0.069	0.071	0.072
1000	0.021	0.021	0.021	0.020	0.020	0.020	0.022	0.021	0.022

ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.270	0.257	0.263	0.385	0.410	0.401	0.561	0.562	0.580
10	0.351	0.347	0.354	0.317	0.312	0.322	0.193	0.191	0.194
30	0.133	0.137	0.136	0.124	0.129	0.132	0.116	0.122	0.123
50	0.108	0.109	0.109	0.099	0.099	0.098	0.093	0.096	0.098
100	0.069	0.071	0.069	0.065	0.062	0.065	0.071	0.074	0.073
1000	0.022	0.021	0.020	0.021	0.021	0.021	0.019	0.020	0.021
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.248	0.260	0.262	0.316	0.316	0.310	0.349	0.375	0.368
10	0.192	0.191	0.197	0.226	0.235	0.243	0.234	0.234	0.239
30	0.114	0.114	0.110	0.120	0.120	0.125	0.122	0.121	0.123
50	0.084	0.081	0.081	0.090	0.088	0.093	0.090	0.090	0.095
100	0.061	0.063	0.062	0.080	0.076	0.079	0.068	0.070	0.073
1000	0.019	0.019	0.019	0.021	0.020	0.020	0.020	0.021	0.021
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.491	0.494	0.492	0.415	0.400	0.412	0.289	0.286	0.281
10	0.163	0.151	0.153	0.263	0.268	0.276	0.228	0.235	0.237
30	0.127	0.129	0.126	0.120	0.120	0.120	0.115	0.113	0.109
50	0.099	0.100	0.102	0.100	0.100	0.099	0.113	0.111	0.112
100	0.071	0.070	0.067	0.068	0.068	0.069	0.065	0.065	0.063
1000	0.021	0.019	0.022	0.020	0.020	0.020	0.021	0.020	0.020

Table 9: Posterior Standard Deviation for $\rho_x = 0.7$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.554	0.567	0.554	0.349	0.351	0.365	0.562	0.586	0.559
10	0.205	0.208	0.205	0.252	0.252	0.252	0.218	0.215	0.218
30	0.112	0.110	0.116	0.126	0.123	0.125	0.143	0.132	0.139
50	0.092	0.096	0.099	0.106	0.107	0.105	0.092	0.092	0.092
100	0.063	0.064	0.064	0.062	0.064	0.060	0.067	0.067	0.065
1000	0.021	0.020	0.020	0.020	0.020	0.019	0.020	0.021	0.019
ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.552	0.544	0.516	0.238	0.238	0.225	0.397	0.401	0.383
10	0.323	0.325	0.308	0.347	0.343	0.327	0.192	0.188	0.187
30	0.131	0.126	0.131	0.116	0.111	0.114	0.135	0.127	0.136
50	0.096	0.102	0.096	0.091	0.092	0.089	0.083	0.079	0.079
100	0.066	0.068	0.068	0.059	0.058	0.058	0.063	0.064	0.065
1000	0.020	0.020	0.020	0.020	0.021	0.021	0.020	0.020	0.020

Advances In Multidisciplinary & Scientific Research
AIMS Research Journal
A Multidisciplinary & Interdisciplinary Journal
Volume 6, 2020 Edition

ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.202	0.198	0.206	0.647	0.618	0.620	0.326	0.317	0.317
10	0.250	0.254	0.264	0.162	0.158	0.162	0.227	0.235	0.235
30	0.151	0.143	0.145	0.117	0.116	0.114	0.117	0.117	0.115
50	0.105	0.105	0.105	0.093	0.093	0.088	0.097	0.095	0.094
100	0.065	0.066	0.066	0.066	0.071	0.070	0.060	0.062	0.062
1000	0.021	0.020	0.020	0.020	0.020	0.020	0.021	0.022	0.021
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.635	0.619	0.657	0.224	0.233	0.217	0.393	0.409	0.399
10	0.267	0.238	0.238	0.313	0.330	0.334	0.202	0.207	0.209
30	0.104	0.102	0.102	0.133	0.140	0.137	0.109	0.108	0.103
50	0.112	0.109	0.115	0.083	0.081	0.085	0.107	0.107	0.106
100	0.062	0.067	0.066	0.072	0.069	0.067	0.068	0.068	0.068
1000	0.021	0.021	0.021	0.021	0.021	0.020	0.022	0.021	0.021
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.212	0.221	0.220	0.360	0.359	0.365	953	0.960	0.973
10	0.170	0.177	0.167	0.219	0.220	0.214	0.318	0.317	0.320
30	0.121	0.118	0.125	0.107	0.104	0.106	0.103	0.102	0.102
50	0.092	0.089	0.092	0.092	0.089	0.092	0.083	0.082	0.087
100	0.073	0.075	0.075	0.069	0.070	0.067	0.064	0.066	0.063
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.291	0.306	0.308	0.370	0.363	0.384	0.390	0.374	0.389
10	0.353	0.349	0.370	0.280	0.285	0.290	0.224	0.226	0.231
30	0.108	0.115	0.105	0.116	0.124	0.120	0.093	0.102	0.0960
50	0.093	0.095	0.097	0.102	0.106	0.105	0.099	0.097	0.096
100	0.065	0.063	0.065	0.068	0.065	0.067	0.062	0.063	0.065
1000	0.020	0.020	0.020	0.021	0.020	0.021	0.020	0.020	0.020

Table 10: Posterior Standard Deviation for $\rho_x = 0.9$ when ρ_e = 0.0, 0.2, 0.4, 0.6, 0.8 and 0.9 at Different Sample Sizes

ρ = 0.0	Equation 1			Equation 2			Equation 3		
	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.2483	0.2578	0.2631	0.3045	0.3042	0.3188	0.2857	0.2834	0.2814
10	0.2278	0.2341	0.2324	0.1806	0.1837	0.1867	0.2898	0.2865	0.2914
30	0.1142	0.1148	0.1150	0.1456	0.1527	0.1443	0.1419	0.1475	0.1399
50	0.0842	0.0856	0.0855	0.0889	0.0864	0.0934	0.1304	0.1278	0.1321
100	0.0599	0.0625	0.0618	0.0605	0.0612	0.0583	0.0646	0.0623	0.0625
1000	0.0209	0.0211	0.02014	0.02011	0.0192	0.0197	0.0214	0.0210	0.0209

ρ = 0.2	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.4314	0.4219	0.4235	0.4236	0.4015	0.4200	0.2230	0.2296	0.2318
10	0.2102	0.2130	0.2107	0.2084	0.2057	0.2015	0.2988	0.3113	0.2990
30	0.1104	0.1111	0.1049	0.1202	0.1232	0.1138	0.1100	0.1117	0.1065
50	0.1014	0.0996	0.0992	0.0955	0.0980	0.0965	0.0962	0.0962	0.0916
100	0.0631	0.0650	0.0666	0.0661	0.0671	0.0661	0.0593	0.0621	0.0610
1000	0.0202	0.0208	0.0209	0.0211	0.0212	0.0218	0.0198	0.0188	0.0202
ρ = 0.4	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.4279	0.4092	0.4305	0.7211	0.7288	0.6989	0.4135	0.4297	0.4042
10	0.2068	0.2059	0.2057	0.1715	0.1739	0.1783	0.3132	0.3171	0.3291
30	0.1107	0.1119	0.1102	0.1298	0.1327	0.1352	0.1207	0.1235	0.1188
50	0.1001	0.0997	0.0996	0.0886	0.0887	0.0873	0.0982	0.0981	0.1024
100	0.0630	0.0633	0.0635	0.0673	0.0703	0.0655	0.0680	0.0715	0.0664
1000	0.0197	0.0205	0.0191	0.0209	0.0207	0.0205	0.0207	0.0206	0.0200
ρ = 0.6	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.2693	0.2682	0.2663	0.3986	0.4167	0.4055	0.3726	0.3751	0.3851
10	0.1602	0.1611	0.1664	0.3188	0.3286	0.3329	0.2760	0.2747	0.2667
30	0.1257	0.1235	0.1267	0.1315	0.1304	0.1323	0.1300	0.1281	0.1224
50	0.0957	0.0951	0.0931	0.0956	0.0989	0.0986	0.0823	0.0840	0.0834
100	0.0650	0.0674	0.0657	0.0653	0.037	0.0615	0.0657	0.0688	0.0672
1000	0.0213	0.0213	0.0212	0.0204	0.0207	0.0199	0.0207	0.0205	0.0212
ρ = 0.8	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.2483	0.2429	0.2468	0.2915	0.2969	0.3087	0.3467	0.3378	0.3526
10	0.2312	0.2400	0.2390	0.2291	0.2330	0.2324	0.1824	0.1862	0.1748
30	0.1187	0.1177	0.1227	0.1287	0.1282	0.1276	0.1167	0.1170	0.1227
50	0.0936	0.0957	0.0970	0.1003	0.1003	0.1024	0.0921	0.0942	0.0940
100	0.0686	0.0688	0.0688	0.0644	0.0603	0.0629	0.0654	0.0694	0.0663
1000	0.0204	0.0210	0.0206	0.0202	0.0215	0.0212	0.0205	0.0206	0.0197
ρ = 0.9	β0	β1	β2	β0	β1	β2	β0	β1	β2
5	0.7676	0.7858	0.8159	0.5900	0.5846	0.6183	0.5016	0.4891	0.5310
10	0.2154	0.2160	0.2320	0.3686	0.3506	0.3546	0.2792	0.2646	0.2580
30	0.1176	0.1154	0.1174	0.1098	0.1140	0.1142	0.1001	0.1038	0.1052
50	0.0924	0.0900	0.0917	0.0821	0.0815	0.0848	0.0955	0.0949	0.0996
100	0.0643	0.0618	0.0622	0.0639	0.0643	0.0660	0.0660	0.0669	0.0646
1000	0.0208	0.0202	0.0212	0.0211	0.0209	0.0193	0.0214	0.0208	0.0208

3.2 Discussion of Results

In Table 1, the posterior means were generally irregular across the different values of ρ and increasing sample sizes. However, for equation 1, when ρ = 0.0, the posterior mean of the β increases with increasing sample sizes. Also in Table 2, the posterior means are generally inconsistent across the different values of ρ and increasing sample sizes. However, there seems to be an alternating pattern, of increase and decrease in the posterior means, with increasing sample size. A similar pattern was observed in Table 3, the posterior means are generally inconsistent across the different values of ρ and increasing sample size.

There seems to be an alternating pattern, of decrease and increase in posterior means, with increasing sample sizes, especially for Equation 1. In Tables 4 and 5, the posterior means are also generally inconsistent across the different values of ρ and increasing sample sizes. However, there seems to be an alternating pattern, of increase and decrease in posterior means, with increasing sample sizes for Equation 1 when ρ_e = 0.0. However, in Table 6, the posterior standard deviations generally decrease as sample size increases across the different values of ρ. Also in Tables 7, 8, 9 and 10, the posterior standard deviations generally decrease as sample size increases across the different values of ρ.

4. CONCLUSION

This study was used to explore the effect of the misspecification of model on seemingly unrelated regression model within the Bayesian context. In econometrics, it is common to exclude important variables, which frequently leads to incorrect model design. This study used the seemingly unrelated regression (SUR) model that was incorrectly described. The explanatory variables were produced from a uniform distribution for sample sizes of 5, 10, 30, 50, 100, and 1000. The independent variables were correlated at different values (0.2, 0.4, 0.5, 0.7, and 0.9) and were taken into consideration. Hyperprior distribution options were investigated. The targeted feature of unbiasedness in the estimated coefficients, which rely on the fitted model's correspondence with the real underpinning data-generating process, was not obtained because the model was wrongly defined. The posterior means and standard deviations showed irregular patterns, except a few high sample sizes.

REFERENCES

1. Alaba, O. O. and Akinrelere, O. (2019): Tests for Aggregation Bias in Seemingly Unrelated Regression with Unequal Observations. *Proceedings of the 18th ISTEAMS Multidisciplinary Cross-Border Conference.* 239- 248pp.
2. Alaba, O. O., Olubusoye, O. E. and Ojo, S.O. (2010). Efficiency of Seemingly Unrelated Regression Estimator over the Ordinary Least Squares. *European Journal of Scientific Research.* Vol.39 No.1: 153-160.
3. Alaba, O. O. and Lawal, A. A. (2019). Bootstrap Bartlett Adjustment on Decomposed Variance-Covariance Matrix of Seemingly Unrelated Regression Model. *Afrika Statiska* Vol 14. No. 1: 1891 – 1902.
4. Alaba, O. O. Adepoju, A. A. and Olaomi, O. (2019). Seemingly Unrelated Regression with Decomposed Variance-Covariance Matrix: A Bayesian Approach. *Journal of the Nigerian Association of Mathematical Physics* Vol. 51: 137 - 144.
5. Aigner, D. (1974). An Appropriate Econometric Framework for Estimating a Labor Supply Function from the Seo File. *International Economic Review*, *15*(1), 59–68. https://doi.org/10.2307/2526088
6. Branson, J., and Lovell, C. K. (2000). Taxation and economic growth in New Zealand. *Taxation and the Limits of Government*, 37-88.
7. Bozdogan, H (1987). Model selection and Akaike's Information Criterion (AIC): The general theory and its analytical extensions. *Psychometrika* 52, 345–370. https://doi.org/10.1007/BF02294361
8. Clarke, K. A. (2005). The Phantom Menace: Omitted Variable Bias in Econometric Research. *Conflict Management and Peace Science*, *22*(4), 341–352.
9. https://doi.org/10.1080/07388940500339183
10. Das, S. and Chatterjee, S. (2011). Multicollinearity Problem – Root Cause, Diagnostics and Way Outs. http://dx.doi.org/10.2139/ssrn.1830043
11. Graham, M. H. (2003). Confronting Multicollinearity in Ecological Multiple Regression, Statistical Report https://doi.org/10.1890/02-3114
12. Hausman, J. A. (1975). An Instrumental Variable Approach to Full Information Estimators for Linear and Certain Nonlinear Econometric Models. *Econometrica*, *43*(4), 727–738. https://doi.org/10.2307/1913081
13. Kim, J. H. (2019). Multicollinearity and misleading statistical results. *Korean J Anesthesiol* 72(6):558-569. https://doi.org/10.4097/kja.19087
14. Lawal, A. and Alaba, O. O. (2019). Exploratory Analysis of Some Sectors of the Economy: A Seemingly Unrelated Regression Approach. *African Journal of Applied Statistics*, 6(1): 649 – 661.
15. Midi, H., Sarkar, S. K. and Rana, S. (2010): Collinearity diagnostics of binary logistic regression model, *Journal of Interdisciplinary Mathematics*, 13(3): 253267.
16. McGuirk, A. M., Driscoll, P., & Alwang, J. (1993). Misspecification Testing: A Comprehensive Approach. *American Journal of Agricultural Economics*, *75*(4), 1044–1055. https://doi.org/10.2307/1243992
17. Rohlfing, I. (2008). What You See and What You Get: Pitfalls and Principles of Nested Analysis in Comparative Research. *Comparative Political Studies*, *41*(11), 1492–1514. https://doi.org/10.1177/0010414007308019

18. Winship, Christopher, and Bruce Western. (2016). "Multicollinearity and Model Misspecification." Sociological Science 3: 627-649
19. Wooldridge, J. M. (2016). Should instrumental variables be used as matching variables? Research in Economics, 70(2): 232-237,
20. https://doi.org/10.1016/j.rie.2016.01.001.
21. Tiao, G. C., & Zellner, A. (1964). On the Bayesian Estimation of Multivariate Regression. *Journal of the Royal Statistical Society. Series B (Methodological)*, *26*(2), 277–285. http://www.jstor.org/stable/2984424
22. Zainodin, H. J. and Yap, S. J. (2013): Overcoming multicollinearity in multiple regression using correlation coefficient *AIP Conference Proceedings* 1557, 416–419