





### 1.5. Thermal Conduction In Spherical Coordinate

The general heat conduction equation in spherical coordinate can be obtained from an energy balance on a volume element in a spherical coordinates (shown in Figure 1.1), by the following steps below. It can also be obtained directly by coordinate transformation using the following relations between the coordinates of a point in rectangular and spherical coordinate systems;

$$x = r \cos \phi \sin \theta, y = r \sin \phi \cos \theta \text{ and } z = r \cos \theta$$

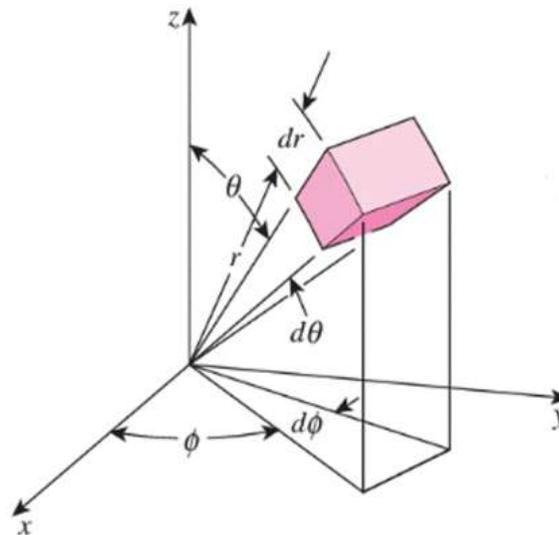


Fig. 1: Thermal Conduction In Spherical Coordinate

### 1.6 Steady State Conduction

Steady state conduction is the form of conduction that happens when the temperature difference during the conduction is constant so that the spatial distribution of temperature in the conducting object does not change any further, thus all partial derivatives of temperature with respect to space may either be zero or having non-zero values, but all derivatives of temperature at any point with respect to time are uniformly zero.

### 1.7 One Dimensional Heat Flow

The term "one dimensional" is applied to heat conduction problems when

- Only one space coordinate is required to describe the temperature distribution within a heat conducting body.
- Edge effects are neglected
- The flow of heat energy takes place along the coordinate measured normal to the surface

### 1.8 Differential Equation (DE)

In mathematics, we call the changing entities as variables and the speed of change of one variable with respect to another as a derivative. Equations expressing a relationship among these variables and their derivatives are known as differentials. In other words, a differential equation originates whenever a universal law is expressed by means of variables and their derivatives.



Any equation containing differential coefficients is called a differential equation. (H.Lee., 2002).

- **Order:** This means the order of the highest derivative appearing in the equation.
- **Degree:** This means that exponent where the maximum derivative is being raised either ODE or PDE.

### 1.9 Ordinary Differential Equation (ODE)

This is a situation where the unknown function depends on one independent variable only.

### 1.10 Partial Differential Equation (PDE)

This is an equation that has more than two independent variables in an unknown function. The power of PDE is the power of the maximum derivative involved. Partial differential equations are used mathematically to formulate problems in engineering, physical and life sciences. For instance, fluid flow, elasticity, transmission of heat, electrostatics, electrostatics,etc. (Unsworth, 1979).

## 2. LITERATURE REVIEW

Makinde (2007) studied the oscillatory flow and heat transfer in a rigid tube of varying cross-section with permeable wall. The flux across any representative section of the tube is viewed as composed of a pulsate part superimposed on a steady part and the effect of fluid absorption through the permeable wall is accounted by prescribing flux as arbitrary function of axial distance and time. The flow downstream of this section is investigated by expanding stream-function, vortices and fluid temperature in asymptotic series about a small parameter,  $\epsilon$ , characterizing the tube aspect ratio. Numerical results computed for wavy wall tube and a tapering tube as presented graphically and discussed quantitatively.

Jha and Ajibade (2012), investigated the natural convection flow of viscous incompressible fluid in a channel formed by two infinite vertical parallel plates. Fully developed laminar flow is considered in a vertical channel with steady-periodic temperature regime on the boundaries. The effect of internal heating by viscous dissipation is taken into consideration. Separating the velocity and temperature fields into steady and periodic parts, the resulting second order ordinary differential equations are solved to obtain the expressions for velocity and temperature. The amplitudes and phases of temperature and velocity are also obtained as well as the rate of heat transfer and the skin-friction on the plates. In presence of viscous dissipation, fluids of relatively small Prandtl number have higher temperature than the channel plates and as such, heat is being transferred from the fluid to the plate.

Jha and Ajibade (2010), presented suction/injection control of free Convective motion of a viscous incompressible fluid between two periodically heated infinite vertical parallel plates. The temperature and velocity fields are separated into steady and periodic parts and the resulting second order differential equations solved to obtain the solution to the problem. The influence of each governing parameter is discussed with maps. The significant result from this study is that temperature is higher near the plate with injection while velocity is more enhanced near the plate with suction. It is also interesting to note that for large values of suction, the nature of the flow is that of constant heating of the plates while for large values of Strouhal number, the flow behaves as if there was no suction/injection at the place (Wang). Adesanya (2012), This paper investigates the combined effect of heat source/sink and radiation of heat transfer to steady flow of a conducting optically thick visco-elastic fluid through a channel filled with saturated porous medium induced by non-uniform wall temperature. Analytical solutions of nonlinear ordinary differential equations governing the flow are obtained using Adomian decomposition method. The effects of different parameters in the model on the temperature and velocity profiles are presented and discussed.



### 3. MATERIAL AND METHODS

Consider a hollow sphere made of spherical material having a constant thermal conductivity. According to the steady one-dimensional heat conduction problem, this is used by obtaining the applicable differential equation and specifying the boundary condition

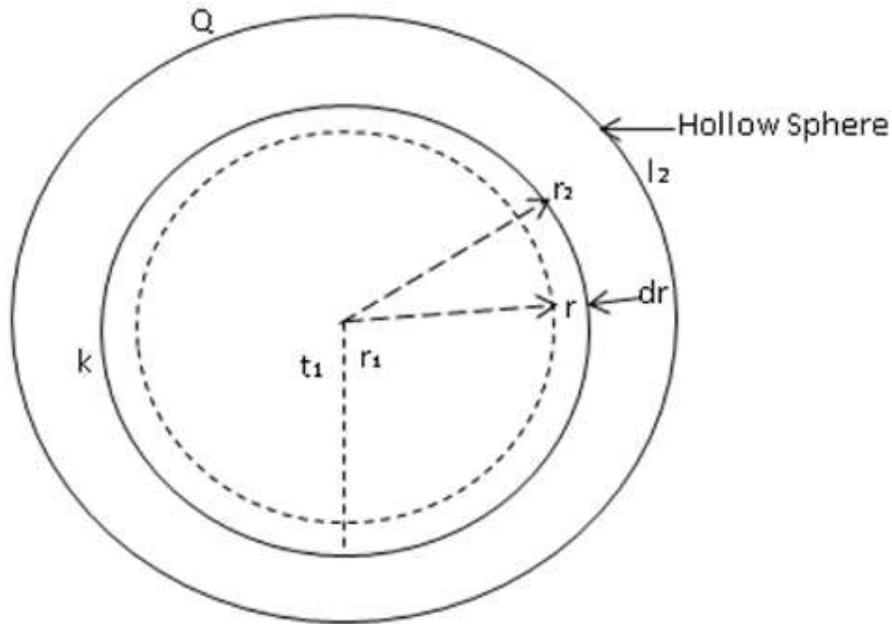


Fig 2: Hollow Sphere Made Of Spherical Material Having A Constant Thermal Conductivity

The governing equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \cdot \frac{\partial T}{\partial \theta} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + q_v = PC_p \frac{\partial T}{\partial t} \quad (3.1)$$

Where parameter description

- K is the material conductivity ( $W m^{-1} k^{-1}$ )
- $q_v$  Is the rate at which energy is generated per unit volume of the medium ( $W m^{-3}$ )
- P is density ( $kg m^{-3}$ )
- $C_p$  Is the specific heat capacity ( $J K^{-1} kg^{-1}$ )
- $r_1$  Inner radii of the Sphere
- $r_2$  Outer radii of the Sphere
- $T_1$  Temperature of the inner surface
- $T_2$  Temperature of the Outer surface
- Q Heat flow radially outward

The equation reduces to

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \cdot \frac{\partial^2 T}{\partial \theta^2} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{q_v}{k} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

Where the property  $\alpha = \frac{k}{\rho C}$  is again the thermal diffusivity of the material. For steady state  $\frac{\partial T}{\partial t} = 0$ , unidirectional heat flow in the radial direction  $\{T = \mathcal{F}(\theta, \phi)\}$  and with no heat generator  $q_g = 0$ .

Equation (3.1) becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \cdot \frac{\partial^2 T}{\partial \theta^2} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) = 0 \quad (3.2)$$

We replaced the partial derivatives by ordinary, the one-dimensional steady heat conduction case since partial and ordinary derivatives of a function are identical with the function depend on a single variable only.

The above equation reduces to

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0 \quad (3.3)$$

$$\frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0 \text{ or } \frac{1}{r^2} \neq 0 \quad (3.4)$$

$$r^2 \frac{d^2 T}{dr^2} + 2r \frac{dT}{dr} = 0 \quad (3.5)$$

Divide through by r

$$r \frac{d^2 T}{dr^2} + 2 \frac{dT}{dr} = 0 \quad (3.6)$$

### 3.1 Methods of solution

#### First Method

Using second order homogeneous Equation

Auxiliary equation,

$$rt^2 + 2t = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$







#### 4. RESULT AND DISCUSSION

To show the approximate numerical values for a spherical shaped vessel under different scenarios, we consider the following case studies:

##### Case I:

Consider a spherical shaped vessel of 1.4m diameter which is 90mm thick. If the temperature difference between the inner and outer surfaces is 220°C. The graph below displays the rate of heat transfer/leakage considering the thermal conductivity of the material of the sphere.

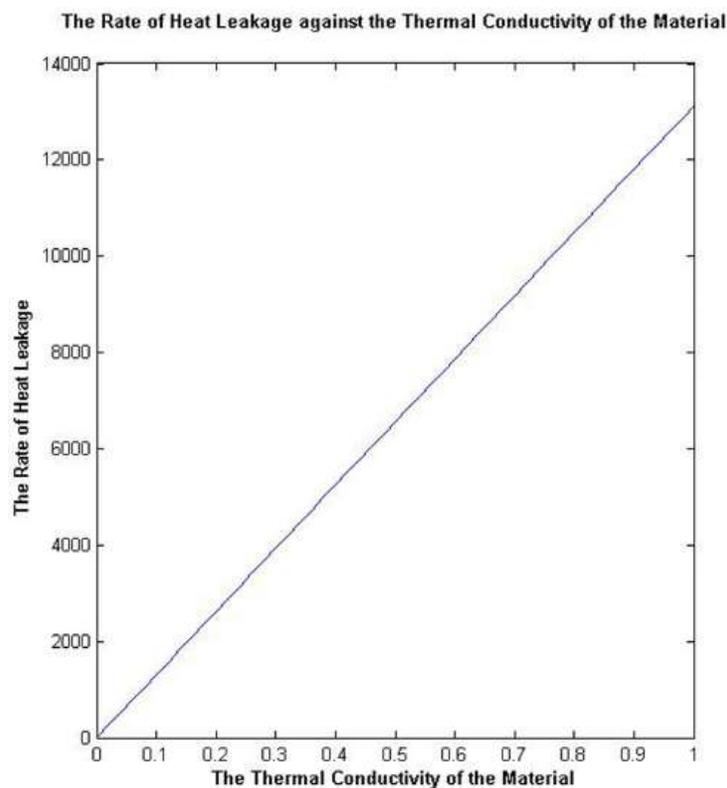
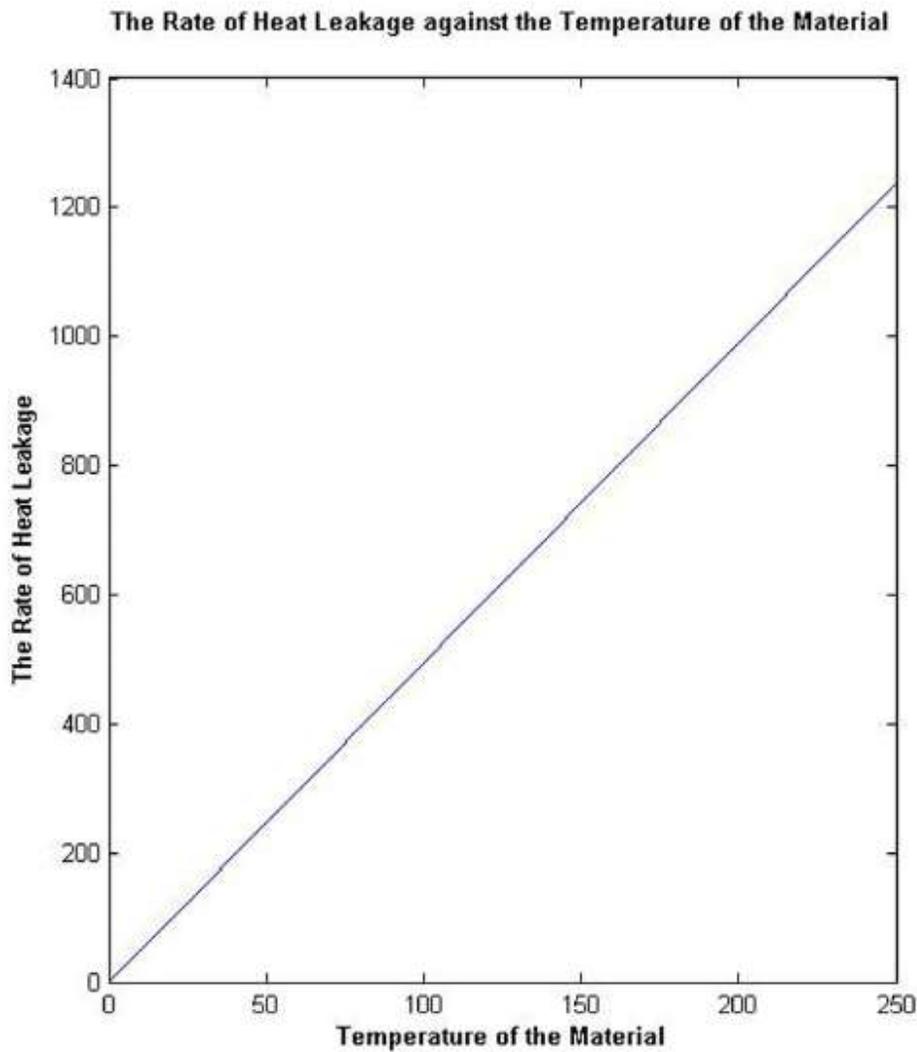


Figure 4.1: The rate of heat transfer/leakage against the thermal conductivity of the material

From Figure 4.1, the rate of heat transfer/leakage increases as the thermal conductivity of the material increases.

**Case II:**

Suppose the thermal conductivity of the material is  $0.083W/m^2c$ . The graph below shows the rate of heat transfer/leakage when the temperature of the materials is being examined.

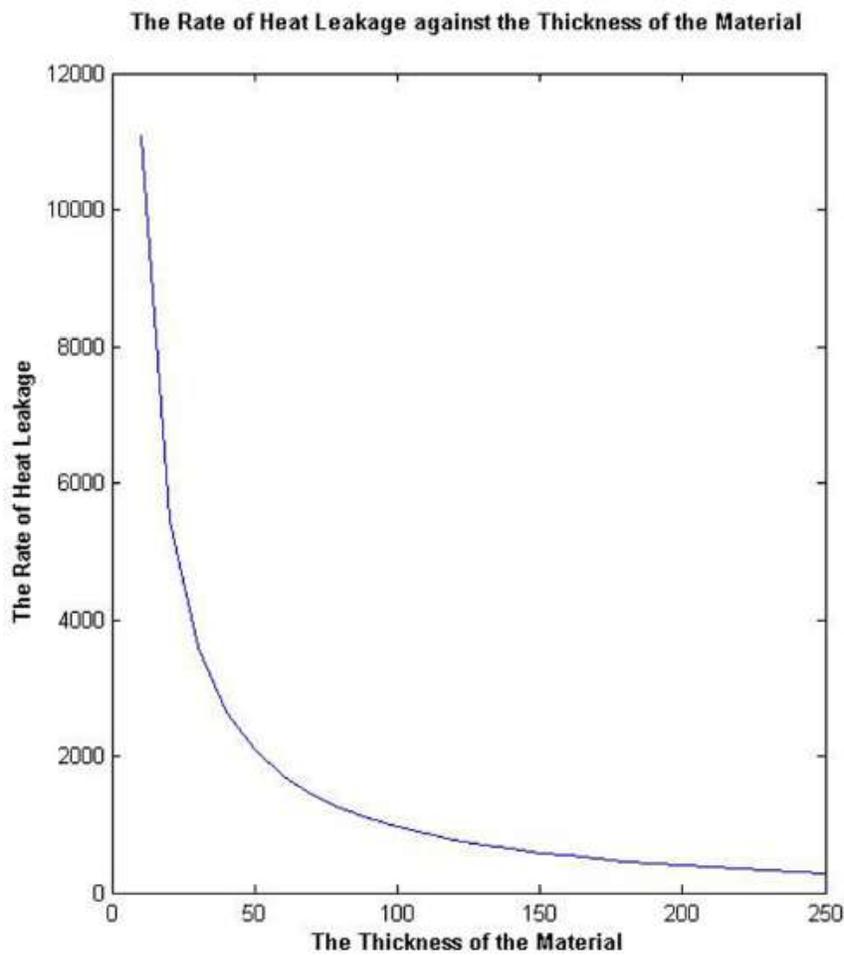


**Figure 4.2: The rate of heat transfer/leakage against the thermal conductivity of the material**

From Figure 4.2, the rate of heat transfer/leakage increases as the temperature of the material increases.

**Case III:**

Suppose the thermal conductivity of the material is  $0.083W/m^2c$  and the temperature difference between the inner and outer surfaces is  $220^{\circ}C$ . The graph below displays the rate of heat transfer/leakage considering the thickness of the spherical shaped vessel which is 1.4mdiameter.

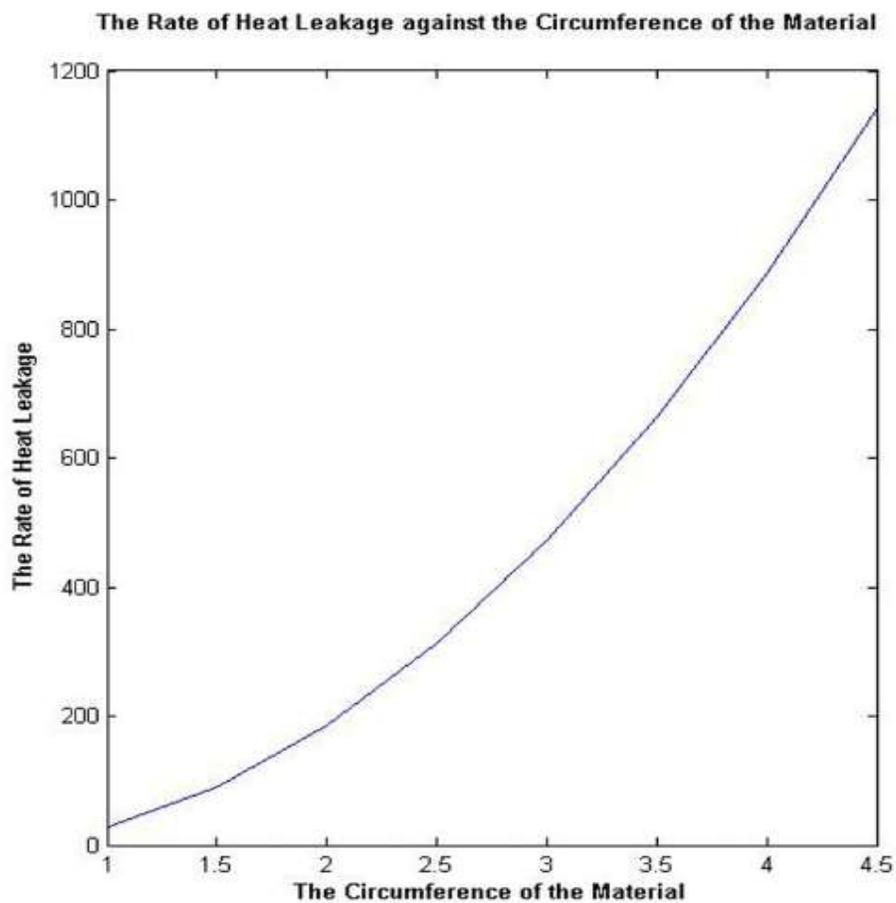


**Figure 4.3: The rate of heat transfer/leakage against the thermal conductivity of the material**

From Figure 4.3, the rate of heat transfer/leakage decreases with increase in the thickness of the material.

**Case IV:**

Suppose the thermal conductivity of the material is  $0.083W/m^2c$  and the temperature difference between the inner and outer surfaces is  $220^{\circ}C$ . The graph below shows the rate of heat transfer/leakage when we examine the circumference of the spherical shaped vessel of thickness  $90mm$ .



**Figure 4.4:** The rate of heat transfer/leakage against the thermal conductivity of the material

From Figure 4.4, the rate of heat transfer/leakage increases with increase in the circumference



## 5. SUMMARY, CONCLUSION AND RECOMMENDATION

### 5.1 Summary

Thermal conductivity through hollow sphere was considered in this project work. The conduction of heat transfer rate was determined by replacing the partial derivatives of the spherical coordinate in one-dimensional steady case (as regarded as our governing equation i.e. (3.1) in this project) by ordinary derivatives since the partial and ordinary derivatives of a function are identical with the function on a single variable only. Numerical values for the conduction of heat transfer rate for a spherical shaped vessel under different scenarios were considered.

### 5.2 Conclusion

It can be concluded from Figure 4.1-4.4 that the rate of heat leakage increases linearly as the thermal conductivity of the material increases, likewise for the temperature of the material but decreases drastically to zero as the thickness of the material increases. Considering the circumference of the material as it increases, the rate of heat leakage also increases.

### 5.3 Recommendation for Further Study

It was recommended further that research can be carried out using two-dimensional steady state and unsteady state technique to solve the problem under consideration.

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